- **<u>Theme Music:</u> George Fox**
  Do the math

- **<u>Cartoon:</u> Carlos Castellanos & Hector Cantú - *Baldo***

Announcements

- Don’t forget to go to hands-on Tues – Thurs!
- Reading questions for Chapter 2 due tomorrow night!
- LON-CAPA input issues:
  - I opened a “Input Practice” problem for 0 points at the front of the Ch1&2 homework
  - If you enter something that LON-CAPA can’t understand you should see a little “?” button which will give you more information

Dimensions and units

- The simplest mathematical model we use in science is we assign numbers to physical quantities by measurement.
- Each kind involves an arbitrary choice of scale.
  - Different types ↔ **dimensions**
    - Distance, time, mass, …
  - Equations that represent physical relationships must maintain their equality even when we change our arbitrary choice.
- The quantity we create by adding a unit is NOT just a number but a blend.

Consider two mathematical models of real world things:

1. Distance
2. Time

We map positions and times into numbers. What kinds of numbers are we mapping to?

A. All numbers
B. Non-negative numbers only
C. Positive only

Foothold ideas: Dimensional and unit analysis

- We label the kinds of measurement that go into assigning a number to a quantity like this:

  [x] = L means "x is a length"

  [t] = T means "t is a time"

  [m] = M means "m is a mass"

  [v] = L/T means "you get v by dividing a length by a time

- Units specify which particular arbitrary measurement we have chosen.
  - Units should be manipulated like algebraic quantities.
  - Units can be changed by multiplying by appropriate forms of "1" e.g. 1 = (1 inch)/(2.54 cm)

Foothold ideas: Dimensional analysis

- In physics we have different kinds of quantities depending on how measurements were combined to get them. These quantities may change in different ways when you change your measuring units.
- Only quantities of the same type may be equated (or added) otherwise an equality for one person would not hold for another. Equating quantities of different dimensions yields nonsense.
- Dimensional analysis tells us *how* something changes when we either
  - Change our arbitrary scale (passive change)
  - Change the scale of the object itself (active change)

Which equation represents the quantity on the left?

