Make sure you grab a whiteboard!

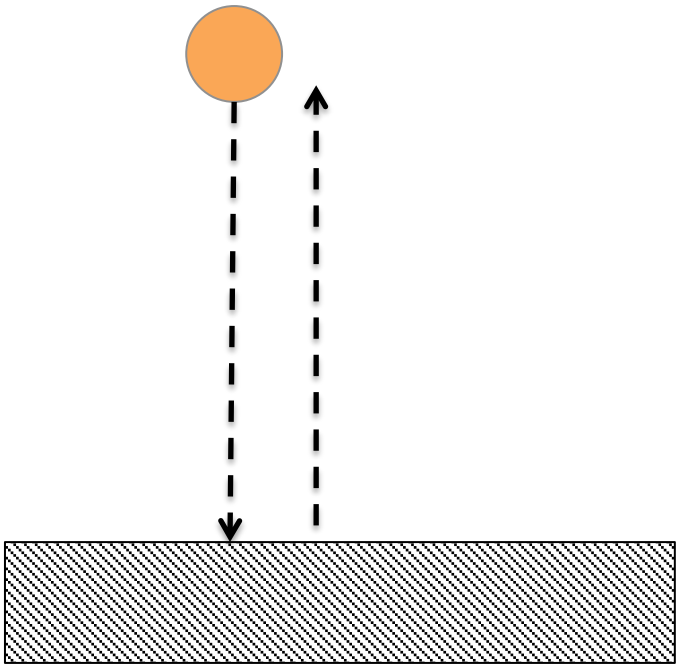
(We have a lot of graphs to work with today)

A ball bounces up and down on a floor with perfect elastic bounces, so that the ball bounces forever. Is this an example of simple harmonic motion?

(Vote silently)

A. Yes

B. No



Simple Harmonic Motion (SHM)

1. There is a restoring force proportional to the distance displaced from the equilibrium — $F \propto \Delta x$
2. The potential energy is proportional to the square of the displacement — $U \propto (\Delta x)^2$
3. The period or frequency (1/T) is independent of the amplitude of the motion
4. The position, x, the velocity, v, and the acceleration are all sinusoidal in time

Anything that looks like sine, cosine, or anything in between

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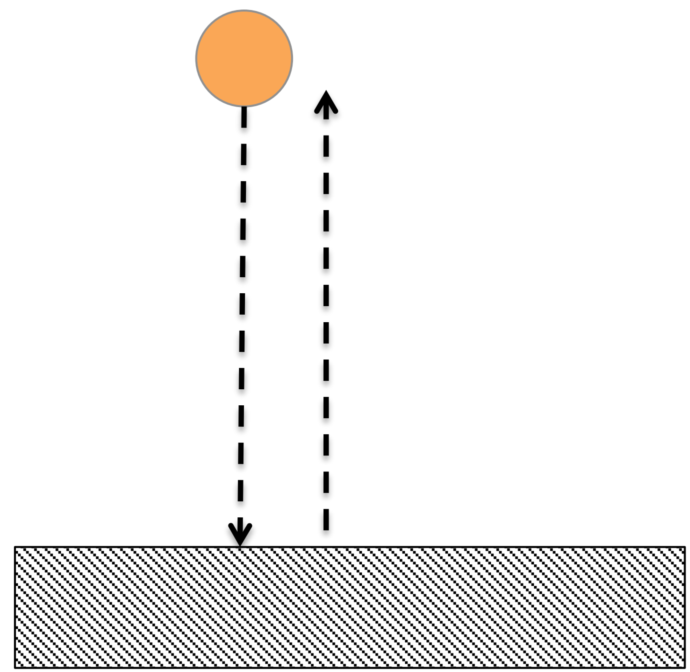

A ball bounces up and down on a floor with perfect elastic bounces, so that the ball bounces forever. Is this an example of simple harmonic motion?

