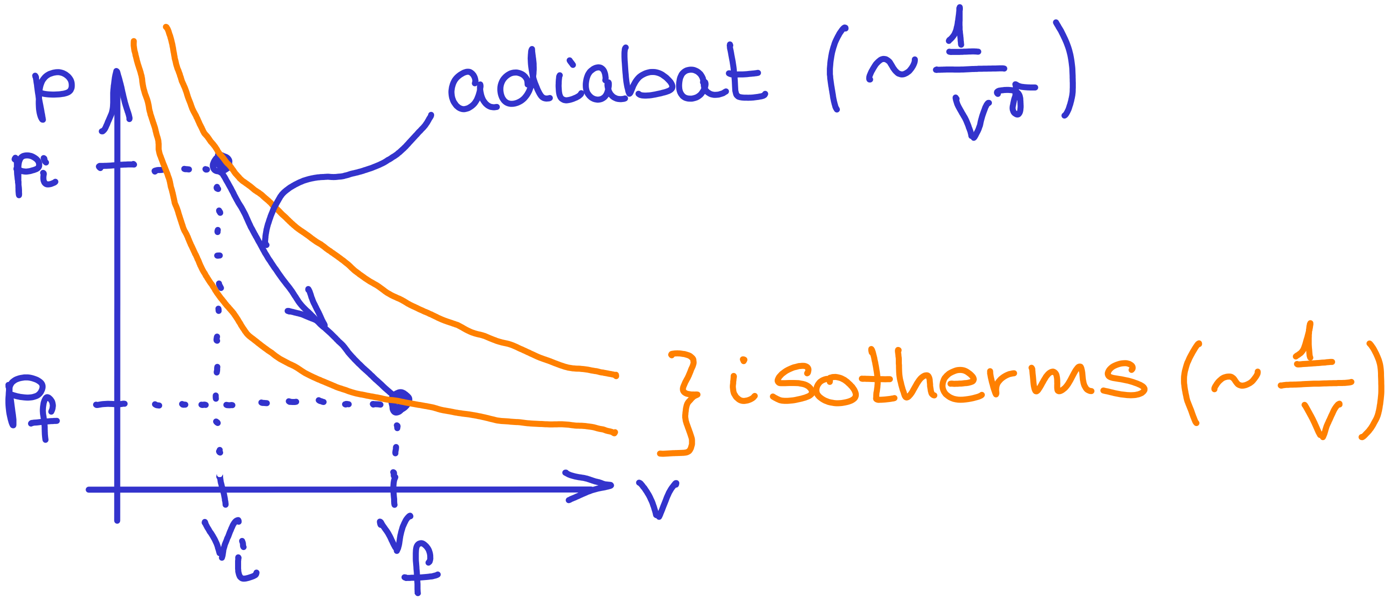
Adiabatic expansion



→ $V_f > V_i$ b/c it is an expansion.

→ $p_f < p_i$ b/c $p_i V_i^\gamma = p_f V_f^\gamma$: increasing the volume decreases the pressure.

