Ideal Gas Law

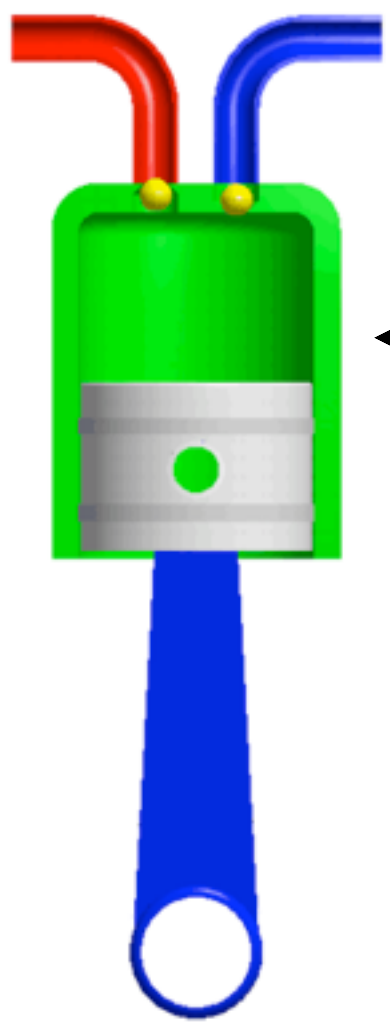
PHY 215
Thermodynamics and Modern Physics

Fall 2025
MSU

Outline:
The Ideal Gas Law

- Systems of interest: Gases
- Ideal Gas Laws
- Kinetic Theory of Gases
- Equipartition
- Molar Specific Heat of Gases

Systems of Interest



“Working Fluid”:
Gas

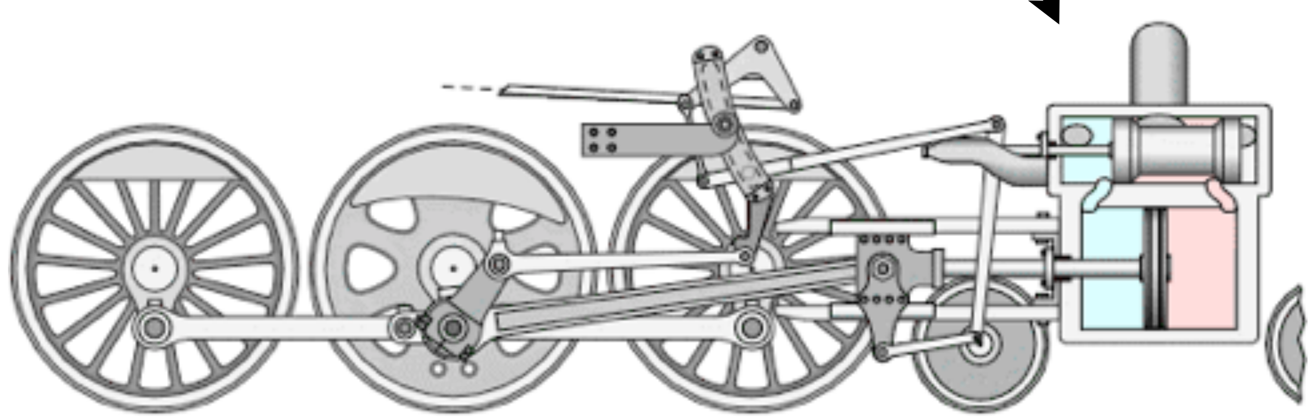
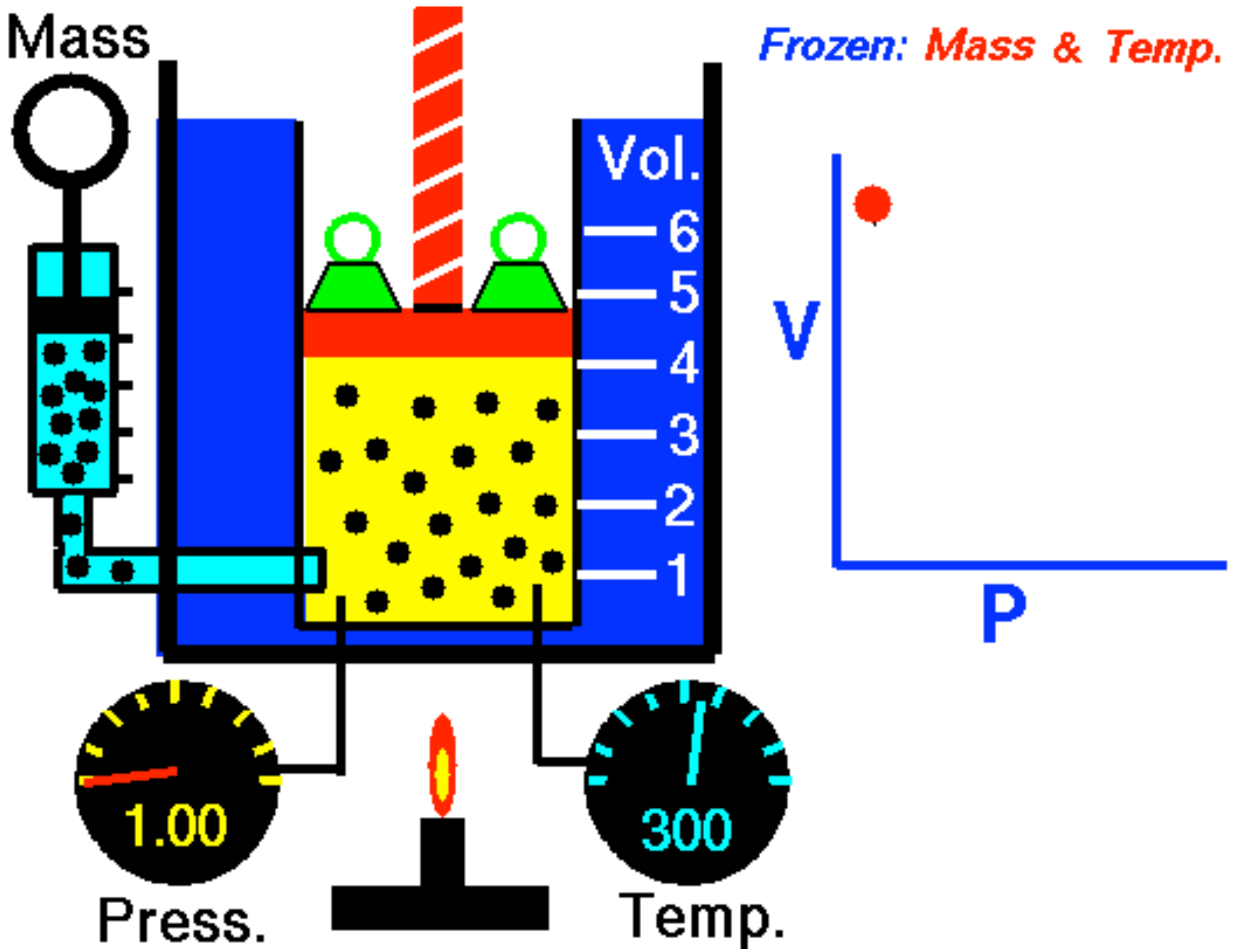