A. The area of a circle.

B. The volume of a sphere.

C. The circumference of a circle.

D. The surface area of a sphere.

1. $2\pi R$

2. $4\pi R^2$

3. $\frac{4}{3}\pi R^3$

4. πR^2

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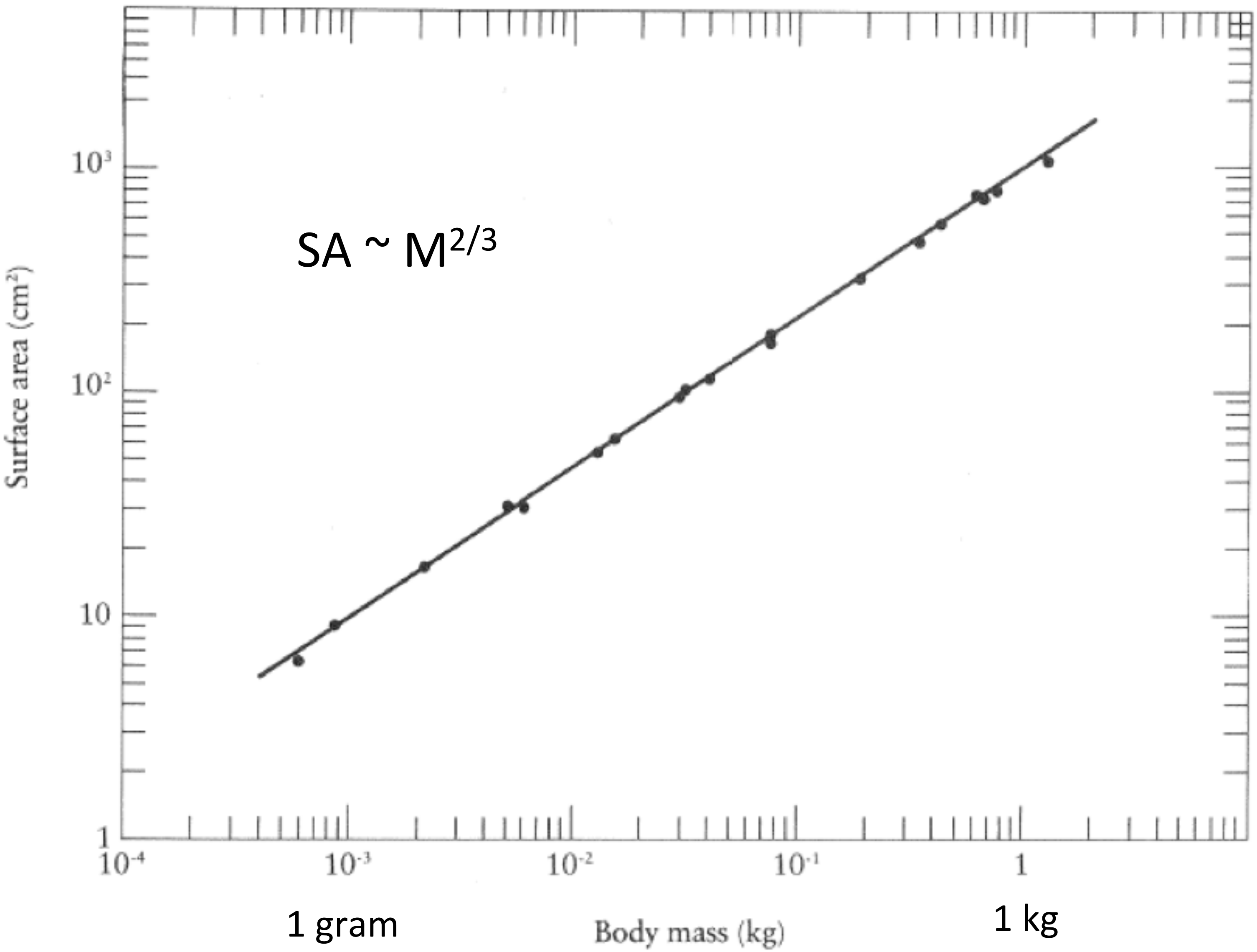
The dwarf siren salamander



McMahon & Bonner, 1981

Implications?

- Mass grows with volume, or size^3
- Area available to support weight goes as size^2
- Difficulty of supporting your weight ~ Mass/Area or size^3/size^2 = size
- So big organisms have more difficulty supporting their weight!

A sculptor builds a model for a statue of a Spartan for an honor's project. She discovers that to cast her small scale model she needs 2 kg of bronze. When she is done, she finds that she can give it two coats of finishing polyurethane varnish using exactly one small can of varnish. The final statue is supposed to be 5 times as large as the model in each dimension. How much bronze will she need? How much varnish should she buy?

(*Hint*: If this seems difficult, you might start by writing a simpler question that is simpler to work on before tackling this one.)

Worked solution on next slide

Worked solution:

I know that mass is proportional to volume, so

2kg = 1 unit of volume, I also know that volume has dimensions of $[L^3]$

Now, when I scale the model up, I need the statue to be 5x the size of the original, so that means the surface area should be 5x bigger

$SA_{statue}=5\ SA_{model}$

So now I need to figure out how to get from a measure of surface area to a measure of volume (or vice versa) so I can compare the model to the statue

$[SA] = [L^2]$ and $[V] = [L^3]$, so if I want to go from SA to Volume:

$[SA] * [L] = [V] \sim [M]$

So since $L_{statue} = 5L_{model}$ I can see the following:

$V_{statue}=SA_{statue}*L_{statue}= 5*SA_{model}*5*L_{model}= 25*SA_{model}*L_{model}=25V_{model}$

$V_{statue}=25V_{model}$

And since I know that volume is proportional to mass, then the

$M_{statue} \sim 25*M_{model}$

How much bronze do I need? 25 * 2kg = 50kg

How many cans of varnish do I need? 25*1 can (for two coats) = 25 cans for two coats (or 13 cans for 1 coat!)