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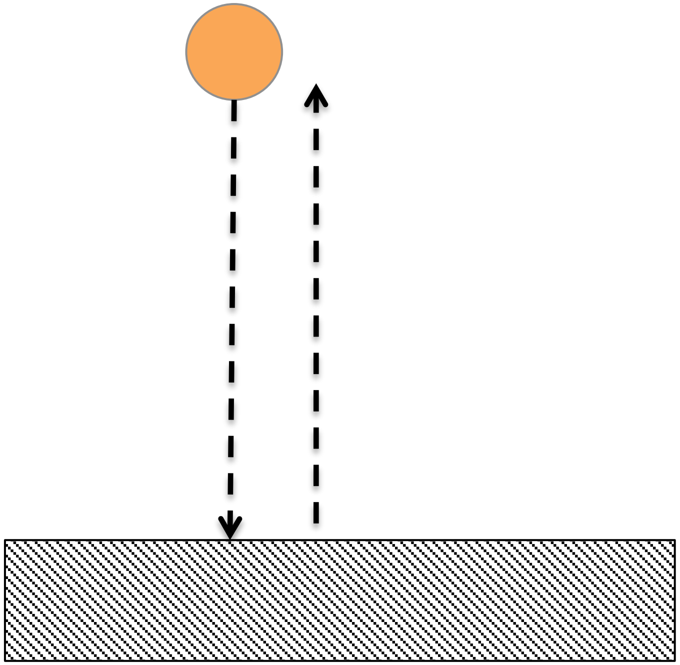
If we graph the vertical position of the ball, we see it has a sinusoidal shape.

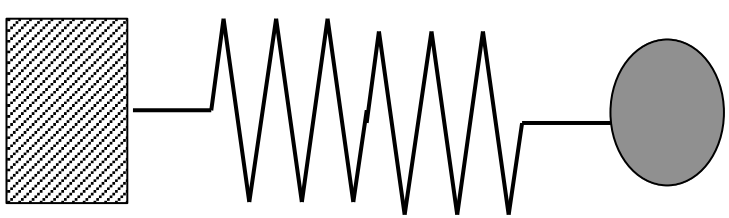


The simplest kind of SHM: Mass on a spring

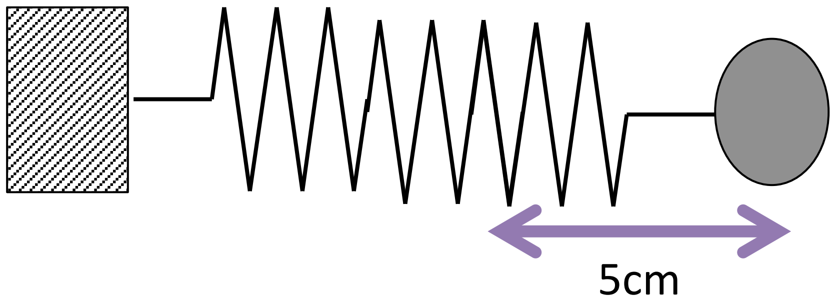
- Hooke's Law

$$F^{s}_{spring \to object} = -k\Delta x$$

How much you stretched it from equilibrium.



I have an object attached to a spring, and I've stretched it out 5cm from it's equilibrium point. Which way will the mass move if I let it go?



A. To the left
B. To the right
C. It won't move

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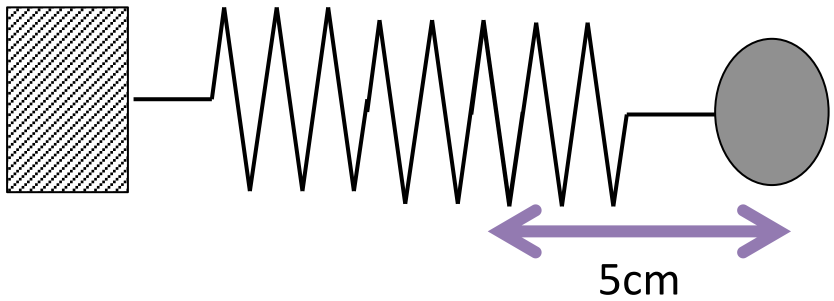