Image: http://en.wikipedia.org

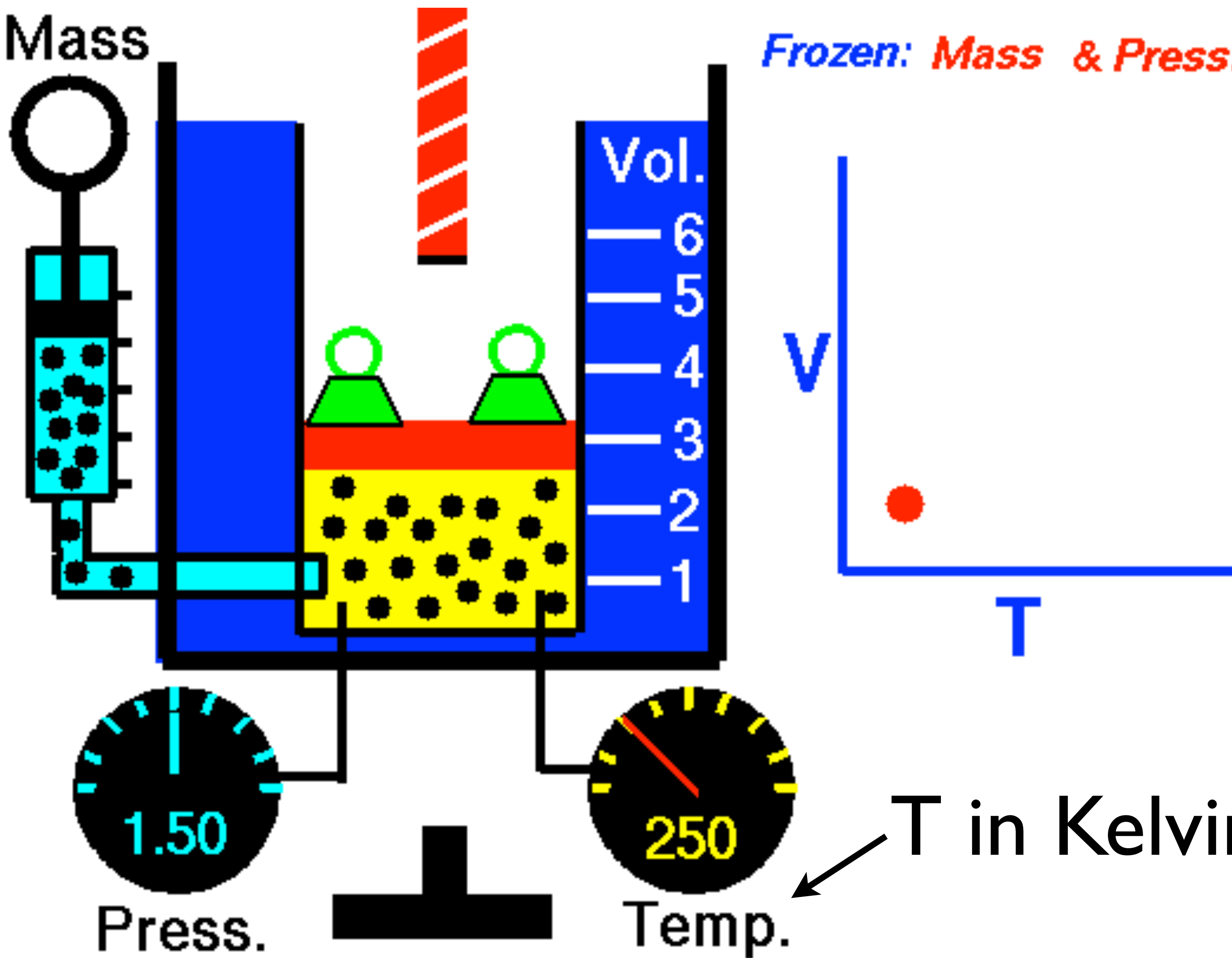
Boyle's Law



pV=const, at fixed T and n

Image: http://en.wikipedia.org

Charles's Law



T in Kelvin!

$P \propto T$ at fixed V
Guy-Lussac's Law

$V \propto T$, at fixed P and n

Image: http://en.wikipedia.org

Combined

THE IDEAL GAS LAW

$$PV = nRT$$ ← "Equation of State"

where n is the number of moles of gas and R is the gas constant,

$$R = 8.31\ \mathrm{J/(mol \cdot K)}$$

One mole of an ideal gas at standard temperature (0 °C) and pressure (1 atm) ("STP") occupies approx. 22.4 liters.

$$[p] = 1\,\mathrm{Pa} = 1\,\mathrm{N/m^2}$$
$$[V] = 1\,\mathrm{m^3} = 10^6\,\mathrm{l}$$
$$[T] = ^\circ\,\mathrm{K}$$
$$[n] = \mathrm{moles}$$

Molecular Version of Ideal Gas Law

Alternate form:

