MAGNETIC DIPOLES IN EQUILIBRIUM

Consider two identical magnetic dipoles. What is the equilibrium configuration?

The figure shows the two dipole moments pointing in the same direction, and aligned with the line from one dipole to the other. The torque on either dipole, due to the field of the other dipole, is 0, so this is an equilibrium.

If the angles of the dipole moments with respect to the line are $\theta_1$ and $\theta_2$, then the potential energy is

$$U = -m_2 \cdot B_1 = \frac{\mu_0}{4\pi r^3} [m_1 \cdot m_2 - m_1 \cdot \hat{r} m_2 \cdot \hat{r}]$$

$$= \frac{\mu_0 m_0^2}{4\pi r^3} [\cos(\theta_1 - \theta_2) - \cos \theta_1 \cos \theta_2] = \frac{\mu_0 m_0^2}{4\pi r^3} \sin \theta_1 \sin \theta_2.$$

The equilibria are states with

$$\frac{\partial U}{\partial \theta_1} = \frac{\partial U}{\partial \theta_2} = 0.$$

The stable equilibrium states are $\theta_1 = \theta_2 = 0$ (shown in the figure) and $\theta_1 = \theta_2 = \pi$ (both moments pointing to the left).

Now add a third identical dipole, such that the three dipoles are located at the vertices of an equilateral triangle. What are the equilibria? The dipoles are free to rotate but pinned at the corners of the triangle.

Or, consider $n$ identical dipoles located at the corners of a regular polygon with $n$ vertices. What are the equilibria?
Three dipoles at the corners of an equilateral triangle

By symmetry it is obvious that there are four special orientations of the three moments:

1. All radially outward
2. All radially inward
3. All tangent to the circle clockwise
4. All tangent to the circle counterclockwise

(The circle means the smallest circle that encloses the triangle; the triangle inscribes the circle.) States 1 and 2 are maxima of potential energy, i.e., unstable equilibria. States 3 and 4 are minima, i.e., stable equilibria.

State 4 is shown in the figure.

The photograph below shows three very strong magnets set on pins at the vertices of an equilateral triangle. Note that they are in equilibrium, with their dipole moments tangent to the circle in which the triangle is inscribed.