A. To the left
B. To the right
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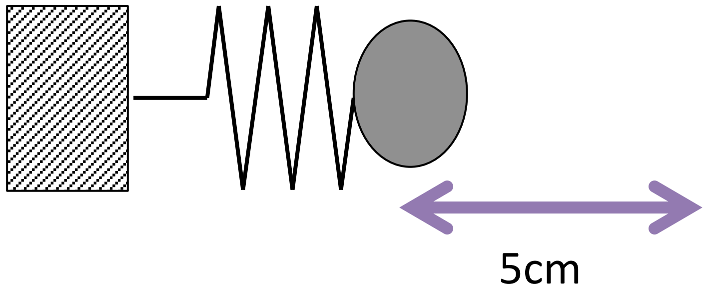
The force will always pull back to the equilibrium.

I have an object attached to a spring, and now I’ve compressed it 5cm from it’s equilibrium point. Which way will the mass move if I let it go?



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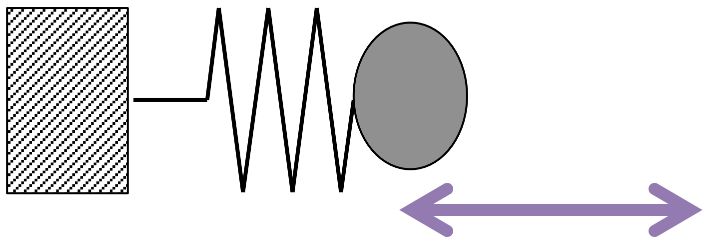

A. To the left
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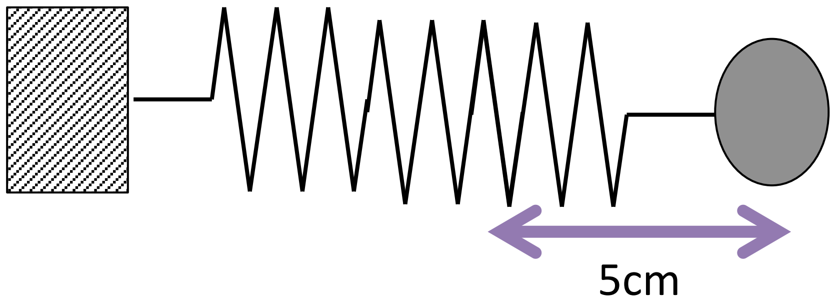
C. It won't move

The force will always pull back to the equilibrium.

Same spring and mass, now I stretch it out 5cm and let go. Graph the position over time assuming no friction.



