Thermodynamics

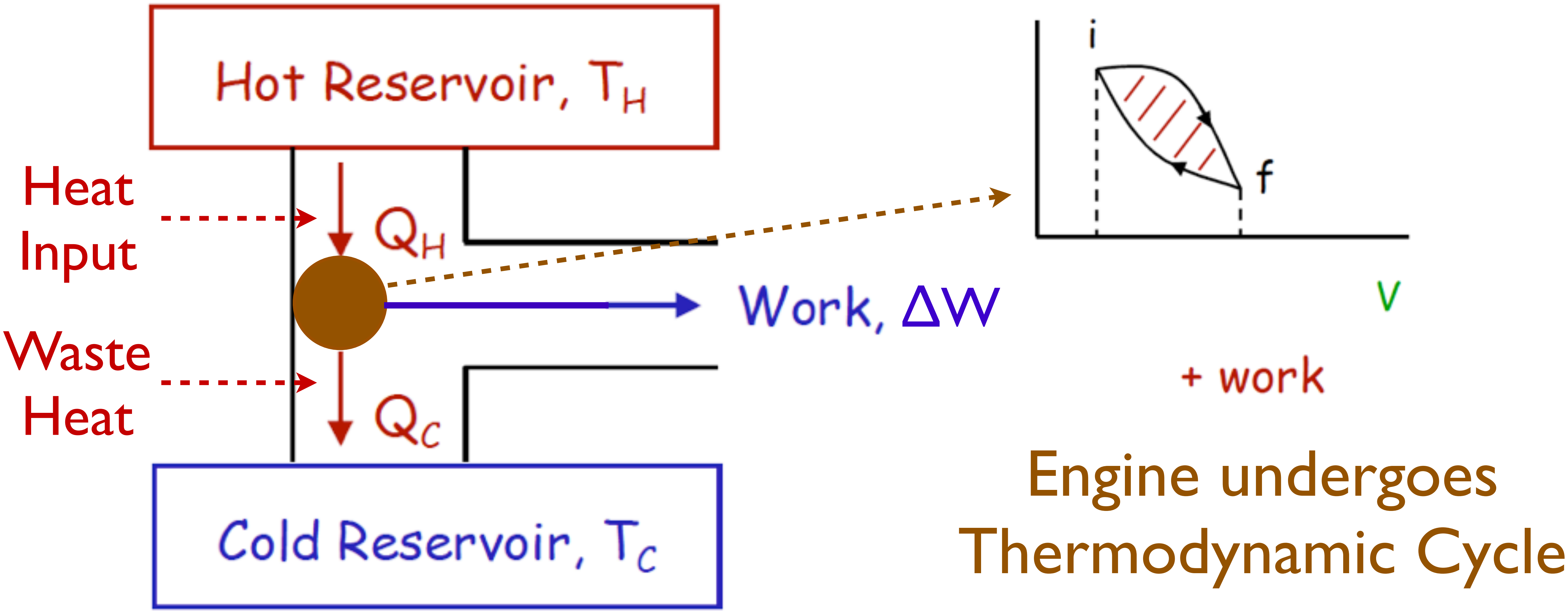
PHY 215
Thermodynamics and Modern Physics

Fall 2025
MSU

Outline

- Heat Engines
- Analyze a Cycle: compute η
- 2^{nd} Law in Kelvin Form
- The Carnot Cycle
    - Carnot Efficiency
- 2^{nd} Law in Carnot Form
- The Otto Cycle

Heat Engine: Heat to Work

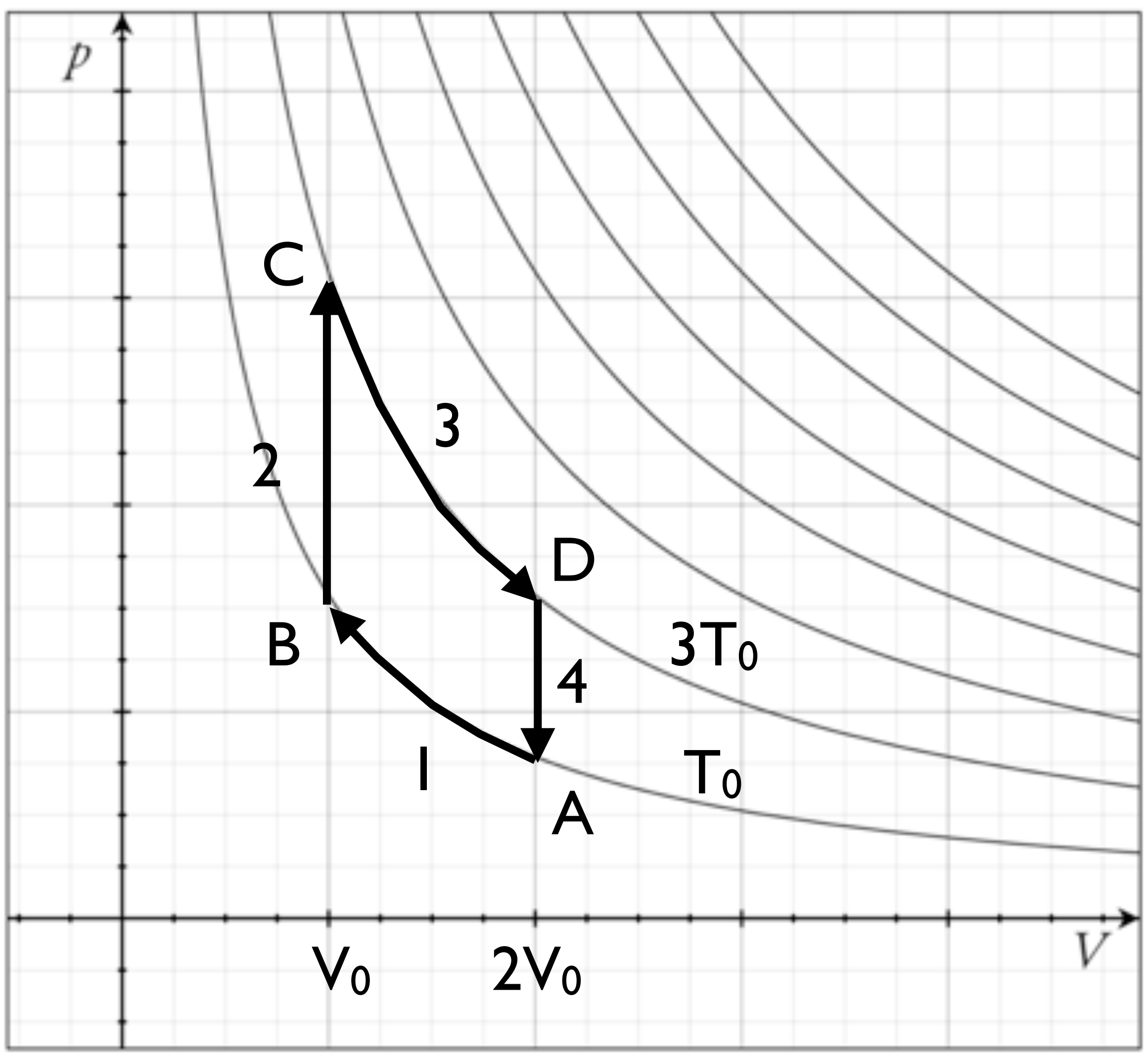


Efficiency=Work done/Heat Input

$$\eta = \frac{\Delta W}{Q_H}$$

Concept Test

An ideal gas undergoes the 4 step cycle shown, which is a combination of isochoric and isothermal processes.