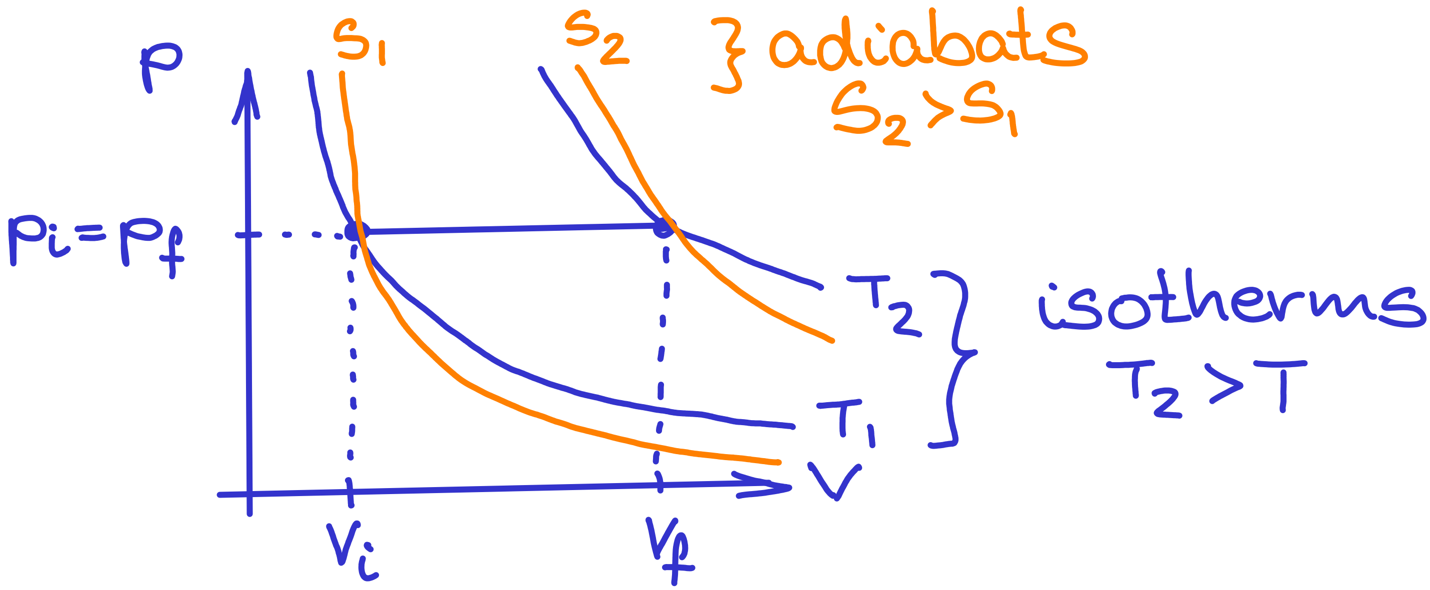
→ $T_f < T_i$ b/c adiabats fall steeper than isotherms.

→ $U_f < U_i$ b/c internal energy depends only on the temperature : $U = \frac{f}{2} nRT$. If the temperature drops, then the internal energy decreases.

→ $S_f = S_i$ b/c $Q = 0$: no heat is emitted or absorbed.

adiabatic = isentropic or isoentropic

Isobaric expansion



→ $V_f > V_i$ b/c it is an expansion.

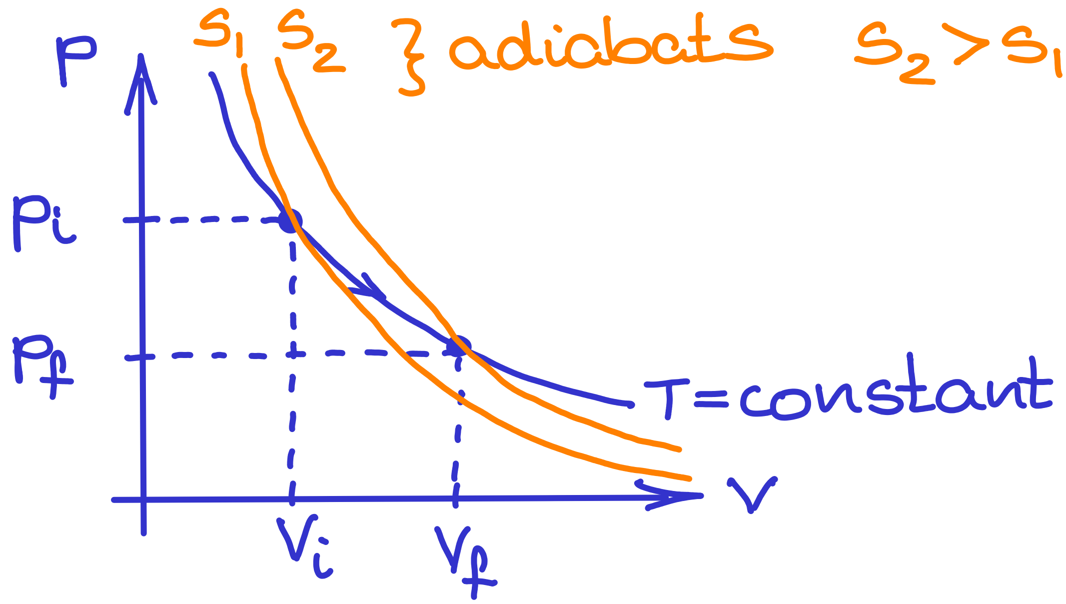
→ $p_f = p_i$ b/c it is isobaric.