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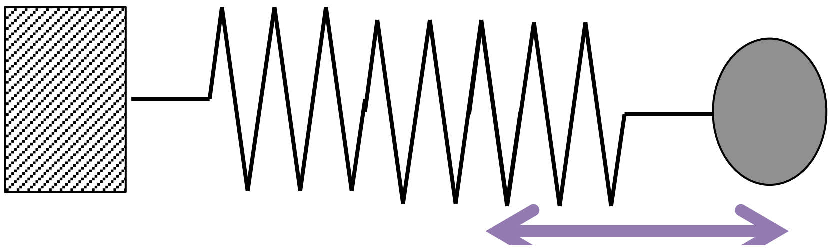
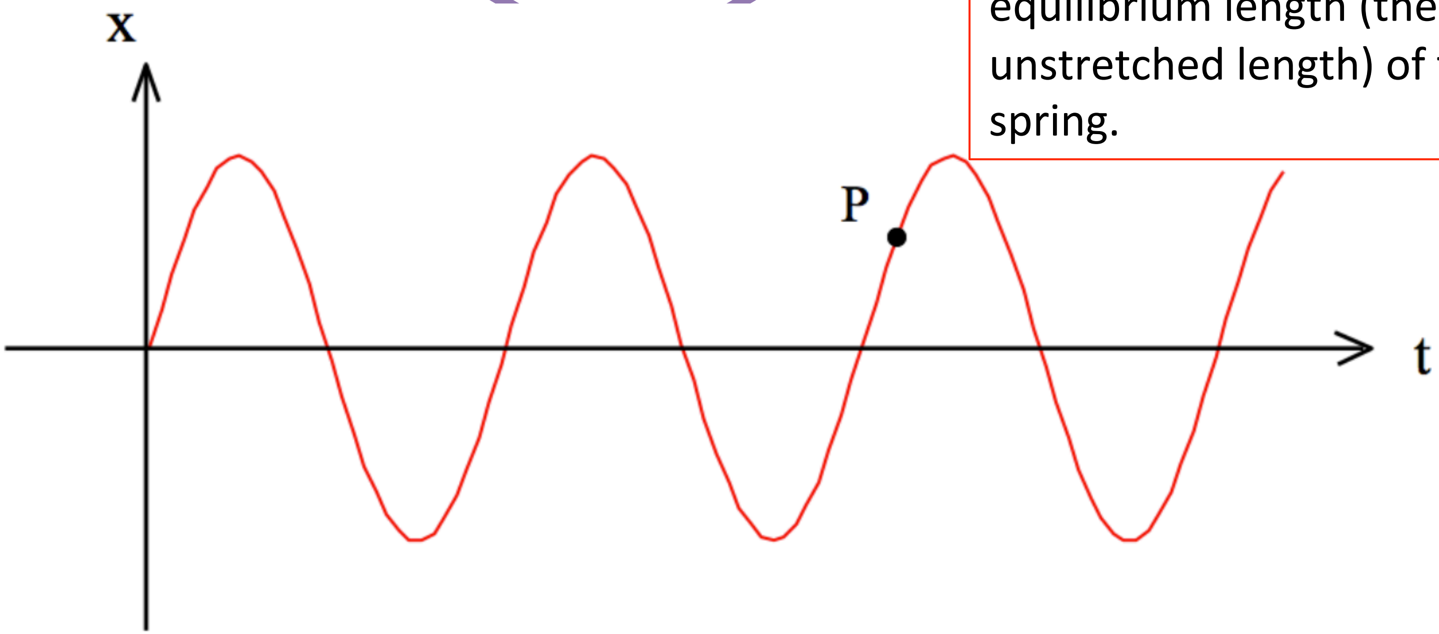


If we set the 0 point to be the equilibrium length (the unstretched length) of the spring.



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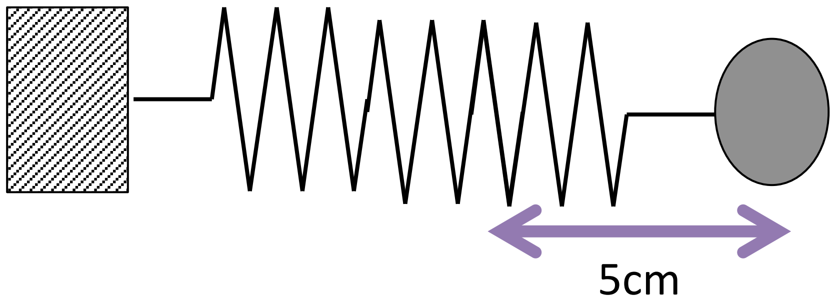
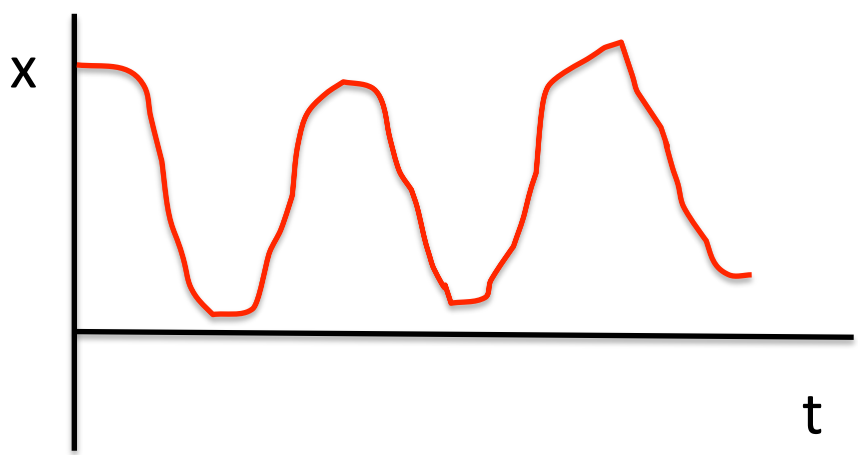