$$PV = NkT$$

where N is the number of molecules in the gas

and k is the Boltzmann's constant,

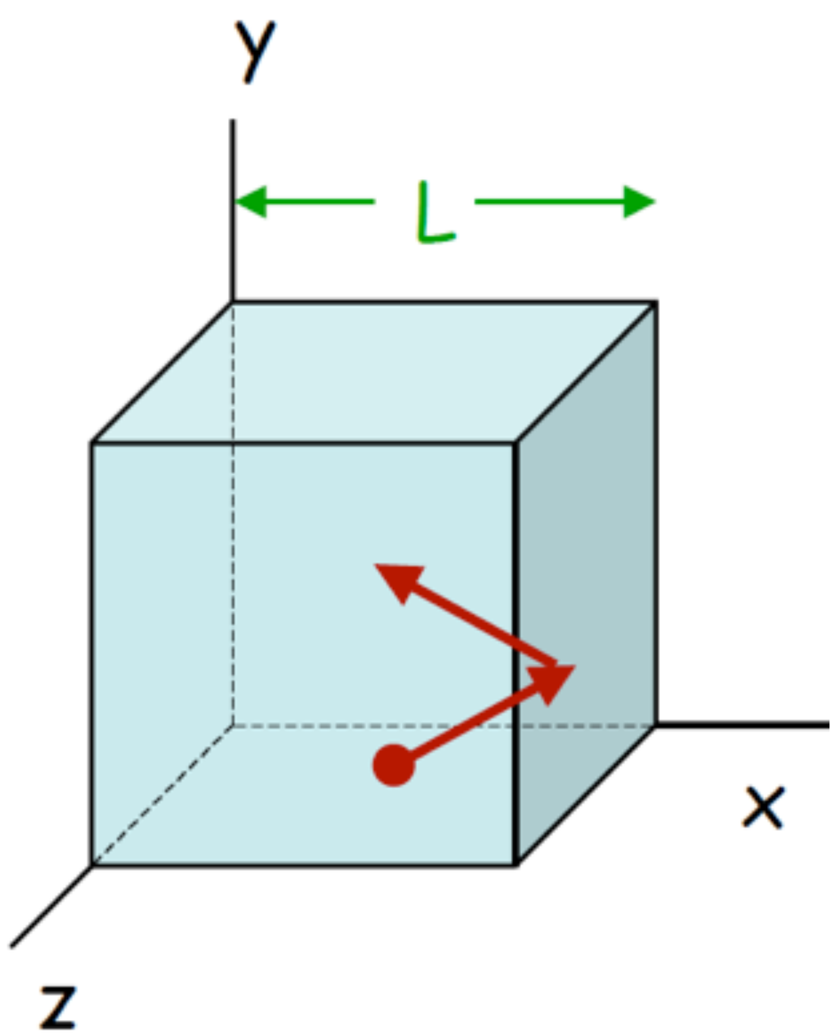
$$k = 1.38\times10^{-23}\ J/K$$

N_A=Avagadro's Number
$=6.022 \times 10^{23}$

(Comparing the two forms gives $R=N_A k$.)

Kinetic Theory of Gases

Consider N molecules (n moles with $n = N/N_A$) in a cubical box of side L, i.e. Volume = L^3.



