PHY215-01f: Special Relativity

C.-P. Yuan

September 9, 2025

1 Special Relativity

In Einstein's special relativity, the word "special" indicates that this theory applies to a specific, restricted set of reference frames—namely, inertial reference frames. These are frames where an object with no net force acting on it moves at a constant velocity (including being at rest).

The "special" here does not refer to a single "preferred" or absolute reference frame (unlike the old idea of a "luminiferous ether" as a universal frame). Instead, it means the theory is "specialized" to inertial frames, as opposed to general relativity (which later extended the theory to include non-inertial frames and gravity, making it "general" in scope).

2 Differences Between Special Relativity and General Relativity

Special relativity and general relativity, both formulated by Einstein, differ significantly in their scope, foundational principles, and the phenomena they describe. Below is a concise comparison:

2.1 Scope of Application

- **Special Relativity**: Applies exclusively to *inertial reference frames*—frames where objects move at a constant velocity (with no acceleration) and no net force acts upon them. It does not take gravity into account.
- **General Relativity**: Extends to *all reference frames*, including non-inertial (accelerating) ones. It incorporates gravity as a fundamental component of the theory.

2.2 Foundational Principles

- **Special Relativity** is based on two postulates:
  1. The laws of physics are identical in all inertial frames (relativity of inertial motion).
  2. The speed of light in a vacuum is constant for all observers, irrespective of their motion relative to the light source.

- **General Relativity** is built on the *equivalence principle*: The effects of gravity are indistinguishable from the effects of acceleration in a straight line (e.g., the "weight" felt in an accelerating elevator is equivalent to the force of gravity).

  The equivalence of inertial mass (the mass that appears in Newton's second law, $F = ma$) and gravitational mass (the mass that appears in Newton's law of gravitation, $F = G_N * (m * M)/r^2 = mg$) is a cornerstone of general relativity. This equivalence means that the way an object resists acceleration (inertia) is the same as the way it responds to gravitational forces. That's why, for instance, all objects fall at the same rate in a gravitational field when other factors like air resistance are negligible.

2.3 Key Phenomena Explained

- **Special Relativity** explains: Time dilation, length contraction, relativistic mass-energy equivalence ($E = mc^2$), and the relativity of simultaneity.

- **General Relativity** explains: Gravitational time dilation, the bending of light by gravity (gravitational lensing), the expansion of the universe, black holes, and planetary orbits (e.g., correcting Mercury's orbital precession, which Newtonian physics could not fully account for).

In summary, special relativity is a "special case" (restricted to inertial frames and excluding gravity), while general relativity is a "general theory" (applicable to all frames and including gravity as a curvature of spacetime).

3 Some remarks about General Relativity

3.1 GPS satellite clocks run faster than Earth clocks

The GPS abbreviation stands for Global Positioning System. It refers to the U.S.-owned utility and navigation system that uses signals from satellites to provide precise location, navigation, and timing services to receivers on Earth

GPS relies on a network of satellites that orbit the Earth and transmit signals to receivers on the ground. These signals contain the time the message was sent and the satellite's position. The receiver calculates its position by determining how long it took for the signals to arrive from multiple satellites.

Based on Eistein's Special Relativity and General Relativity theories, GPS (and similar) satellite clocks run *faster* than clocks on Earth by approximately

$$\Delta t \approx 38\ \mu\text{s per day},$$

.

Breakdown of the effect

Two relativistic contributions determine the rate difference:

- **Gravitational time dilation:** At orbital altitude, the gravitational potential is weaker, so clocks run *faster*. This effect contributes about

$$+45\ \mu\text{s/day}.$$

- **Special-relativistic time dilation:** Because satellites are moving relative to Earth's surface, their clocks run *slower*, by about

$$-7\ \mu\text{s/day}.$$

Adding the two effects gives

$$+45\ \mu\text{s/day} - 7\ \mu\text{s/day} \approx +38\ \mu\text{s/day}.$$

Consequences for time and distance

After one Earth day, a satellite clock is ahead by about 38 μs compared to an identical Earth clock. Since light travels at

$$c \approx 2.9979 \times 10^8\,\text{m/s},$$

the corresponding range error is

$$\Delta L = c\,\Delta t = (2.9979 \times 10^8\ \text{m/s})(38 \times 10^{-6}\ \text{s}) \approx 1.14 \times 10^4\ \text{m},$$

that is, about 11.4 km per day.

Equivalently, this offset corresponds to a fractional frequency difference of about

$$\frac{\Delta f}{f} \sim 4.4 \times 10^{-10},$$

meaning that the satellite clock ticks faster by $\sim$ 0.44 parts per billion.

Practical note

If left uncorrected, this $\sim$ 11 km/day error would ruin positioning accuracy. In practice, GPS and other GNSS (Global Navigation Satellite System) systems *pre-correct* satellite clock rates and apply relativistic corrections in their algorithms so that users observe synchronized timing.