PHY215-02f: Speed of Light in a medium

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1 Why does light appear to slow down in a medium like glass?

In vacuum, photons always move at the universal constant

$$c \approx 3.0 \times 10^8 \,\mathrm{m/s}.$$

This is true everywhere: massless photons never travel slower than c.

1.1 Propagation in a medium

When light enters a medium such as glass, it does not simply pass through unaffected. Instead, the electromagnetic field of the photon interacts with the electrons bound in the atoms of the medium. These electrons are driven into oscillation, and in turn they re-radiate electromagnetic waves.

The observed wave inside the medium is therefore a *superposition* of the original incident wave and the secondary waves emitted by the oscillating charges. This interference produces a collective wave that appears to propagate with a reduced velocity—the so-called *phase velocity*,

$$v_{\mathrm{phase}} = \frac{c}{n},$$

where n is the refractive index of the medium. For typical glass, $n \approx 1.5$, giving $v_{\mathrm{phase}} \approx 2 \times 10^8 \,\mathrm{m/s}$.

1.2 Do photons really slow down?

Between interactions, each photon always travels at the fundamental speed of light c. The apparent reduction in speed arises because repeated interactions with the medium introduce delays, so that the net progress of the light wavefront is slower. Thus, the "slowing down" refers to the effective propagation of the electromagnetic wave, not to an actual reduction in the photon's velocity.

1.3 Group velocity and causality

The speed at which a wave packet or signal propagates is given by the *group velocity*, which is generally less than c in a medium. Meanwhile, the *phase velocity* v_{phase} (the speed of the wave crests) is reduced by the refractive index n when $n > 1$. Importantly, the causal limit—the maximum speed at which information can travel—remains bounded by c.

1.4 Summary

Light appears to slow down in glass because its electric field excites electrons in the medium, which then re-emit radiation. The interference between the incident and re-emitted waves produces a collective wave that propagates more slowly. However, any individual photon always travels at c between interactions; it is the *effective* progress of the light signal that is reduced.

2 X-rays in Matter and the Frequency Dependence of the Refractive Index

2.1 Phase velocity of X-rays in glass

For visible light in glass, the refractive index satisfies $n > 1$, so the phase velocity is reduced according to

$$v_{\text{phase}} = \frac{c}{n} < c.$$

However, for X-rays in matter (including glass), the situation is different. At such high frequencies, the refractive index is slightly less than unity:

$$n \approx 1 - \delta, \qquad \delta \sim 10^{-6} - 10^{-5}.$$

Consequently,

$$v_{\text{phase}} = \frac{c}{n} > c.$$

This superluminal phase velocity does not violate special relativity, because no information or energy is transmitted at the phase velocity. The relevant speed for signal propagation is the *group velocity*, which always remains less than or equal to c.

2.2 Origin of the frequency dependence of $n(\omega)$

The refractive index arises from how bound electrons in the medium respond to the oscillating electric field of the light wave. A simple description is provided by the Lorentz oscillator model. An electron of mass m and charge $-e$ bound with natural frequency ω_0 satisfies

$$m\ddot{x} + m\gamma\dot{x} + m\omega_0^2 x = -eE_0 e^{-i\omega t},$$

where γ represents damping and $E_0 e^{-i\omega t}$ is the driving field. Solving this equation gives the induced displacement $x(\omega)$, and hence a polarization $P(\omega)$ proportional to the electric field.

The polarization determines the dielectric function:

$$\varepsilon(\omega) = 1 + \frac{Ne^2}{\varepsilon_0 m} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega},$$

where N is the electron density. The refractive index then follows as

$$n(\omega) = \sqrt{\varepsilon(\omega)}.$$

2.3 Frequency regimes

- **Visible light:** For frequencies below strong electronic resonances, the denominator $(\omega_0^2 - \omega^2)$ is positive, yielding $n > 1$. This is the familiar regime where visible light appears to "slow down" in glass.

- **X-rays:** For X-ray frequencies, $\omega \gg \omega_0$, so the electrons cannot follow the field oscillations. In this limit,

  $$\varepsilon(\omega) \approx 1 - \frac{\omega_p^2}{\omega^2},$$

  where ω_p is the plasma frequency of the medium. Thus,

  $$n(\omega) \approx 1 - \frac{\omega_p^2}{2\omega^2} < 1.$$

  Hence, the phase velocity of X-rays in matter exceeds c.

2.4 Summary

The refractive index is frequency dependent because bound electrons in the medium respond differently depending on the driving frequency of the light. For visible light, $n > 1$ and light slows down in glass. It also explains why different colors of light bend differently when passing through a prism: higher-frequency light (like blue) interacts more strongly with the medium and is therefore slowed down to a greater extent than lower-frequency light (like red). However, any individual photon always travels at the speed of light c between interactions; it's the "effective progress" of the light signal that's reduced.

For X-rays, $n < 1$, giving a phase velocity larger than c, but without any violation of relativistic causality.