If we set the 0 point to be at the wall.

Foothold ideas: Harmonic oscillation

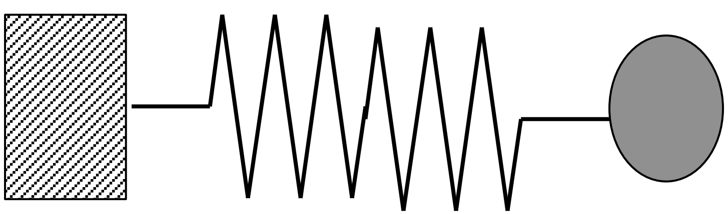
- There is an equilibrium (balance) point where the mass can stay without moving.
- Whichever way the mass moves,
  the force is in the direction of
  pushing it back to its equilibrium position.
- When it gets back to its equilibrium,
  it's still moving so it overshoots.

The simplest kind of SHM: Mass on a spring

- Hooke's Law

$$F^{s}_{spring\ ->object} = -k\Delta x$$

- Amplitude: $|x_{max}| = |x_{min}|$ the mass oscillates between these two extreme positions



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Simple Harmonic Motion

$$F^{net} = -kx \qquad a = \frac{1}{m}F^{net} \qquad a = \frac{-k}{m}x$$

Notation:

$$\omega_0^2 = \frac{k}{m}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$a = -\omega_0^2 x$$

$$\frac{d^2x}{dt^2} = -\omega_0^2 x$$

This is the differential equation that describes simple harmonic motion.

Consider a variable x that depends on θ, $x = x(\theta)$. Now consider the differential equation $d^2x/d\theta^2 = -x$. Here are some proposed solutions:

I) $x = \sin(\theta)$ II) $x = \cos(\theta)$, III) $x = e^{\theta}$

How many of them are actual solutions?