The work done by the gas is:

A. Positive ←

B. Zero

C. Negative

The net change in internal energy is:

A. Positive

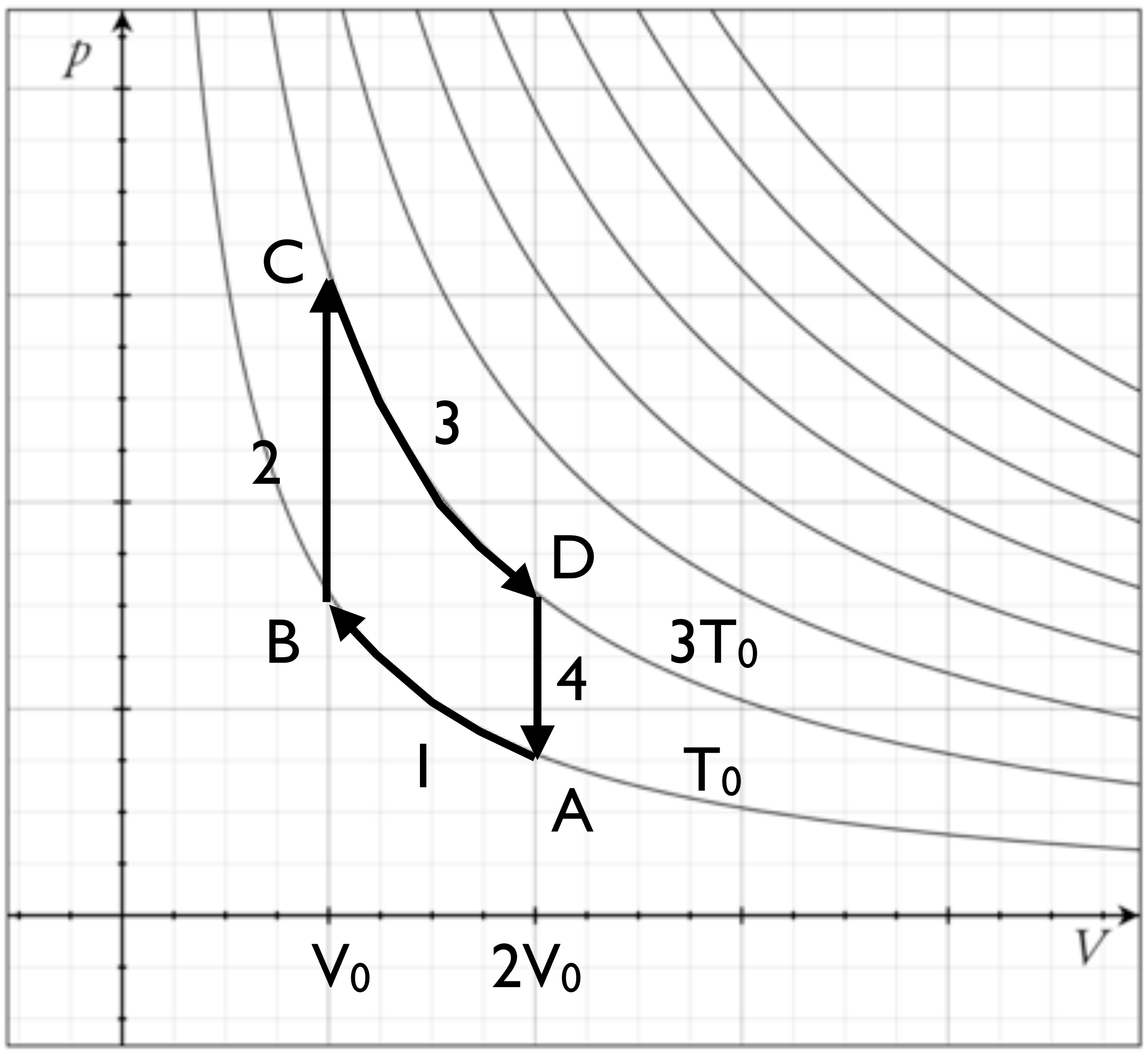
B. Zero ←

C. Negative

Image: http://en.wikipedia.org

Analyze Simple Cycle I

An ideal gas undergoes the 4 step cycle shown, which is a combination of isochoric and isothermal processes.

