the Fermi-Dirac and Bose-Einstein distributions reduce to the classical Maxwell-Boltzmann distribution when $B_i \exp(\beta E)$ is much greater than 1* (in that case the normalization constant $A = 1/B_i$). This means that the Maxwell-Boltzmann factor $A \exp(-\beta E)$ is much less than 1 (that is, the probability that a particular energy state will be occupied is much less than 1). This is consistent with our earlier use of Maxwell-Boltzmann statistics for a dilute, noninteracting system of particles. See Table 9.2 for a summary of the properties of the three distribution functions.

A comparison of the three distribution functions is shown in Figure 9.8, with each one graphed as a function of energy. The normalization constants for the distributions (A for the Maxwell-Boltzmann, B_{FD} for the Fermi-Dirac, and B_{BE} for the Bose-Einstein) depend on the physical system being considered. For convenience, we set them all equal to 1 for this comparison. Notice that the Bose-Einstein factor F_{BE} is higher than the Fermi-Dirac factor F_{FD} at any given energy. Mathematically, this is due to the difference between +1 and −1 in the denominators of the two functions. Physically, the higher value of F_{BE} results from the fact that bosons do not obey the Pauli exclusion principle, so more bosons are allowed to fill lower energy states. Another thing to notice in Figure 9.8 is that the three graphs coincide at high energies—the classical limit. That is why Maxwell-Boltzmann statistics may be used in the classical limit, regardless of whether the particles in the system are fermions or bosons.

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Austrian physicist **Ludwig Boltzmann** (1844–1906) worked independently of Maxwell on developing the laws governing the statistical behavior of classical particles. Boltzmann is remembered most for his work on the statistical nature of entropy, and he supported the notion that the equipartition theorem is a fundamental part of statistical physics and thermodynamics. On Boltzmann's Vienna tombstone is carved his famous formula for entropy: $S = k \log W$, where W is the number of possible ways a state can be configured and k is the constant that was named in Boltzmann's honor.

*This happens at high temperatures and low densities. A good rule of thumb is to compare the interparticle spacing with the average de Broglie wavelength. If the interparticle spacing is much greater than the de Broglie wavelength, then Maxwell-Boltzmann statistics are fairly accurate. Otherwise one should use the quantum statistics.

Table 9.2 Classical and Quantum Distributions

Distributors	Properties of the Distribution	Examples	Distribution Function
Maxwell-Boltzmann	Particles are identical but distinguishable	Ideal gases	$F_{MB} = A\exp(-\beta E)$
Bose-Einstein	Particles are identical and indistinguishable with integer spin	Liquid ^{4}He, photons	$F_{BE} = \dfrac{1}{B_{BE}\exp(\beta E) - 1}$
Fermi-Dirac	Particles are identical and indistinguishable with half-integer spin	Electron gas (free electrons in a conductor)	$F_{FD} = \dfrac{1}{B_{FD}\exp(\beta E) + 1}$

b = 1/ kT

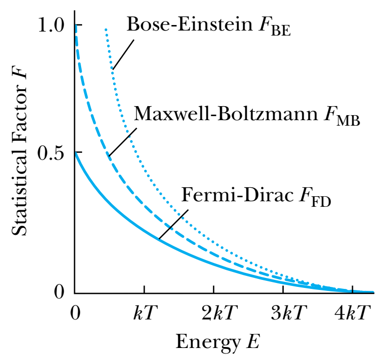

Figure 9.8 A comparison of the three distribution functions, each drawn as a function of energy over the same range. The normalization constants A, B_{FD}, and B_{BE} have been set equal to 1 for convenience. The Bose-Einstein distribution is higher than the Fermi-Dirac distribution, because bosons do not obey the Pauli principle. At high energies, the three distributions are close enough so that the classical Maxwell-Boltzmann distribution can be used to replace either quantum distribution.