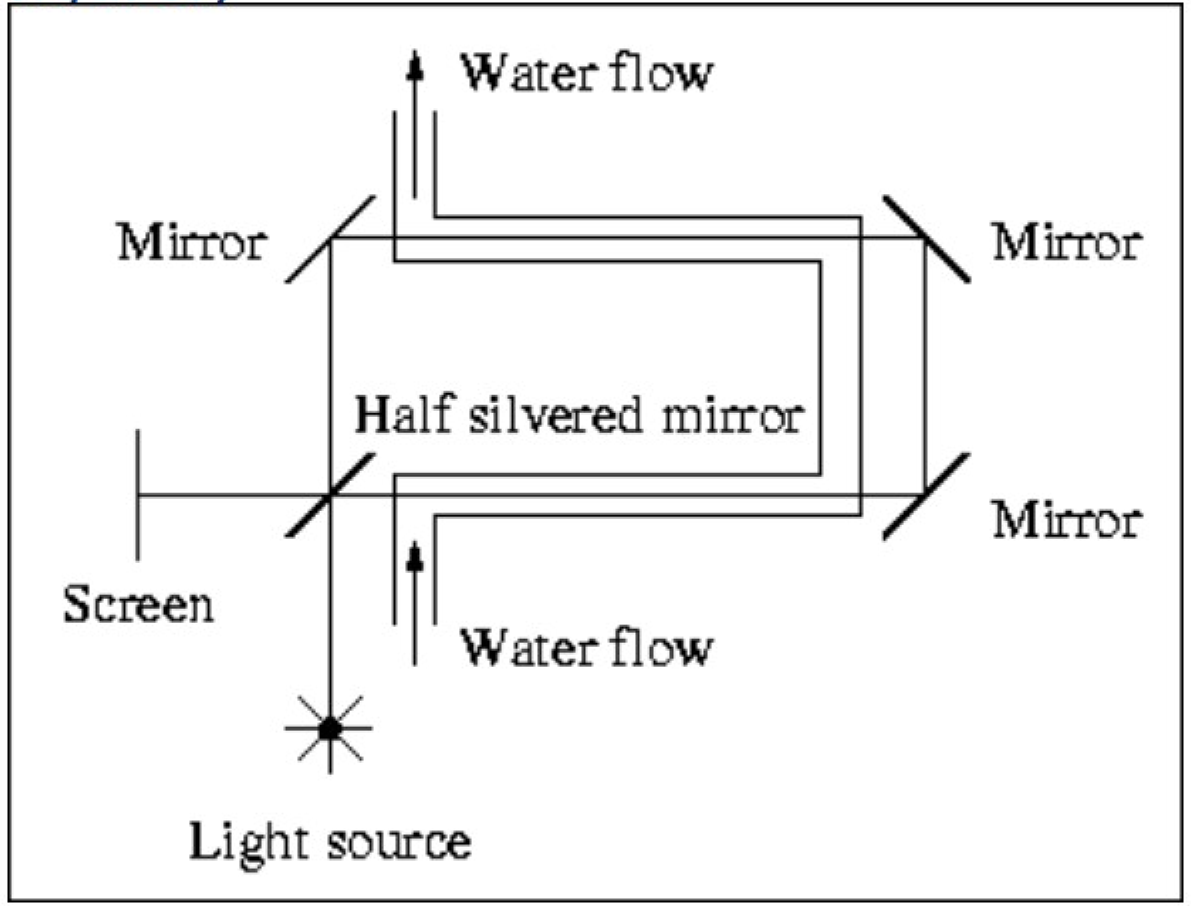
Hippolyte L. Fizeau in 1851 in France found experimentally that classical Galilean velocity addition is not valid for light traveling through flowing water. In his experiment a light beam is split into two beams by a half silvered mirror. (See figure.)



One of the light beams travels in the water with the flow, the other beam travels against the flow. The two beams are then united and they form an interference pattern on the observation screen. When the water is set into motion, the fringes in the interference pattern shift with respect to the case when the water is at rest. From this fringe shift the difference in light speed in the two paths can be determined very accurately. In this experiment the water flows at a speed of 2.90 m/s. The index of refraction for water is 1.33.

Fizeau's experiment

Light travels with a speed of $\frac{c}{n}$ in a material with an index of refraction n. For water $n = 1.33$.

Therefore the speed of light in water is $c_w = \frac{c}{n} = 2.26 \cdot 10^8\ m/s$ which is still a very high speed, therefore we will need relativistic physics, to deal with it.

The water in Fizeau's interferometer flows with a speed of $v = 1\text{-}3\ \frac{m}{s}$.

How should we add and subtract c_w and v: classically or relativistically?

Classical method:

$$\left.\begin{aligned} u_+ &= \frac{c}{n} + v \\ u_- &= \frac{c}{n} - v \end{aligned}\right\} \Delta u_{cl} = u_+ - u_- = 2v$$

Relativistic method:

$$\left.\begin{aligned} u_+ &= \frac{c}{n} + \left(1 - \frac{1}{n^2}\right)v \\ u_- &= \frac{c}{n} - \left(1 - \frac{1}{n^2}\right)v \end{aligned}\right\} \begin{aligned} \Delta u_{rel} &= u_+ - u_- = \\ &= 2\left(1 - \frac{1}{n^2}\right)v \end{aligned}$$

Fizeau's experiment 2.

Where did we get the $\left(1-\frac{1}{n^2}\right)$ term?

Relativistic addition:

$$u_+ = \frac{\frac{c}{n}+v}{1+\frac{\frac{c}{n}v}{c^2}} = \frac{\frac{c}{n}+v}{1+\frac{1}{n}\cdot\frac{v}{c}} = \frac{c}{n}\cdot\frac{1+\frac{nv}{c}}{1+\frac{v}{nc}}$$

Since $v \ll c$: $\frac{1}{1+\frac{v}{nc}} \cong 1-\frac{v}{nc}$

Therefore: $u_+ \cong \frac{c}{n}\cdot\left(1+\frac{nv}{c}\right)\left(1-\frac{v}{nc}\right) =$

$$= \frac{c}{n}\left(1+\frac{nv}{c}-\frac{v}{nc}-\underbrace{\frac{v^2}{c^2}}_{\text{very small}}\right) \cong$$

$$\cong \frac{c}{n}\left(1+\left(n-\frac{1}{n}\right)\frac{v}{c}\right) =$$

$$= \frac{c}{n}+\left(1-\frac{1}{n^2}\right)v$$

Similarly: $u_- = \frac{c}{n}-\left(1-\frac{1}{n^2}\right)v$

Therefore $\Delta u = u_+ - u_- = 2\left(1-\frac{1}{n^2}\right)v$

The $\left(1-\frac{1}{n^2}\right)$ term is called Fresnel's drag coefficient. For water it is $\left(1-\frac{1}{1.33^2}\right) = 0.435$. Fizeau's experiment confirmed this value.