A. All of them
B. None of them
C. 1 of them
D. 2 of them

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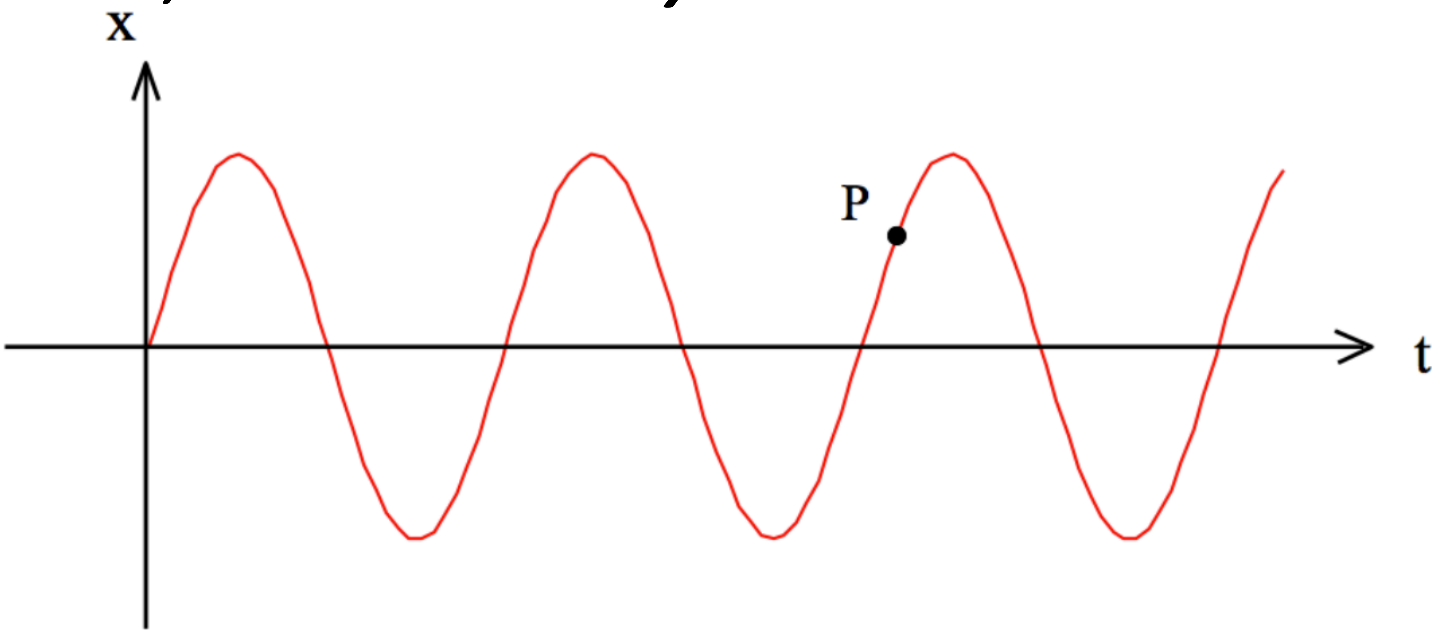
D. 2 of them

Both sin(θ) and cos(θ) satisfy the condition, but e^θ does not because we'll never get the negative.

The position of a mass on a spring as a function of time is shown below. When the mass is at point P, the *velocity* is…



A. v > 0

B. v < 0

C. v = 0

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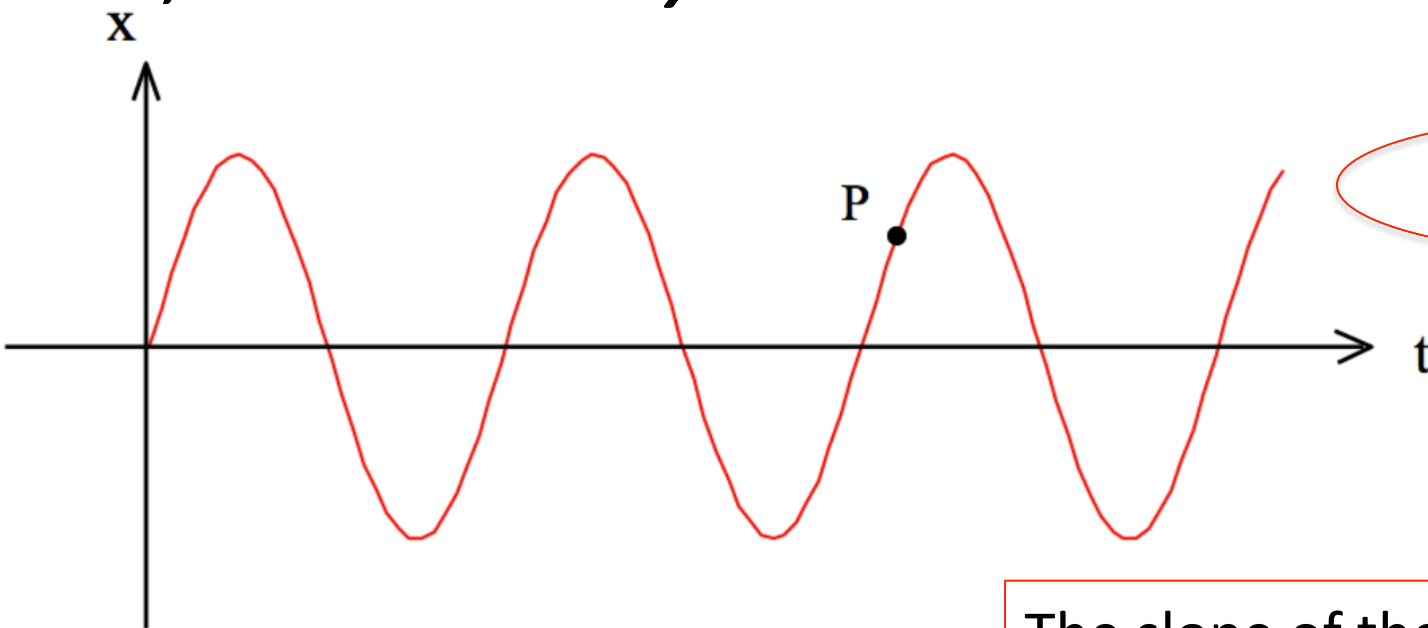