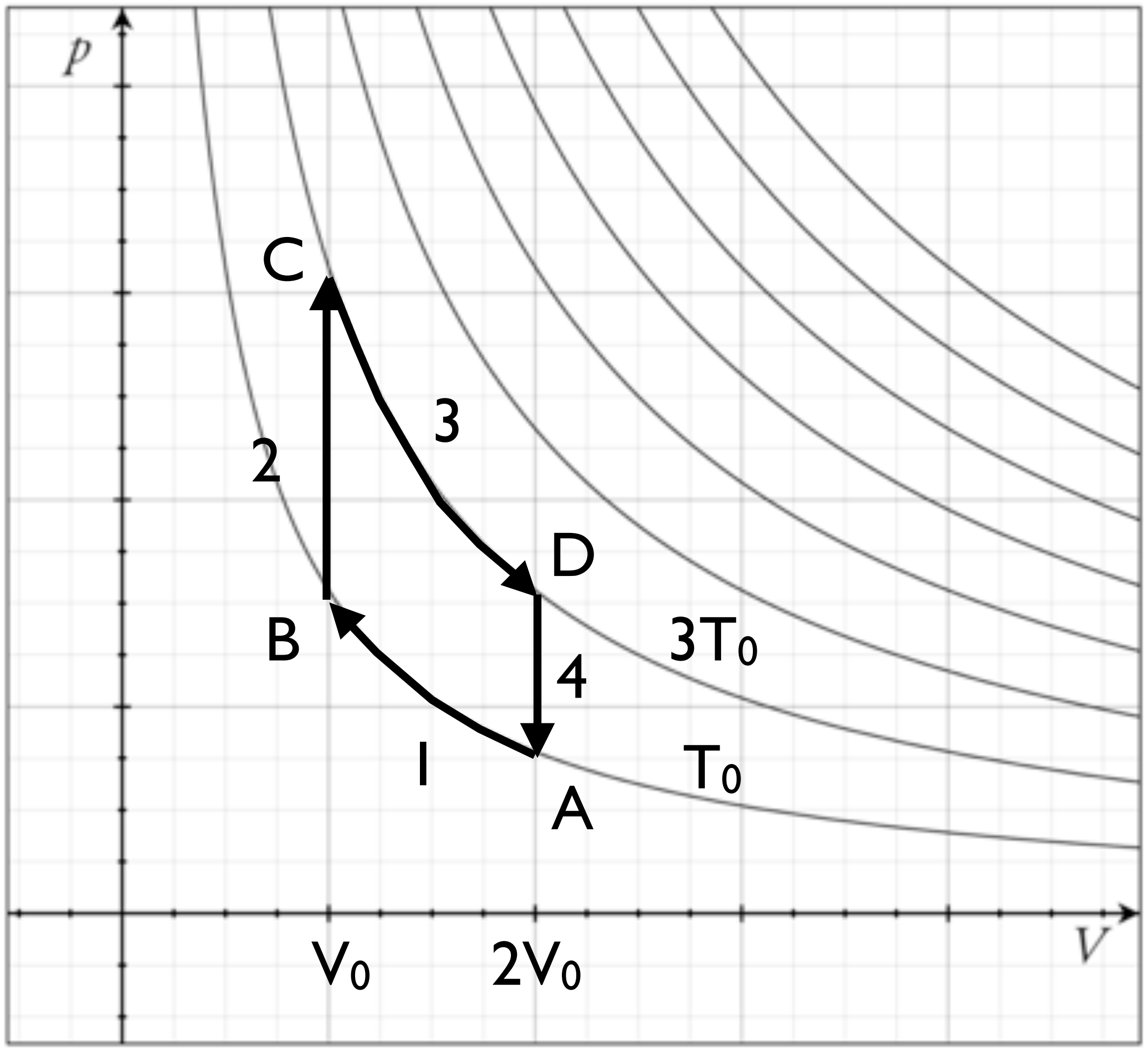


	ΔQ	ΔU	ΔW
1		0	
2			0
3		0	
4			0
total			

An isochoric process does no work;
an isothermal process doesn't change internal energy

Image: http://en.wikipedia.org

Analyze Simple Cycle II



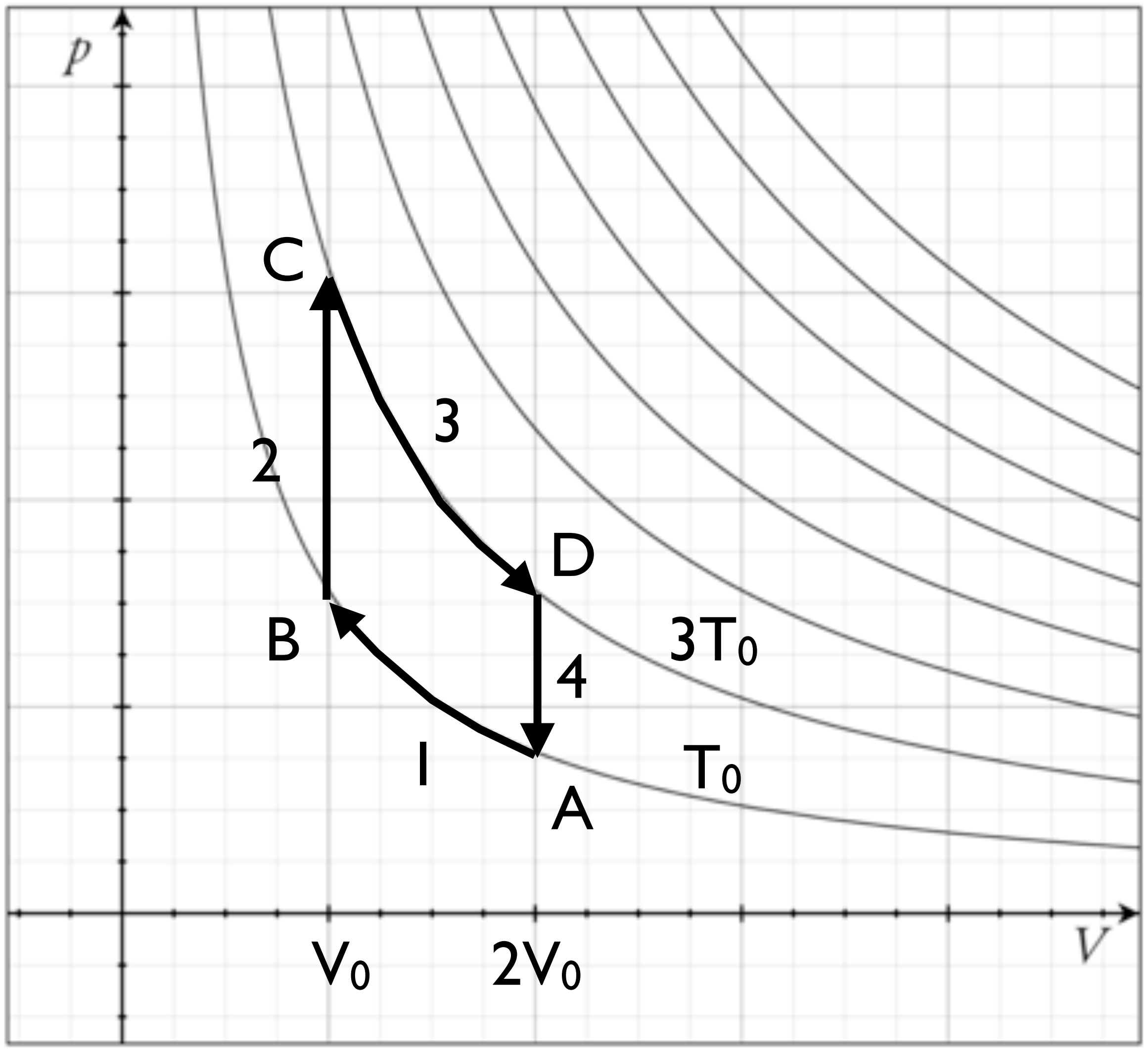
	ΔQ	ΔU	ΔW
1		0	$-nR\,T_0 \log 2$
2			0
3		0	$nR\,3T_0 \log 2$
4			0
total			$nR\,2T_0 \log 2$

Work done during isothermal expansion

$$W = \int_{V_1}^{V_2} (nRT)\frac{dV}{V} = (nRT)\log\left(\frac{V_2}{V_1}\right)$$

