

Today's Topics: Rotational Motion

Comic: Calvin & Hobbes, *Bill Waterson*



How many degrees are in 1 radian?

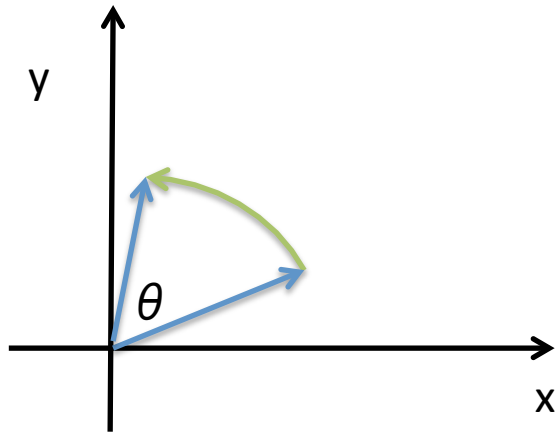


- A. $1 \text{ rad} = 2\pi \text{ degrees}$
- B. $1 \text{ rad} = 180^\circ$
- C. $1 \text{ rad} = 10^\circ$
- D. $1 \text{ rad} = 57.3^\circ$
- E. Radian is not a measure of angle, so the question makes no sense.

Announcements

- Correction problem due NOW
- Moving reading questions for Ch12 to next Tuesday (Nov 4th)
- Homework on section 8.3 & 9.5 due on Friday at midnight

Angular velocity (ω)



We usually talk about the object's motion relative to the x-axis

- So counterclockwise is positive
- Clockwise is negative

We talk about the motion as an object has moved through a distance s (arc length) through an angle (θ)

A pocket watch and Big Ben are both keeping perfect time. Which minute hand has the larger magnitude angular velocity ω ?

- A. Pocket watch's
- B. Big Ben's
- C. Same ω on both



A pocket watch and Big Ben are both keeping perfect time. Which minute hand has the larger magnitude linear velocity?

- A. Pocket watch's
- B. Big Ben's
- C. Same v on both



A student sees the following question on an exam:

A flywheel with mass $M=120\text{kg}$, and a radius $r=0.6\text{m}$, starting at rest, has an angular acceleration of $\alpha=.1\text{ rad/s}^2$. How many revolutions has the wheel undergone after 10 s ?

Which formula should the student use to answer the question?

A. $\omega = \omega_0 + \alpha t$

B. $\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

C. $\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$



Angular Acceleration

- What happens if the angular velocity changes?
 - $\alpha = \Delta\omega/\Delta t$ $\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$
- Rate of change of the angular velocity

Newton's 2nd Law

AKA: "The Momentum Principle"

- Momentum: a vector quantity that depends on both mass and velocity of the object of interest

$$\vec{p} = m\vec{v}$$

- Changes in momentum are achieved by exerting a net force on the system

$$\Delta \vec{p} = \vec{F}_{\text{net}} \Delta t$$

- If a system experiences a net force it will undergo either:
 - a change in the magnitude of its momentum
 - a change in the direction of its momentum
 - a change in both the magnitude and direction of its momentum

What we have so far

Linear Motion

- $v_f = v_0 + at$
- $\Delta x = v_0 t + \frac{1}{2} at^2$
- $v_f^2 = v_0^2 + 2a \Delta x$

Angular Motion

- $\omega_f = \omega_0 + \alpha t$
- $\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$
- $\omega_f^2 = \omega_0^2 + 2\alpha \Delta \theta$

What we have so far

Linear Motion

- $v_f = v_0 + at$
- $\Delta x = v_0 t + \frac{1}{2} at^2$
- $v_f^2 = v_0^2 + 2a \Delta x$

- $\vec{F}_{\text{net}} = m\vec{a}$
- $\vec{p} = m\vec{v}$
- $KE_{\text{linear}} = \frac{1}{2} mv^2$