Change in momentum at the x-wall is

$\Delta p_x = 2\ m\ v_x$

Time between collisions with the x-wall is

$\Delta t = 2\ L\ /\ v_x$

Origin of Pressure

Average rate of change of momentum in x-direction:

$\Delta p_x/\Delta t = (2mv_x)/(2L/v_x) = m\, v_x^2/ L$

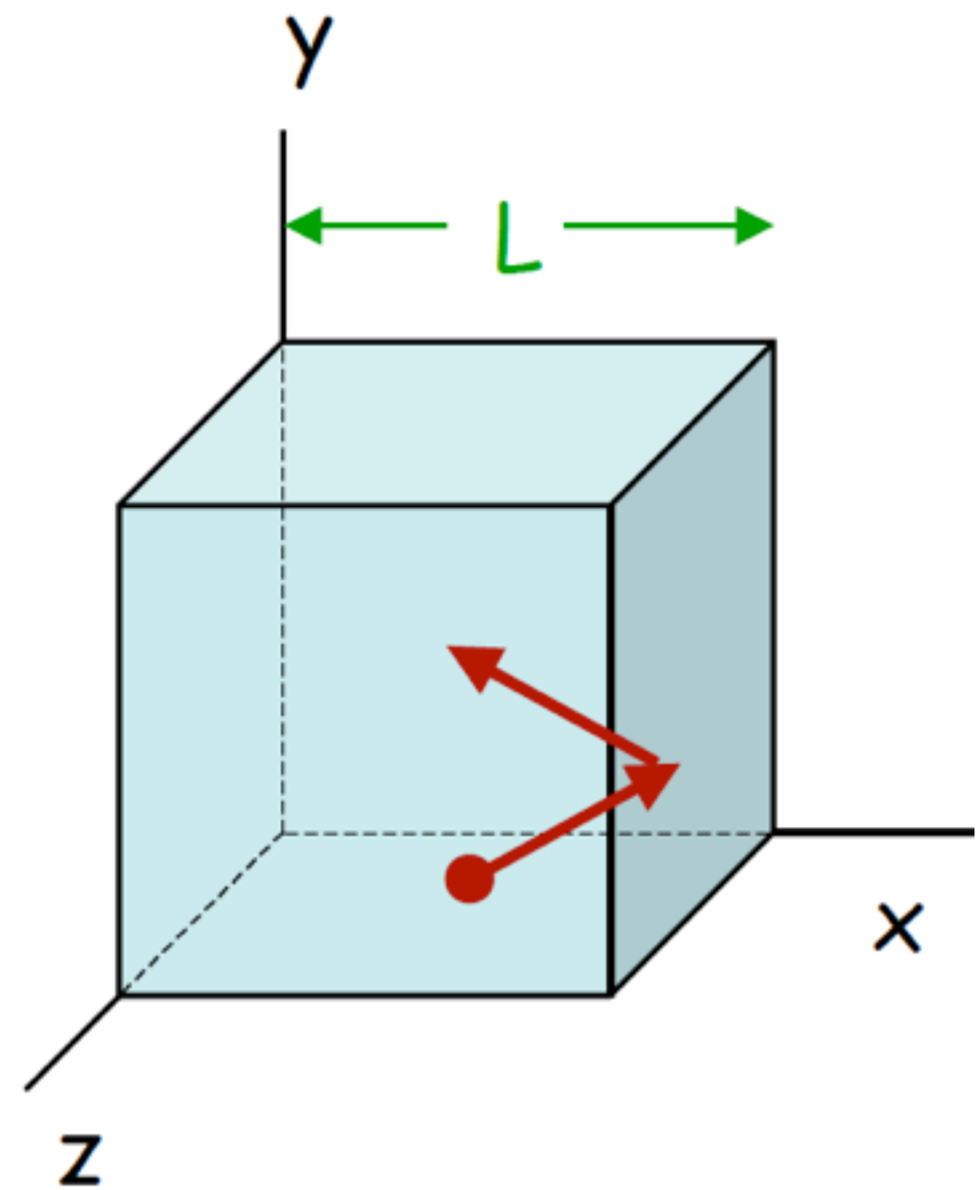
This is force exerted by the molecule.