Image: http://en.wikipedia.org

Analyze Simple Cycle III



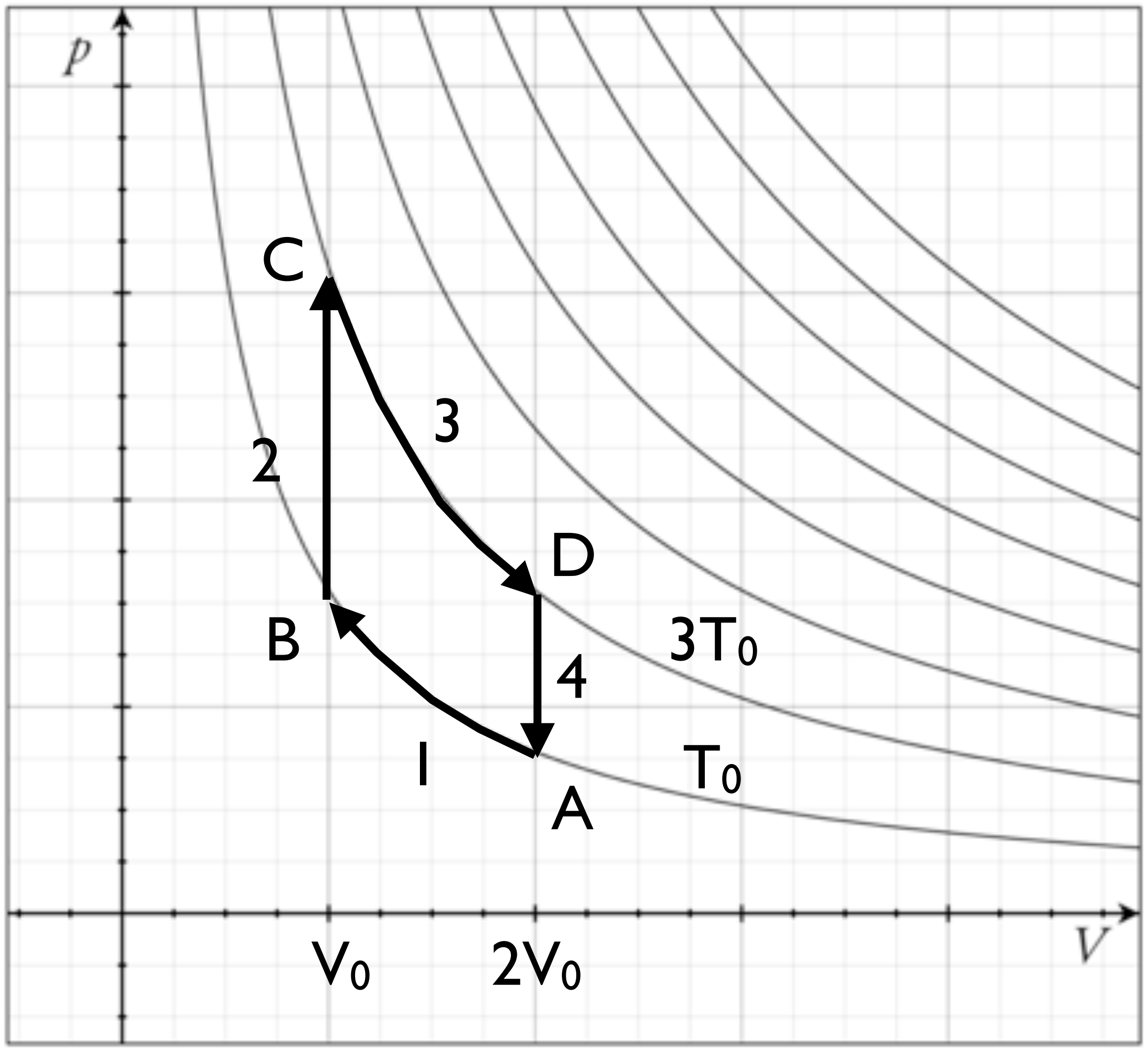
	ΔQ	ΔU	ΔW
I		0	$-nR\,T_0 \log 2$
2		$nC_V\,2T_0$	0
3		0	$nR\,3T_0 \log 2$
4		$-nC_V\,2T_0$	0
total		0 ✓	$nR\,2T_0 \log 2$

Change in internal energy
of an ideal gas

$$\Delta U = nC_V \Delta T$$

Image: http://en.wikipedia.org

Analyze Simple Cycle IV



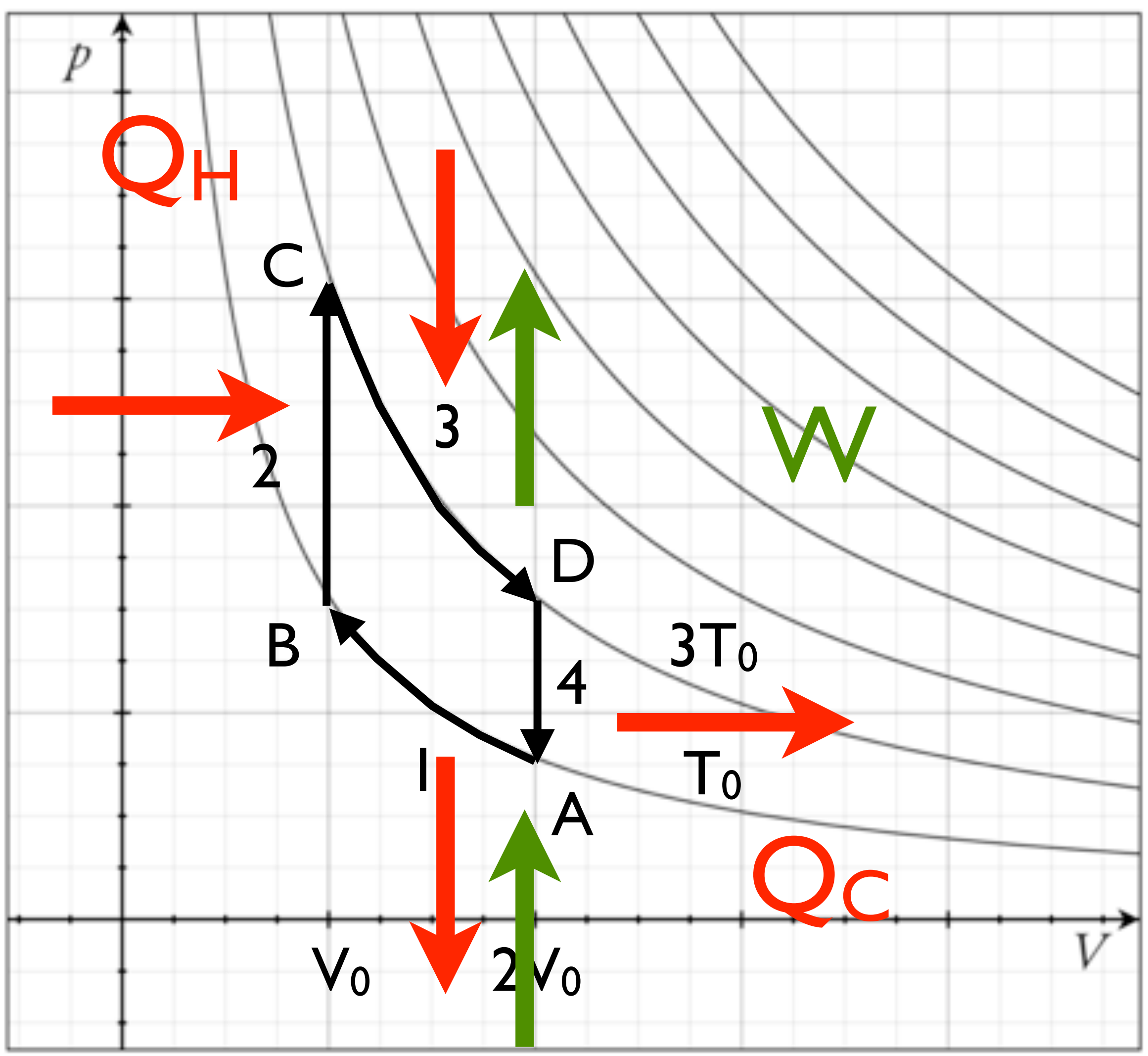
	ΔQ	ΔU	ΔW
1	$-nR\,T_0 \log 2$	0	$-nR\,T_0 \log 2$
2	$nC_V\,2T_0$	$nC_V\,2T_0$	0
3	$nR\,3T_0 \log 2$	0	$nR\,3T_0 \log 2$
4	$-nC_V\,2T_0$	$-nC_V\,2T_0$	0
total	$nR\,2T_0 \log 2$	0	$nR\,2T_0 \log 2$