→ $T_f > T_i$ b/c $\frac{V_f}{T_f} = \frac{V_i}{T_i}$ or b/c we are crossing isotherms moving toward higher temperatures.

→ $U_f > U_i$ b/c $T_f > T_i$ and $U = \frac{f}{2} nRT$.

→ $S_f > S_i$ b/c $\underbrace{\Delta U}_{+} = Q + \underbrace{W}_{-}$ therefore Q must be positive: $Q = \Delta U - W =$ $= \underbrace{\Delta U}_{+} + \underbrace{W_{gas}}_{+}$. Or we are crossing a set of adiabats in the p-V plane moving toward the higher entropies.

Isothermal expansion



→ $V_f > V_i$ b/c it is an expansion.

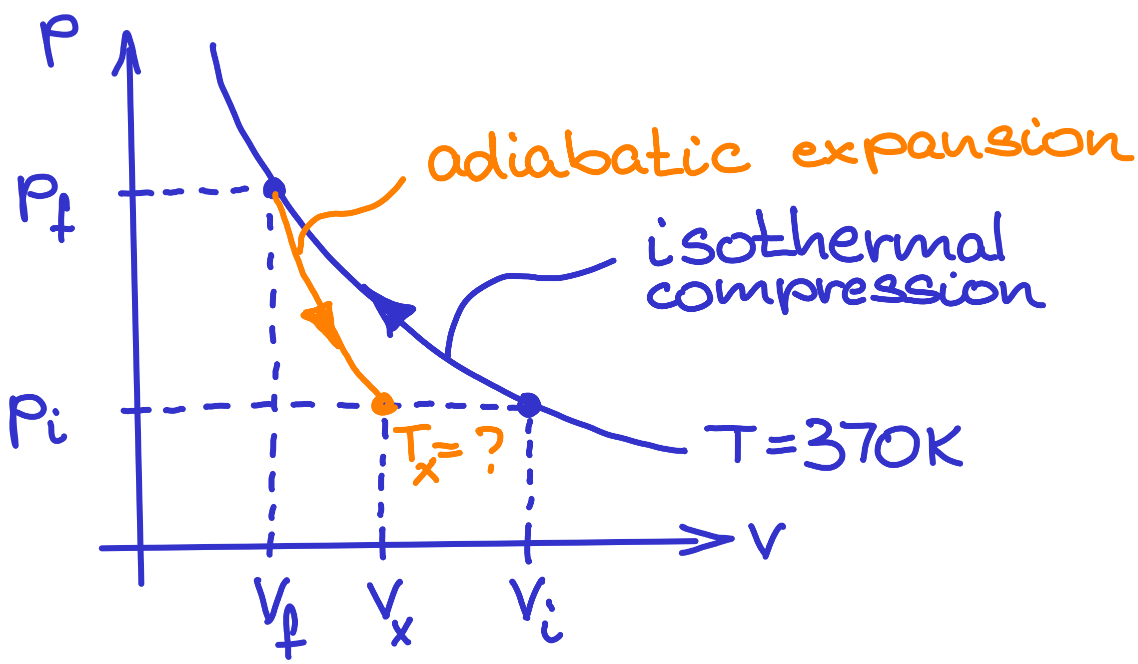
→ $p_f < p_i$ b/c of $p_i V_i = p_f V_f$ (BM law or Ideal Gas Law).

→ $T_f = T_i$ b/c it is an isothermal process.

→ $U_f = U_i$ b/c $T_f = T_i$ and $U = \frac{f}{2} nRT$

→ $S_f > S_i$ b/c $\underbrace{\Delta U}_{=0} = Q + \underbrace{W}_{-}$ therefore Q must be positive. Or we are crossing a set of adiabats moving toward the higher entropies.