Total Force = $\sum_{i=1}^{N} (m\, v_x^2)_i \,/\, L$

Pressure P = Force/Area = F/L^2

$= (m/L^3) \sum (v_x^2)_i$

$\Rightarrow \quad P = (m/L^3)\, N \langle v_x^2 \rangle$ ← average



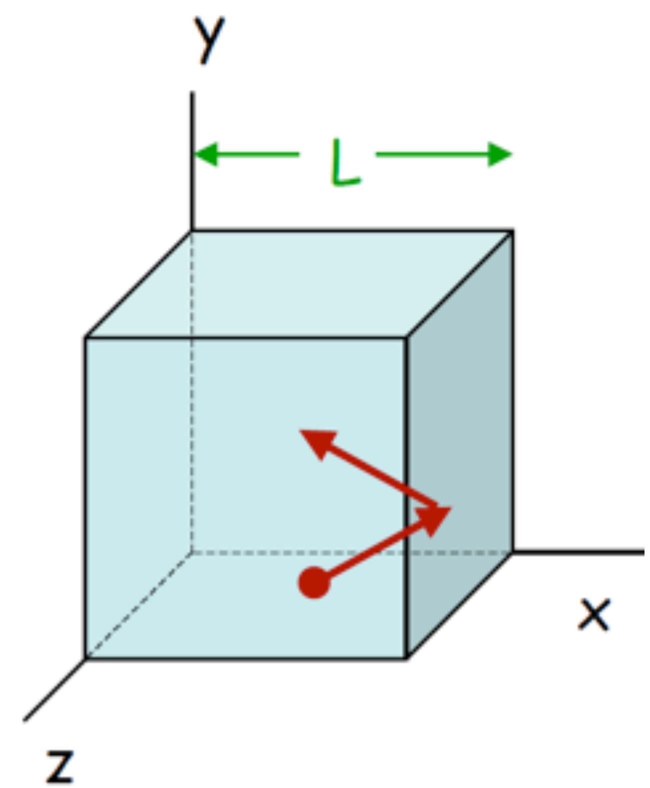
mN = nM is the total mass.

where n = # of moles
M = molar mass

$$P = (nM/V)\,\langle v_x^2 \rangle$$

Relation to Gas Law

For any molecule: $v^2 = v_x^2 + v_y^2 + v_z^2$



$\Rightarrow \langle v_x^2 \rangle = (1/3) \langle v^2 \rangle$

$\Rightarrow P = (nM/3V) \langle v^2 \rangle$

Define root-mean-square speed v_{rms}:

$$v_{rms} = \sqrt{\langle v^2 \rangle}$$

$\Rightarrow PV = (nM/3)\, v^2_{rms}$

From ideal gas law: $PV = nRT$

$\Rightarrow (nM/3)\, v^2_{rms} = nRT$

$$\Rightarrow \boxed{v_{rms} = \sqrt{3RT/M}}$$

Internal Kinetic Energy

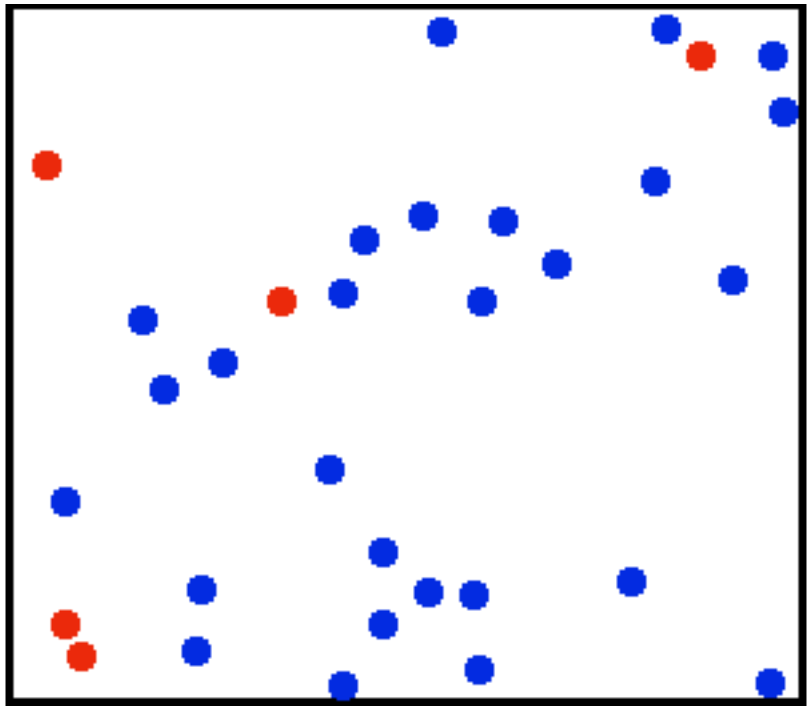
Average (translational) kinetic energy per molecule