1st Law of Thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

Image: http://en.wikipedia.org

Analyze Simple Cycle V



	ΔQ	ΔU	ΔW
1	$-nR\,T_0 \log 2$	0	$-nR\,T_0 \log 2$
2	$nC_V\,2T_0$	$nC_V\,2T_0$	0
3	$nR\,3T_0 \log 2$	0	$nR\,3T_0 \log 2$
4	$-nC_V\,2T_0$	$-nC_V\,2T_0$	0
total	$nR\,2T_0 \log 2$	0	$nR\,2T_0 \log 2$

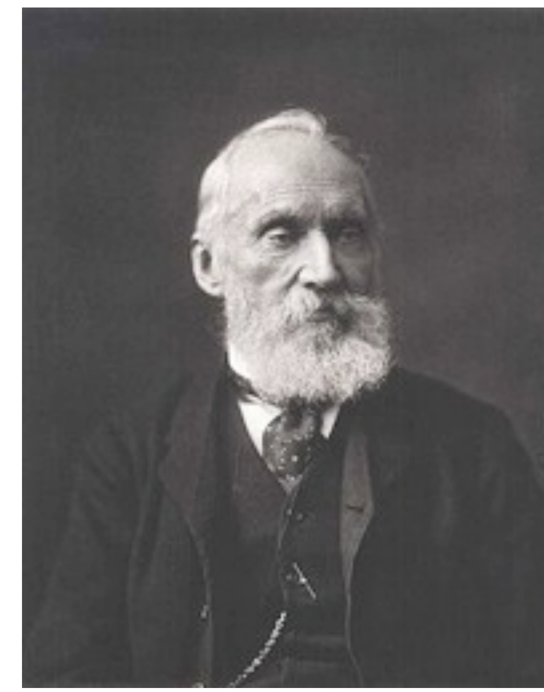
$$\eta = \frac{W}{Q_H} = \frac{2R\log 2}{3R\log 2 + 2C_V} < \frac{2}{3}$$

Image: http://en.wikipedia.org

2nd Law of Thermodynamics

2nd Law of TD (Kelvin form):

It is impossible for a cyclic process to remove thermal energy from a system at a single temperature and convert it to mechanical work without changing the system or surroundings in some other way.



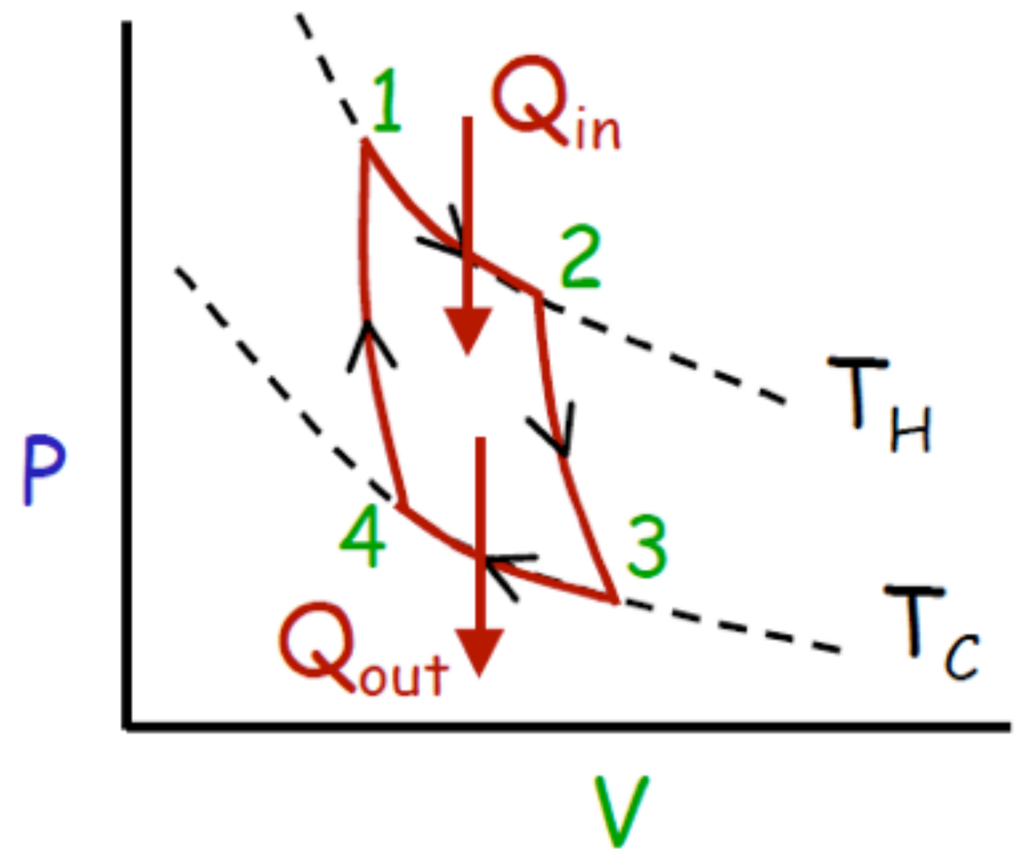
William Thomson, Lord Kelvin

Corollary of the 2nd Law of TD:

It is impossible to make a heat engine whose efficiency is 100%.

How efficient can a heat engine be?

