where \( o(Q) = 0.2k \) is the vertical offset for the \( k \)th \( Q \) bin. As presented in Fig. 102, the theory and NMC data appear to be in qualitative agreement. There are outlying points that do not fit the PDF model, but they appear to be large point-to-point fluctuations within the data rather than systematic deviations between smooth data and the theory. The suggestion from this graph is that the true measurement uncertainties in the NMC data are larger than the numbers used in the analysis.

Figure 103 shows the pull distribution for the NMC data. As in the case of the ZEUS data, the measurement fluctuations are Gaussian. However, the width is noticeably broader than the expected statistical error. Again, one interpretation is that the measurement errors are larger than the numbers used in the analysis.

### A.4 The tolerance criterion

The standard fit occurs at the point in the 20 dimensional parameter space where \( \chi^2 \) is minimum. Nearby points would also be acceptable fits to the data. But how near is near enough to be acceptable? We explore the neighborhood of the minimum using the eigenvectors of the Hessian matrix as the basis vectors. (The Hessian is \( \partial^2 \chi^2 / \partial x_k \partial x_{k'} \) where \( x_k \) = distance from the minimum along the \( k \)th parameter axis.) By calculating the variations of \( \chi^2 \) for all the experiments (labeled by \( e \)) as functions of distance along a particular eigenvector, we learn how far from the minimum is tolerable to each experiment for displacements in that direction. The distance along an eigenvector should be scaled by the square root of the eigenvalue.

To illustrate the method, Figure 104 shows the results for Eigenvector 4. For each of the 16 data sets, a range is shown—the distance \( D \) from the minimum in units of \( \sqrt{\chi^2} \)—within which \( \chi^2 \) lies within the 90% confidence level of its value at the minimum. (To calculate the confidence level we assume that the renormalized variable \( \chi^2_e / \chi^2_e(0) \) obeys a chi-square distribution with \( N_e \) degrees of freedom.) The increase of the global \( \chi^2 \) from its minimum value is \( D^2 \) at distance \( D \) from the minimum. For Eigenvector 4 we see that the strongest bounds on \( D \) come from the data sets H1a (low \( Q \) H1 data) and BCDMSd (BCDMS deuteron scattering) for negative \( D \), and from the data sets CCFRF3 (CCFR measurements of \( F_3 \)) and NMCrx (NMC ratio of \( d/p \) in DIS) for positive \( D \). Other experiments place weaker bounds on the distance along this eigenvector; some experiments are very insensitive to the particular combination of PDF parameters defined by Eigenvector 4.

There is no unique way to convert the 16 ranges in Fig. 104 into a single uncertainty measure, but reasonable algorithms yield bounds of \( |D| \leq 10 \). This is our tolerance \( T = 10 \) for the variation of the PDF parameters. The corresponding increase in the global \( \chi^2 \) is \( \Delta \chi^2 = T^2 = 100 \). We show Eigenvector 4 in Fig. 104; the other 19 eigenvectors are similar. Of course the pattern of sensitivity to specific experiments is different for each eigenvector, and the tolerance could be different in different directions. But we find that \( T \approx 10 \) is a reasonable tolerance in all cases. Reducing the uncertainty to a single number, equivalent to the criterion \( \Delta \chi^2 < 100 \), is in principle an oversimplification, but in fact a generally reasonable estimate of the uncertainty.

If we want to calculate the PDF uncertainty for a specific observable, we would study the full analysis of the \( \chi^2 \)'s, analogous to Fig. 104 and as described in Ref. [11], rather than
simply applying the criterion \( T = 10 \). But to get a picture of the overall uncertainties of the PDF’s, the simple criterion should be adequate.
Figure 103: Histogram of $\Delta_i$ in (A.7) for the NMC data.

Figure 104: Ranges of allowed distance along Eigenvector 4, for each of the 16 data sets. The dots are the positions of the minimum $\chi^2_e$ for each experiment $e$. The error bars are the 90% confidence-level ranges. The horizontal lines are for distance ±10, in units of $\sqrt{\lambda_i}$, from the global minimum.