$= (1/2)\, m < v^2 > \;=\; (1/2)\, m\, (3RT/M)$

Using $M/m = N_A$,

$< K > \;=\; 3RT/(2N_A) \;=\; (3/2)\, k\, T$

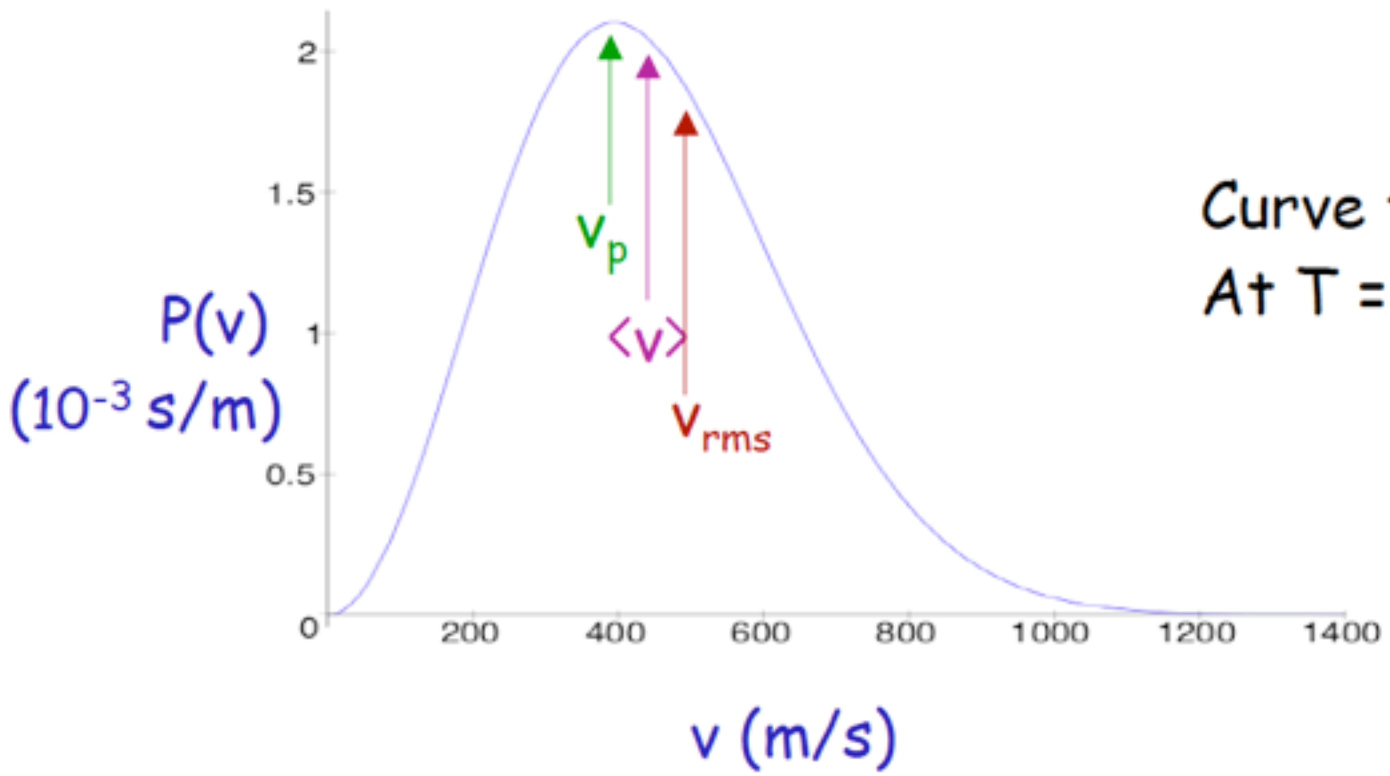
$\Rightarrow \boxed{< K > \;=\; (3/2)\, k\, T}$ ← (1/2)kT "per degree of freedom" (v_x, v_y, v_z)



Temperature is a measure of the average kinetic energy of gas molecules!