Angular Motion

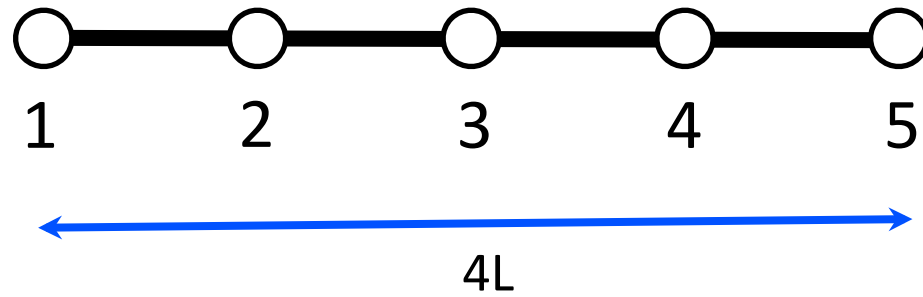
- $\omega_f = \omega_0 + \alpha t$
- $\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$
- $\omega_f^2 = \omega_0^2 + 2\alpha \Delta \theta$

- $\tau_{\text{net}} = I\alpha$
- $L = I\omega$
- $KE_{\text{rot}} = \frac{1}{2} I\omega^2$

Moment of Inertia

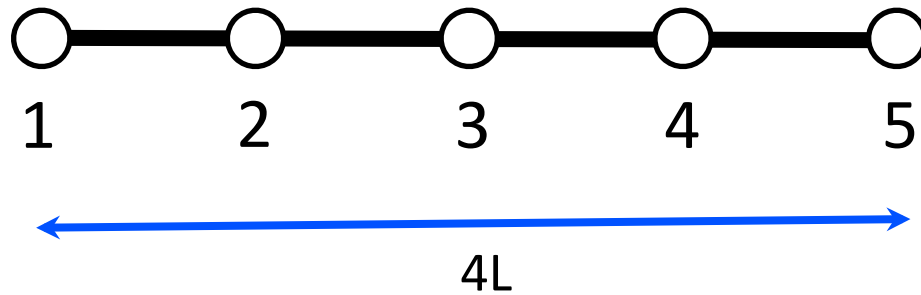
- Center of mass
- Telling us how each piece of mass contributes to the overall resistance to movement

$$I_{\text{tot}} = \sum m_i r_i^2 \quad \rightarrow \int r^2 dm$$



The massless rod shown above has a length of $4L$ and five equidistant point masses, each with mass M . What is the moment of inertia about mass 1, assuming rotation perpendicular to the rod?

- A. $5 ML^2$
- B. $10 ML^2$
- C. $15 ML^2$
- D. $30 ML^2$
- E. None of the above



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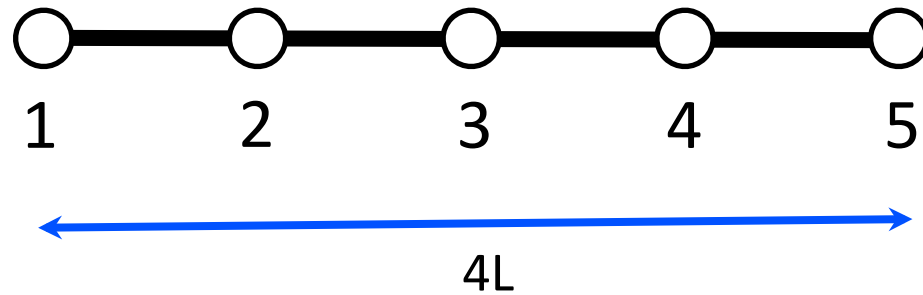
$$I_{\text{tot}} = \sum m_i r_i^2$$

$$I_{\text{tot}} = m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2 + m_5 r_5^2$$