The Carnot Engine



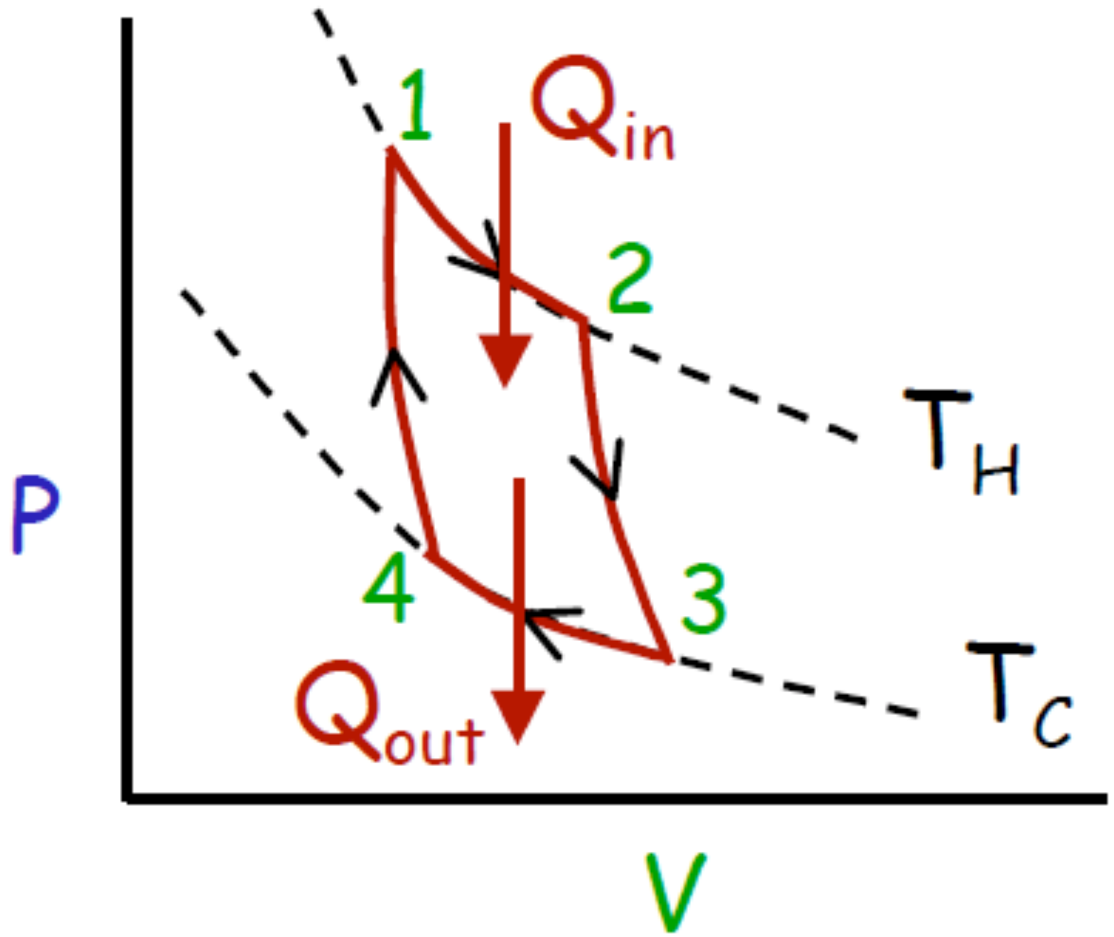
1→2 & 3→4
isothermal

2→3 & 4→1
adiabatic

1. It is reversible: no friction or other dissipative forces.
2. Heat conduction only occurs isothermally at the temperatures of the two reservoirs.

NB: idealized and impractical!

Carnot Cycle



	ΔQ	ΔU	ΔW
1→2	$nRT_H \log \frac{V_2}{V_1}$	0	$nRT_H \log \frac{V_2}{V_1}$
2→3	0	$nC_V \Delta T$	$-nC_V \Delta T$
3→4	$nRT_C \log \frac{V_4}{V_3}$	0	$nRT_C \log \frac{V_4}{V_3}$
4→1	0	$-nC_V \Delta T$	$nC_V \Delta T$
total	?	0	?

Work done during isothermal expansion

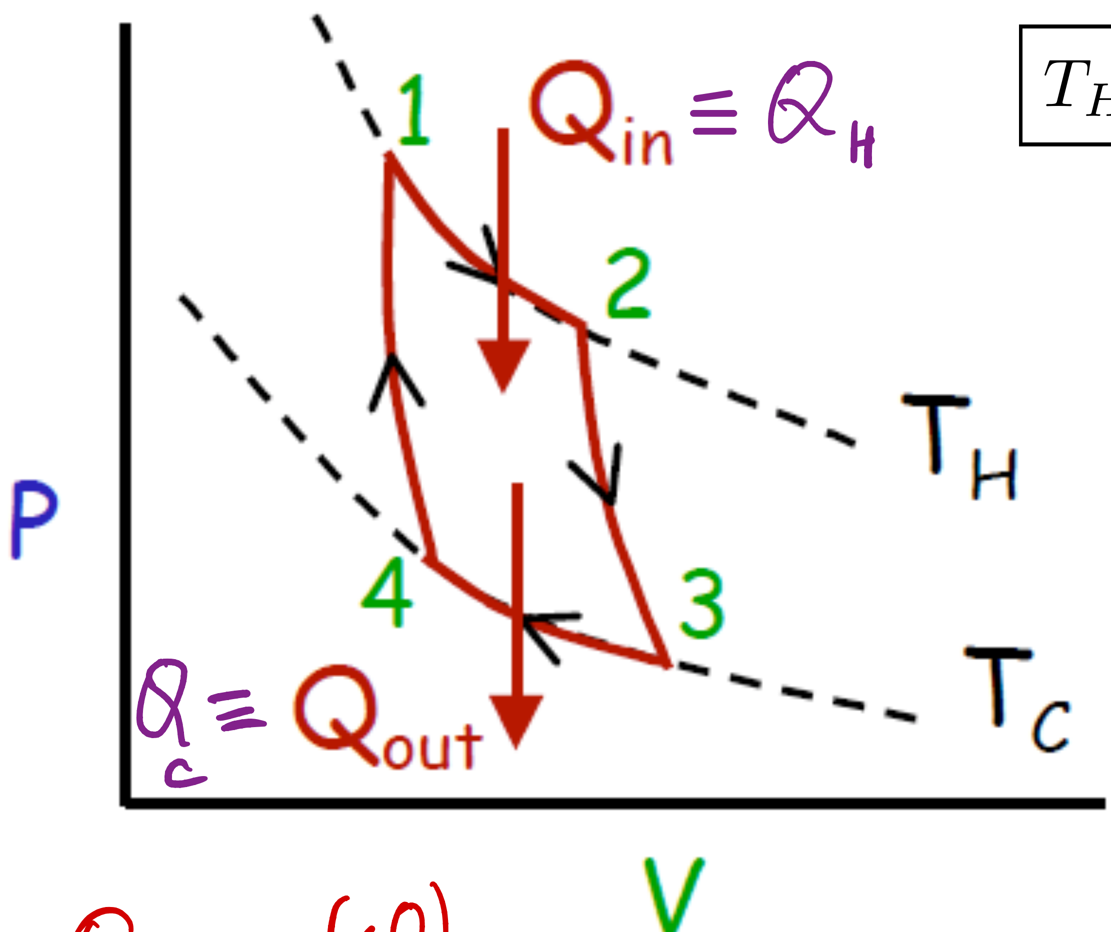
$$W = \int_{V_1}^{V_2} (nRT) \frac{dV}{V} = (nRT) \log \left(\frac{V_1}{V_2} \right)$$