Image: http://en.wikipedia.org

Maxwell Distribution



Curve for O_2
At T = 300 K

$e^{-E_{kin}/kT}$

$$P(v) = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-\frac{Mv^2}{2RT}}$$

P(v) dv is the probability that a molecule has speed between v and v + dv.

Internal Energy: U

- Monatomic gas - Single atoms:

$U = N\,(3/2)\,kT = (3/2)\,nN_AkT = (3/2)\,nRT$

Each atom has 3 Degrees of Freedom.
(K. E. in x, y, or z directions).

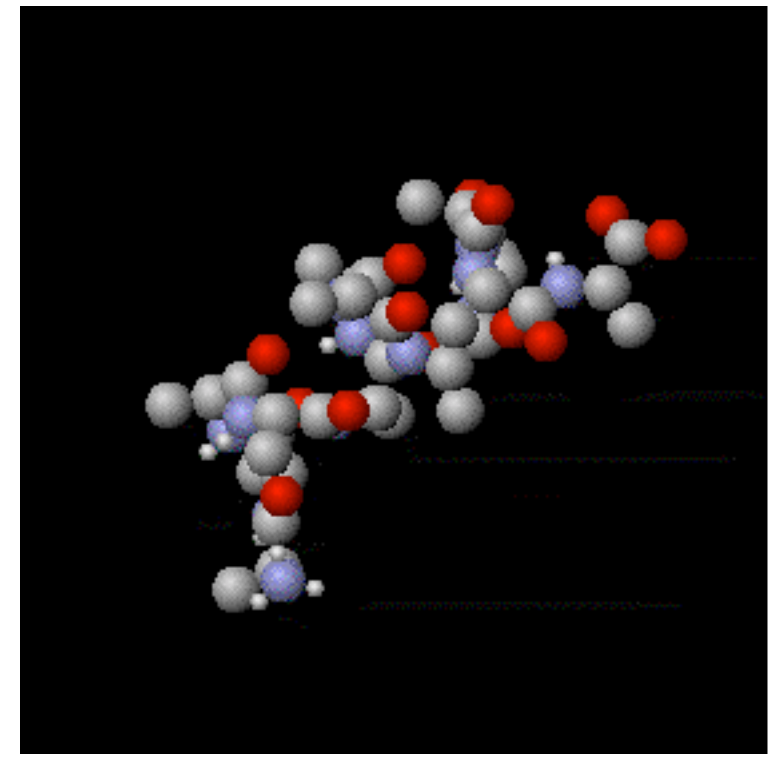
- Diatomic molecule:

  Rotates (in two planes)
  $\Rightarrow$ 5 degrees of freedom.

  $U = (5/2)\,nRT$

- Polyatomic molecule:
  Rotates in all 3 planes
  $\Rightarrow$ 6 degrees of freedom.
  (3 translational + 3 rotational).

  $U = (6/2)\,nRT = 3\,nRT$



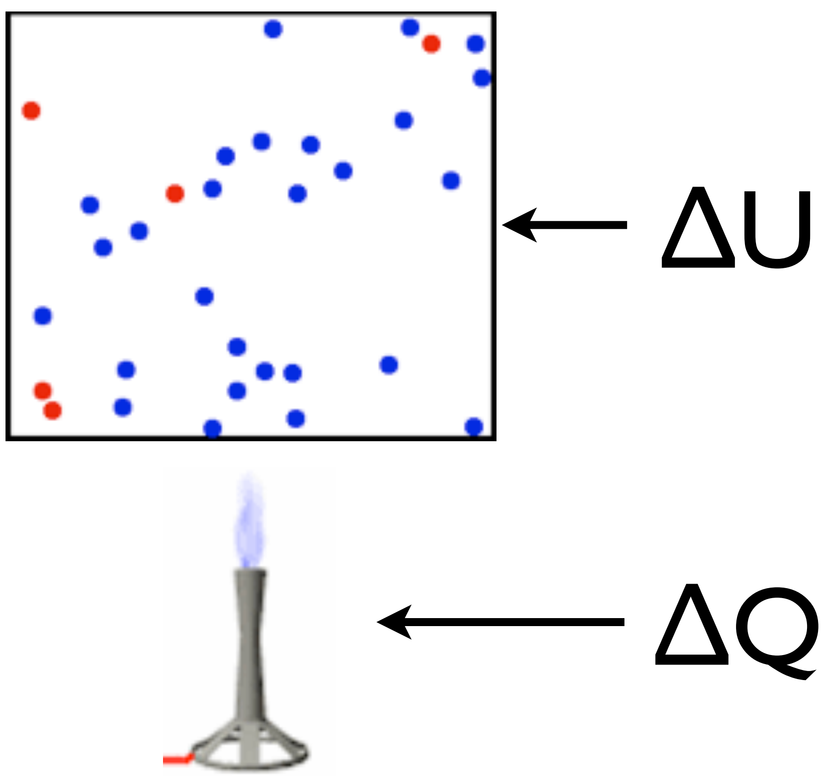
Vibrational degrees of freedom!

