Magnetic Field of a Bar Magnet

Everyone has played with magnets and felt the mystery of their forces. N and S poles attract; N poles repel and S poles repel. These forces come from the magnetic field $\mathbf{B}(\mathbf{x})$. What is the magnetic field of a bar magnet?

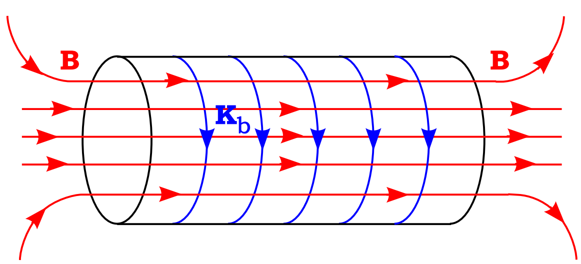
Electric current is one basic source of $\mathbf{B}(\mathbf{x})$, but the field of a bar magnet comes directly from the atoms—from electron spin and orbital states. In a ferromagnet crystal, the *exchange force* (a quantum effect of electrons) causes atomic magnetic moments to align, so that all moments within a single magnetic domain point in the same direction. If the many domains in a sample have a preferred direction, the sample is *magnetized*.

Imagine a perfectly magnetized bar of iron. The magnetization $\mathbf{M}(\mathbf{x})$, defined as the dipole moment density, is a constant, $\mathbf{M} = M_0\widehat{\mathbf{k}}$. The vector potential is

$$\mathbf{A}(\mathbf{x}) = \int \frac{\mu_0 \mathbf{M} \times (\mathbf{x} - \mathbf{x}')}{4\pi \left|\mathbf{x} - \mathbf{x}'\right|^3} d^3x'. \tag{1}$$

[The potential of a single dipole is Eq. (8.77); the integrand in (1) is just that potential for the elemental dipole $\mathbf{M}d^3x'$.] We could try to evaluate the integral, and then $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$, but it is simpler to rewrite it in a clever way. Equation (9.11) shows that $\mathbf{A}(\mathbf{x})$ is just the same as if there exists a current $\mathbf{J}_{\mathrm{b}} \equiv \boldsymbol{\nabla} \times \mathbf{M}$ and a surface current $\mathbf{K}_{\mathrm{b}} \equiv \mathbf{M} \times \widehat{\mathbf{n}}$. These effective currents are *bound currents*, i.e., bound within the atoms. For our ideal bar magnet, $\mathbf{J}_{\mathrm{b}} = 0$ because $\mathbf{M}(\mathbf{x})$ is constant; and $\mathbf{K}_{\mathrm{b}} = M_0\widehat{\boldsymbol{\phi}}$. The azimuthal bound surface current may be considered to be the origin of the magnetic field of the bar magnet.

The field produced by $\mathbf{K}_{\mathrm{b}}$ has the same form as a densely wound solenoid, because both systems (magnet and solenoid) have an azimuthal surface current.

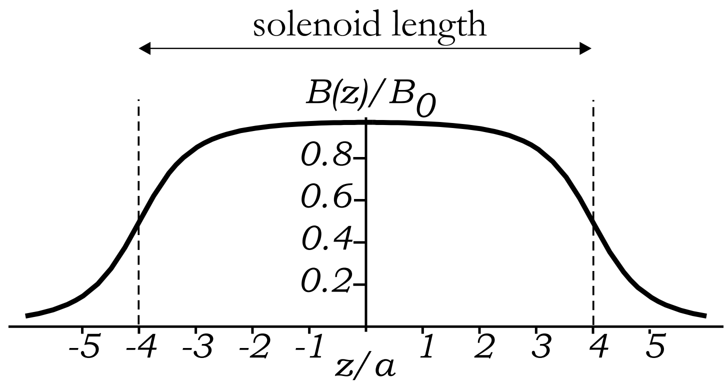


Schematic illustration of $\mathbf{B}(\mathbf{x})$ produced by a cylindrical surface current $\mathbf{K}_{\mathrm{b}}$.

It is possible to calculate $\mathbf{B}(\mathbf{x})$ for points on the symmetry axis of a circular cylinder with $\mathbf{K}_{\mathrm{b}} = M_0\widehat{\boldsymbol{\phi}}$. In Exercise 8-13 the field is found to be

$$\mathbf{B}(0,0,z) = \frac{\mu_0 M_0}{2} \left\{ \frac{z + \ell/2}{\sqrt{a^2 + (z+\ell/2)^2}} - \frac{z - \ell/2}{\sqrt{a^2 + (z-\ell/2)^2}} \right\} \widehat{\mathbf{k}} \tag{2}$$

where ℓ = length and a = radius. For example, the figure following shows a graph of $B_z(0,0,z)$ versus z for a bar magnet with length = $4\times$ diameter, i.e., $\ell = 8a$.



Examples

- For $z \gg \ell$, i.e., far from the magnet, the field is

$$B_z(0,0,z) \sim \frac{\mu_0 m}{2\pi z^3} \quad \text{where} \quad m = M_0 \pi a^2 \ell. \tag{3}$$

(The asymptotic expansion from (2) is nontrivial!) This is the field of a magnetic dipole with dipole moment $\mathbf{m} = \mathbf{M}\Omega$ where $\Omega = \pi a^2 \ell$ is the volume of the magnet. A graph of B_z from (2) on a log scale will reveal that $B_z \propto z^{-3}$ asymptotically.

- If $\ell \gg a$, then the field at the center of the bar magnet is

$$\mathbf{B}(0,0,0) = \mu_0 M_0 \widehat{\mathbf{k}}, \tag{4}$$

and the field at the end is

$$\mathbf{B}(0,0,\ell/2) = \frac{1}{2}\mu_0 M_0 \widehat{\mathbf{k}}. \tag{5}$$

For magnetized iron, $\mu_0 M_0$ is of order 1 T.