$$I_{\text{tot}} = m (L)^2 + m_3 (2L)^2 + m_4 (3L)^2 + m_5 (4L)^2$$

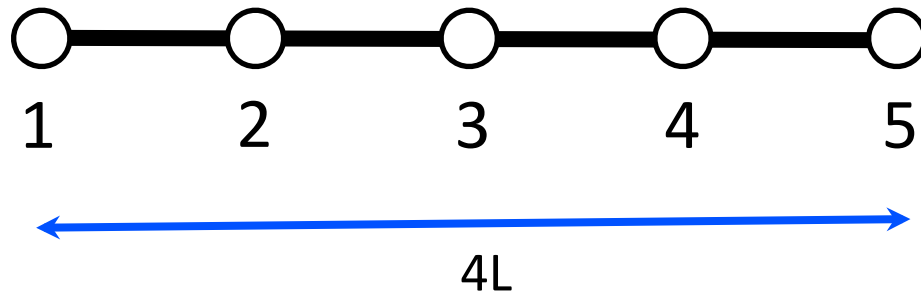
D. $30 ML^2$

E. None of the above



The massless rod shown above has a length of $4L$ and five equidistant point masses, each with mass M . What is the moment of inertia about mass 3, assuming rotation perpendicular to the rod?

- A. $5 ML^2$
- B. $10 ML^2$
- C. $15 ML^2$
- D. $30 ML^2$
- E. None of the above



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D. $I_{\text{tot}} = \sum m_i r_i^2$

E. $I_{\text{tot}} = m_2 r_2^2 + m_1 r_1^2 + m_4 r_4^2 + m_5 r_5^2$

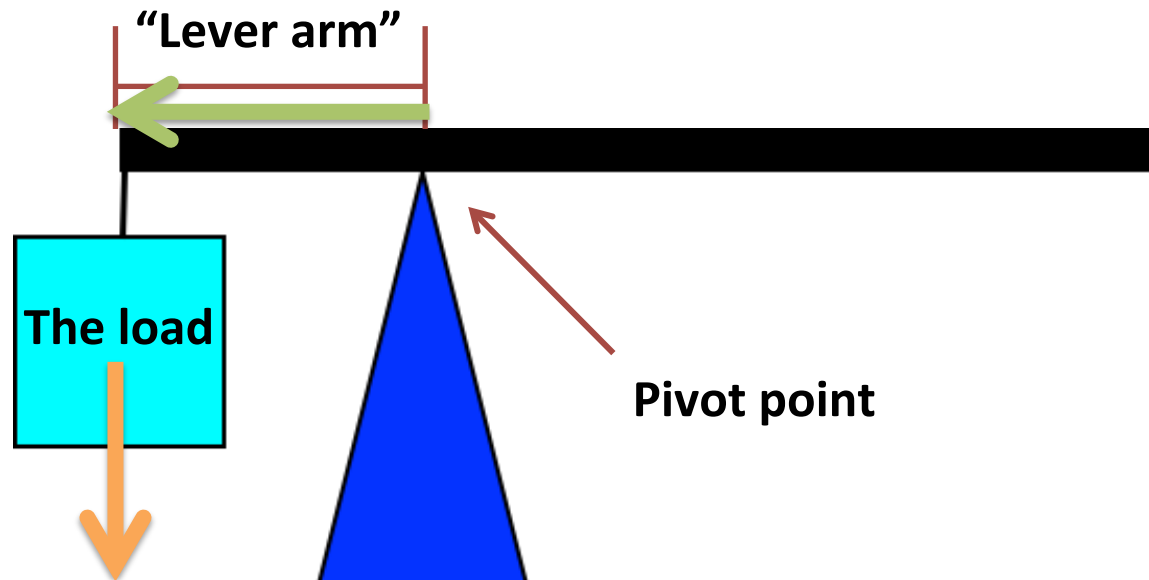
$$I_{\text{tot}} = m_2 (L)^2 + m_1 (2L)^2 + m_4 (L)^2 + m_5 (2L)^2$$