Ideal Gas: U is a Function of T only!

Image: http://en.wikipedia.org

Molar Specific Heat

at constant volume



<u>Monatomic Gas</u>

$$\Delta Q = \Delta U = (3/2)\, nR\Delta T$$

$$\Delta Q = n\, C_V \Delta T$$

molar specific heat

$$C_V = (3/2)\, R = 12.5 \text{ J/(mol} \cdot \text{K)}$$

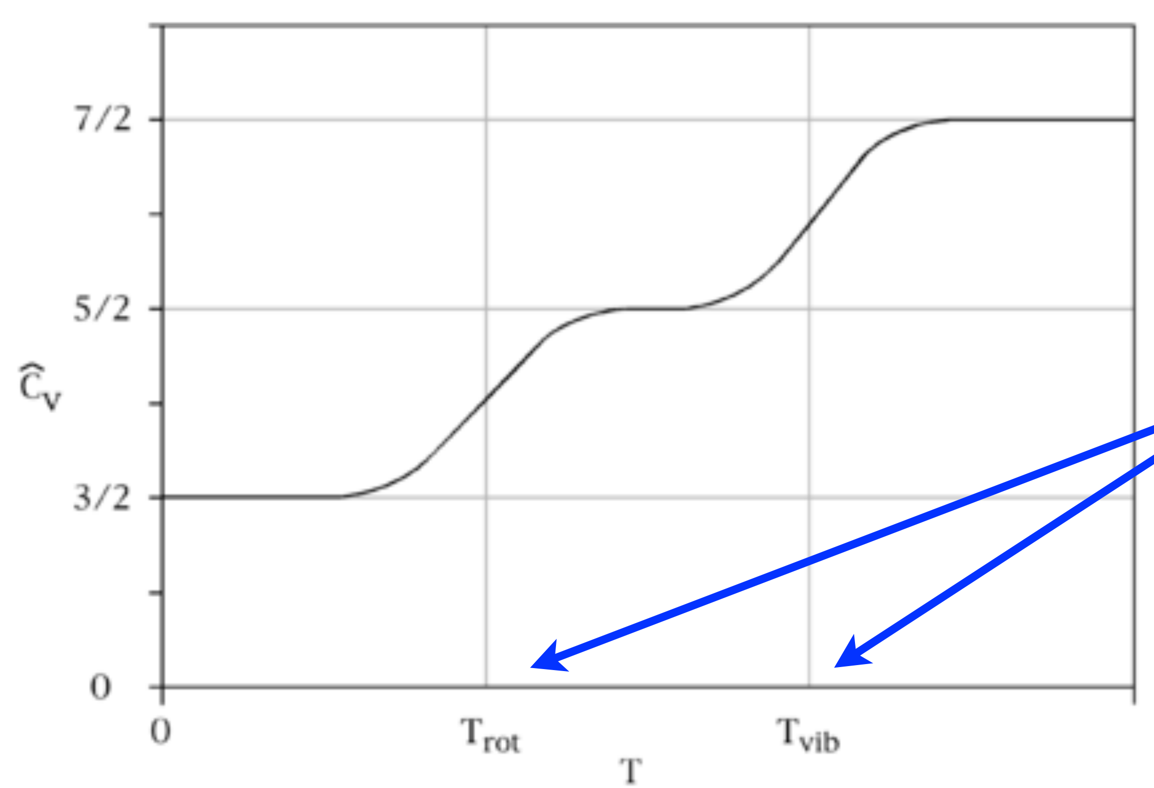
Images: http://en.wikipedia.org, http://www.sciencetime.co.uk

Equipartition

General Case: C_V=(# degrees of freedom/2)R

E=(# degrees of freedom)(kT/2)

Diatomic Molecule



Quantum Mechanics:
Energy Levels Quantized.
Different modes “turn on” at different temperature.

Summary

- Equation of state for a gas: the ideal gas law.
- Kinetic Theory of Gases: pressure and temperature are manifestations of the kinetic motion of gas molecules.
- Maxwell velocity distribution
- Internal energy of a gas: translational, rotational, and vibrational.
- (Molar) specific heat (C_V): (1/2)R per "active" degree of freedom (dof).
- Internal Energy U: (1/2) nRT per dof.