8. Magnetostatics
Chapter Summary

▶ The magnetic field $\mathbf{B}(\mathbf{x})$ may be defined by the force it exerts on a moving charge, $\mathbf{F}_{\text{mag}} = q\mathbf{v} \times \mathbf{B}$. The magnetic force is $\perp \mathbf{B}$, which leads to devices in which charged particles move on circular orbits, e.g., the cyclotron and the mass spectrometer. (See Section 8.2.)

▶ **Biot-Savart law.** A constant electric current produces a magnetic field,

$$\mathbf{B}(\mathbf{x}) = \mu_0 \int \frac{\mathbf{J}(\mathbf{x}') \times (\mathbf{x} - \mathbf{x}')}{4\pi \left|\mathbf{x} - \mathbf{x}'\right|^3} d^3x',$$

where $\mathbf{J}(\mathbf{x}')$ is the current density at $\mathbf{x}'$. In a more concise notation,

$$d\mathbf{B} = \frac{\mu_0 I d\boldsymbol{\ell} \times \widehat{\mathbf{r}}}{4\pi r^2}.$$

Examples 5 and 6 illustrate calculations of $\mathbf{B}$ by integration.

▶ **Ampère's law.** The field equation relating $\mathbf{B}(\mathbf{x})$ and $\mathbf{J}(\mathbf{x})$ may be expressed as a PDE,

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J}; \qquad \text{(static)}$$

or, as the circuital law,

$$\oint_C \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I \qquad \text{(static)}$$

where I is the current through the arbitrary closed curve C. Examples 7 and 8 illustrate calculations of $\mathbf{B}$ from the circuital law.

▶ The magnetic field produced by current I in a long straight wire is

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0 I}{2\pi r} \widehat{\boldsymbol{\phi}}.$$

▶ The magnetic field inside a densely wound solenoid with current I is $\mathbf{B} = \mu_0 n I \,\widehat{\mathbf{k}}$ where $n =$ number of turns per unit length. ($\widehat{\mathbf{k}}$ is the unit vector of the solenoid axis.)

▶ **Vector potential.** The magnetic field obeys $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$. Therefore there exists a function $\mathbf{A}(\mathbf{x})$ such that $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$.

▶ **Magnetic dipole.** The dipole moment of a finite current distribution is defined by $\mathbf{m} = \frac{1}{2} \int \mathbf{x} \times \mathbf{J}\, d^3x$. The asymptotic vector potential, if $\mathbf{m}$ is nonzero, is

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0\, \mathbf{m} \times \widehat{\mathbf{r}}}{4\pi r^2}.$$