Foothold Ideas Underlying Torque



- Start by identifying the pivot point
- Next identify where the push or pull is (sometimes called “the load”)
- Measure the distance between the push and the pivot point (also called the “lever arm”)
- The torque is

$$\vec{\tau} = \vec{r} \times \vec{F}$$



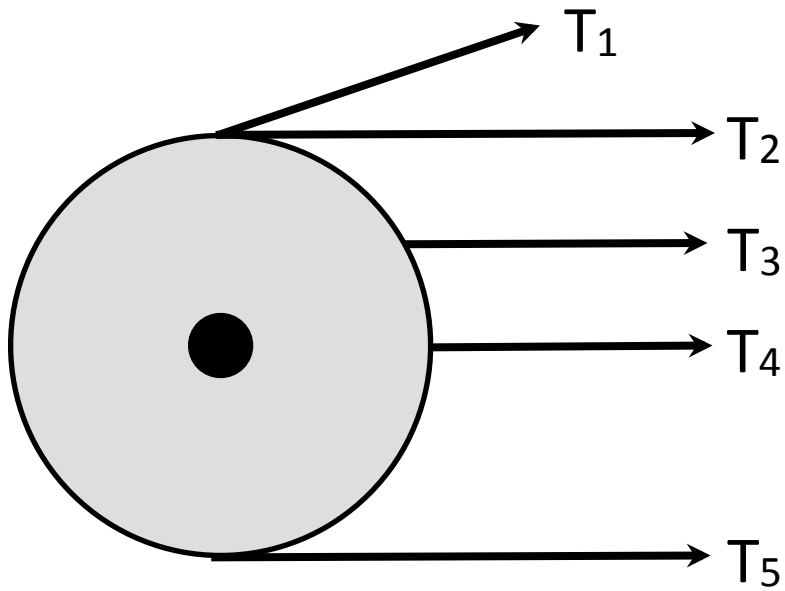
Torque

- Torque is a vector quantity that describes how ω changes
 - Comparable to how F_{net} tells us how acceleration changes

- $\tau_{\text{net}} = I\alpha$



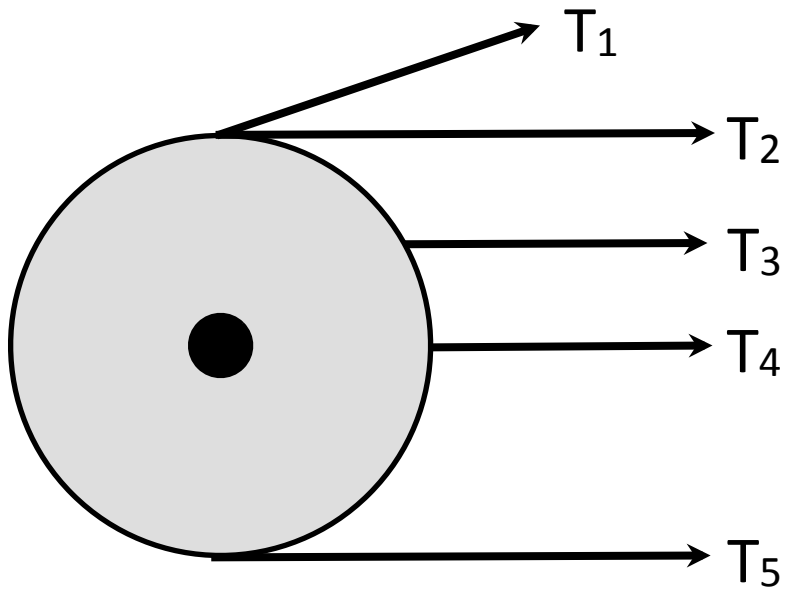
Acts like a “rotational” mass



A solid disk is mounted on an axis, as shown, and is initially at rest. The same force is applied along five different pieces of rope, as shown.

Which rope exerts the **smallest torque** on the disk?

- A. T_1
- B. T_2
- C. T_3
- D. T_4
- E. T_5



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