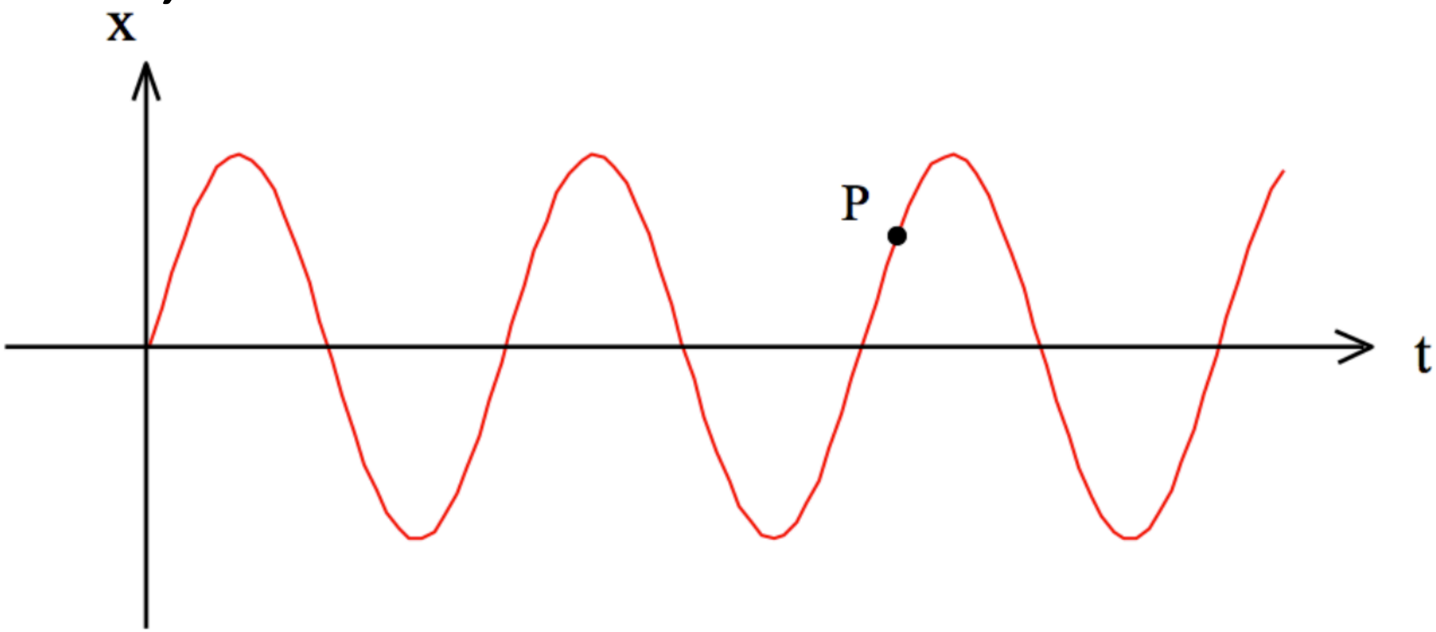
A. $v > 0$

B. $v < 0$

C. $v = 0$

The slope of the line at point P is positive.