Change in internal energy of an ideal gas

$$\Delta U = nC_V \Delta T$$

Carnot Efficiency

$\Delta U = 0$



$$T_H(V_2)^{\gamma-1} = T_C(V_3)^{\gamma-1} \; \& \; T_H(V_1)^{\gamma-1} = T_C(V_4)^{\gamma-1}$$

$$\therefore \quad \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

$$W = nR(T_H - T_C)\log\frac{V_2}{V_1}$$

$$Q_H = Q_{in} = nRT_H \log\frac{V_2}{V_1}$$

$$Q_C = Q_{out} = -nRT_C \log\frac{V_4}{V_3} = nRT_C \log\frac{V_2}{V_1}$$

$Q_{out} = -(\Delta Q)_{3\to4}$

↑ "absorbed" heat

$W = Q_H - Q_C$

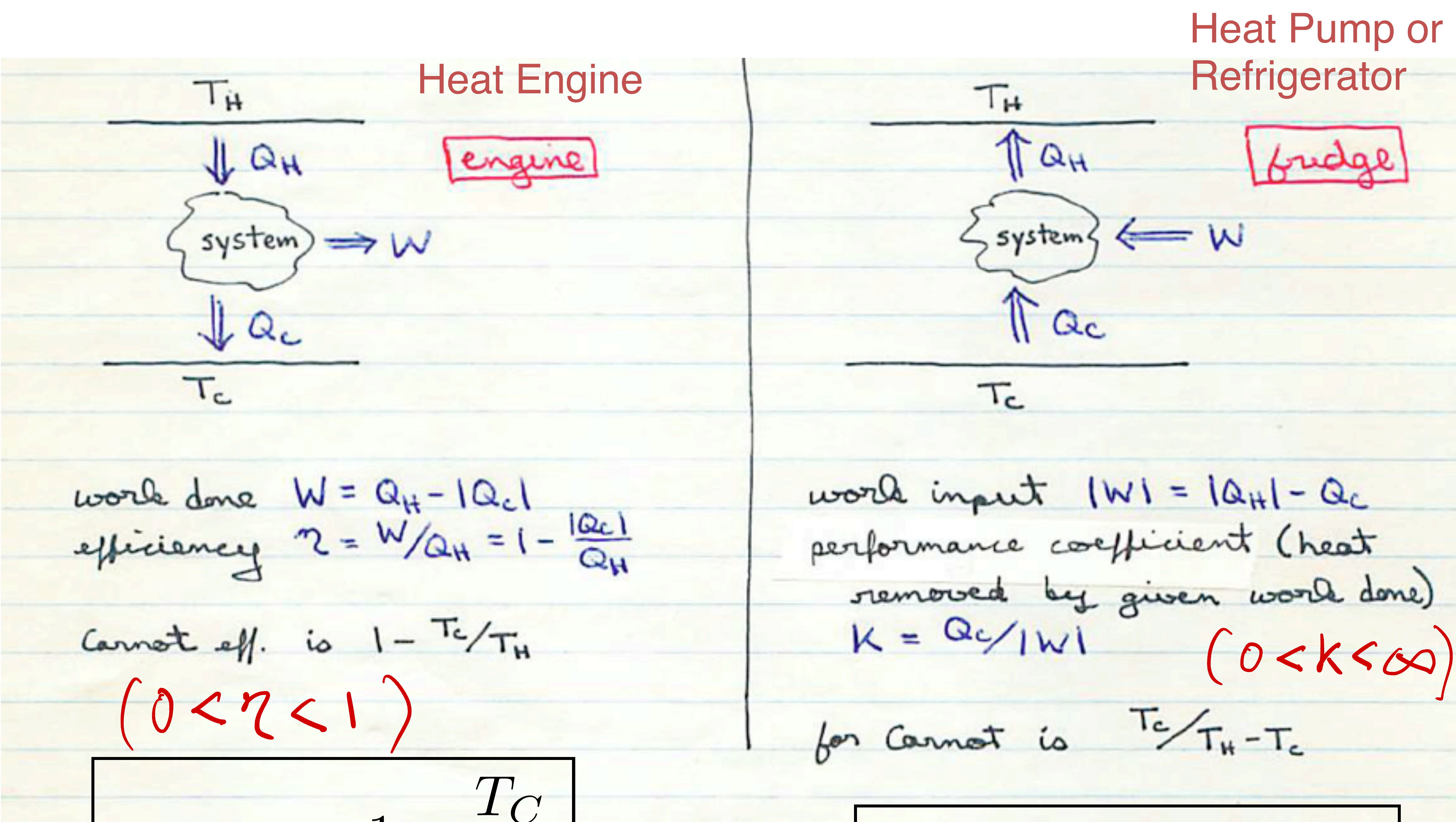
$$\eta_{Carnot} = 1 - \frac{T_C}{T_H}$$

$$\eta = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

Thus for carnot cycle,

$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H}$$

Carnot Engine Reversible!



$$\eta_{Carnot} = 1 - \frac{T_C}{T_H}$$

$$K_{Carnot} = \frac{T_C}{T_H - T_C}$$

2nd Law: Carnot Form

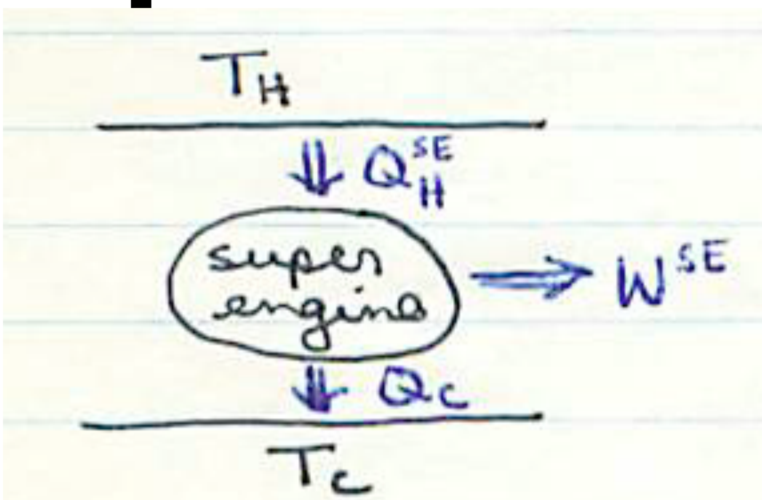
Sadi Carnot

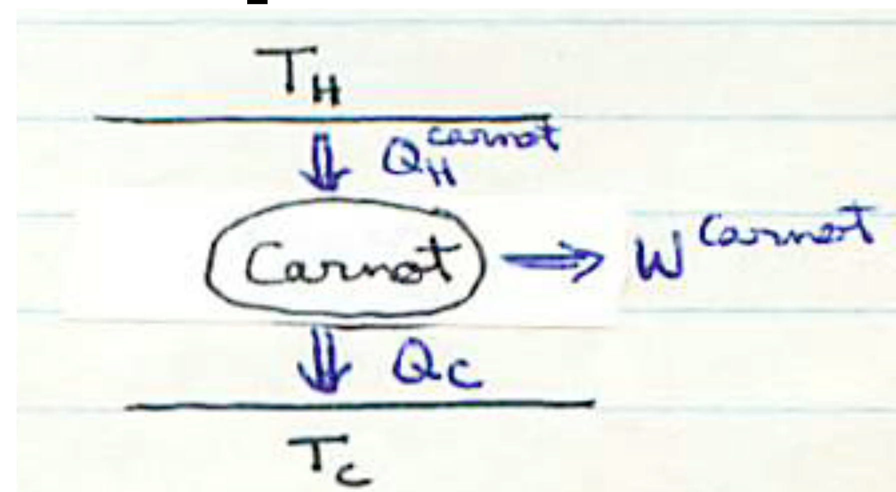
1796 –1832

No engine working between 2 heat reservoirs can be more efficient than ideal engine acting in a Carnot cycle. (Sadi Carnot, 1824)

Image: http://en.wikipedia.org

SuperCarnot Impossible!