Isothermal compression



$n = 3.72$ mol

Hydrogen: $f = 5 \rightarrow \gamma = \frac{f+2}{f} = \frac{7}{5} = 1.4$

$p_i = 335$ kPa $(\approx 3.3$ atm$)$

$p_f = 851$ kPa $(\approx 8.5$ atm$)$

$R = 8.31 \frac{J}{mol K}$

Ideal Gas Law: $pV = nRT$

$$p_i V_i = nRT \rightarrow V_i = \frac{nRT}{p_i} = 34.1\,\ell$$

$$p_f V_f = nRT \rightarrow V_f = \frac{nRT}{p_f} = 13.4\,\ell$$

The work done by the external force is positive, because it is compression. The work done by the gas is negative, because it is being compressed, and these two works above are opposite of each-other.

$$W_{ext} = nRT \cdot \underbrace{\ln\left(\frac{V_i}{V_f}\right)}_{\text{positive b/c } V_i > V_f} \qquad (W_{gas} = -W_{ext})$$

$$W_{ext} = 3.72 \cdot 8.31 \cdot 370 \cdot \ln\left(\frac{34.1}{13.4}\right) = 10.7\ kJ$$

First Law of Thermodynamics:

$$\underbrace{\Delta U}_{=0} = Q + W$$

=0 b/c the temperature T is constant.

$$0 = Q + W$$

$$-Q = W$$

$$Q_{gas} = W \quad \text{because } Q_{gas} = -Q$$

$$Q_{gas} = W = W_{ext} = 10.7\ kJ$$

The external work done on the gas is emitted by the gas as heat.

Since this heat was emitted during a constant temperature (isothermal) process, the entropy emitted by the gas is simply:

$$S_{gas} = \frac{Q_{gas}}{T} = \frac{10.7\ kJ}{370\ K} = 28.9\ \frac{J}{K}$$

Entropy is measured in J/K. (The unit of the Boltzmann constant k_B is J/K.)

Adiabatic expansion back to p_i:

$$p_f \cdot V_f^{\gamma} = p_i \cdot V_x^{\gamma}$$

$$\frac{p_f}{p_i} = \frac{V_x^{\gamma}}{V_f^{\gamma}}$$

$$\left(\frac{p_f}{p_i}\right)^{\frac{1}{\gamma}} = \frac{V_x}{V_f} \Rightarrow V_x = V_f \cdot \left(\frac{p_f}{p_i}\right)^{\frac{1}{\gamma}}$$

Let's put this back to the Ideal Gas Law:

$$p_i V_x = nRT_x \Rightarrow T_x = \frac{p_i V_x}{nR}$$

$$T_x = \frac{p_i \cdot V_f \cdot \left(\frac{p_f}{p_i}\right)^{\frac{1}{\gamma}}}{nR}$$

$$T_x = \frac{335{,}000\,\text{Pa} \cdot 0.0134\,\text{m}^3 \cdot \left(\frac{851}{335}\right)^{\frac{5}{7}}}{3.72\,\text{mol} \cdot 8.31\,\frac{\text{J}}{\text{molK}}}$$

$T_x = 282.6\,\text{K}$ ← lower than the original temperature of the gas: $T = 370\,\text{K}$. This is a way to cool gases. We just ran through half of the Carnot cycle in the refrigerator mode i.e. counter-clock-wise.

Efficiency of a heat engine

Hot reservoir

$Q_H = 92.5\,kJ$

$W = ?$ and $\eta = ?$

$Q_c = 81.8\,kJ$

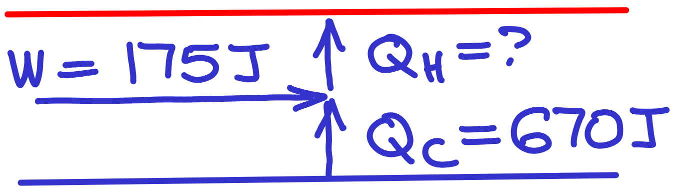
Cold reservoir

$$W = Q_H - Q_c = 92.5\,kJ - 81.8\,kJ = 10.7\,kJ$$

$$\text{Efficiency} = \frac{\text{useful work}}{\text{total heat absorbed}}$$

$$\eta = \frac{W}{Q_H} = \frac{10.7\,kJ}{92.5\,kJ} = 0.116 = 11.6\%$$

Ideal refrigerator



$$Q_H = Q_c + W = 670J + 175J = 845J$$

Coefficient of performance of a refrigerator :

$$K = \frac{\text{heat extracted from the cold compart.}}{\text{work invested}}$$

$$K = \frac{Q_c}{W} = \frac{670J}{175J} = 3.83$$

K is usually left in this form, it is not expressed in percents.