The position of a mass on a spring as a function of time is shown below. When the mass is at point P, the *acceleration* is…



A. a > 0

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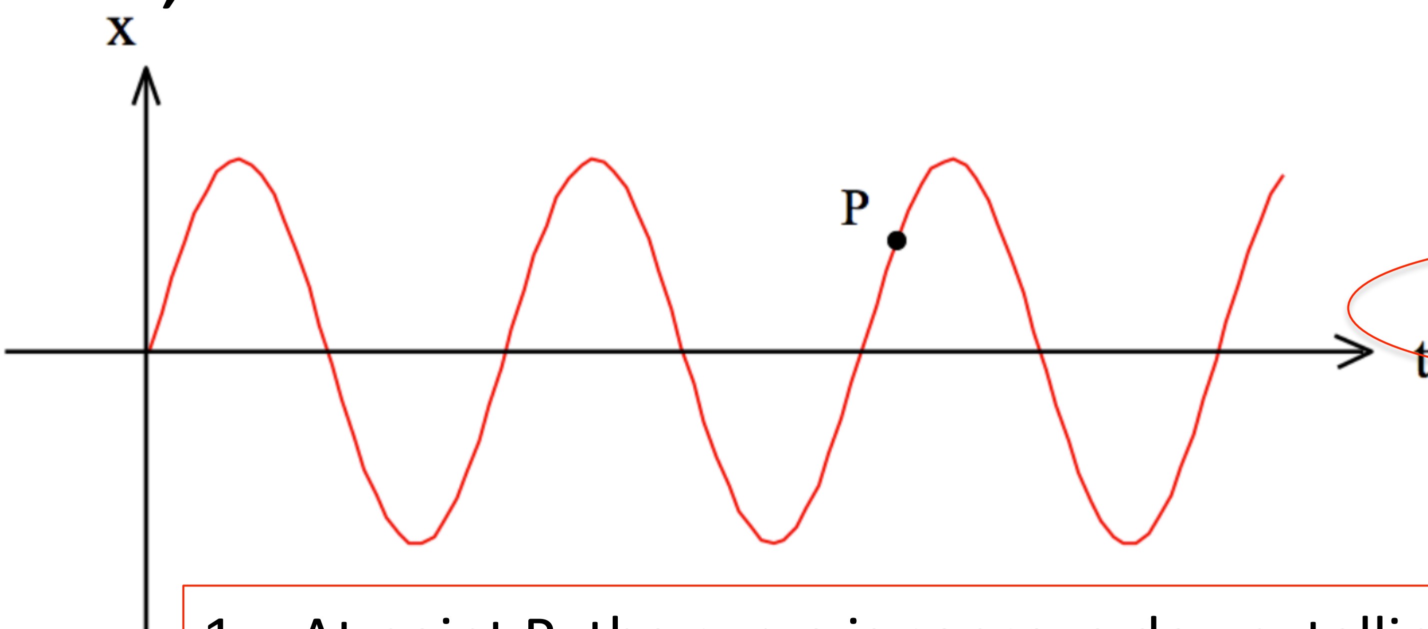

A. $a > 0$
B. $a < 0$
C. $a = 0$

1. At point P, the curve is concave down, telling us the second derivative is negative.
2. At point P, the mass is moving in the positive direction, but slowing down so the acceleration must be negative.

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   $x(t) = A\cos(\omega t)$

   where $\omega = \sqrt{k/m}$)