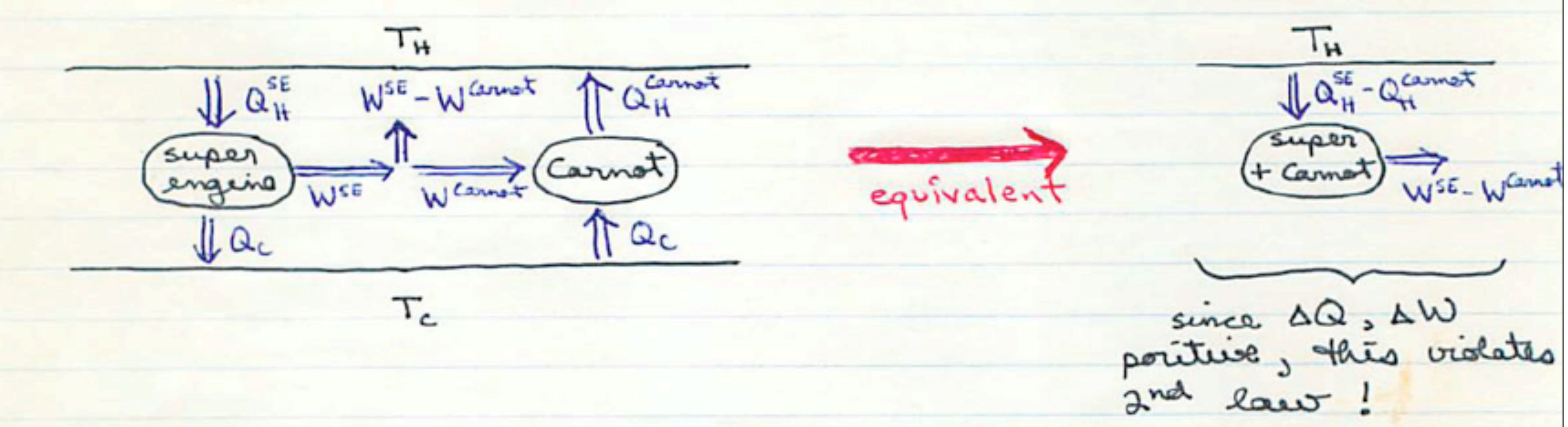


$$\eta^{SE} = 1 - |Q_C| / Q_H^{SE} > \eta^{Carnot} = 1 - |Q_C| / Q_H^{Carnot}$$

therefore $Q_H^{SE} > Q_H^{Carnot}$

and since $\Delta Q = \Delta W$, also $W^{SE} > W^{Carnot}$

Now hook 'em together and run Carnot in reverse:



Concept Test

The Carnot Corollary to the 2nd Law of Thermodynamics implies all of the following EXCEPT:

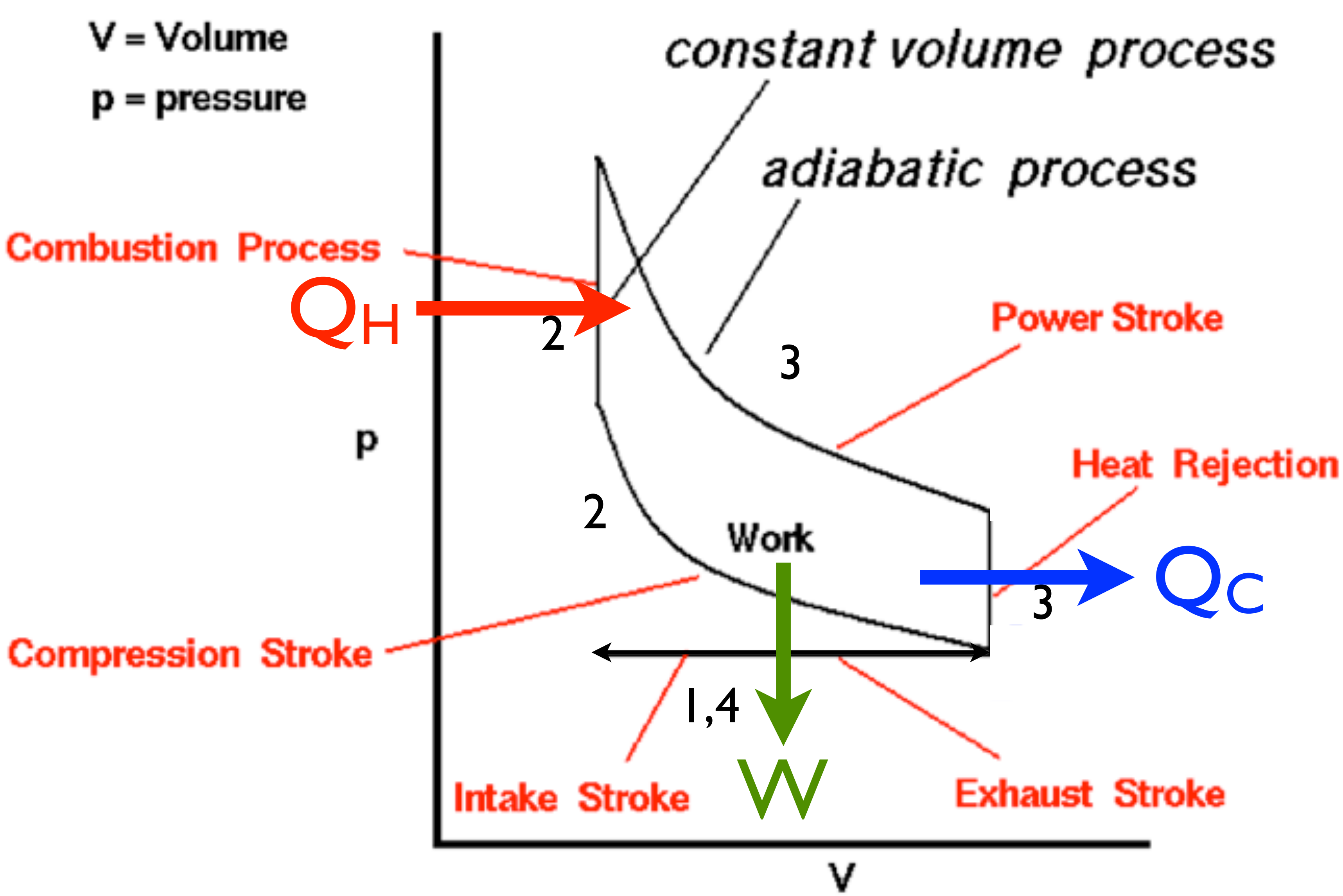
A. No other engine cycle is more efficient than a Carnot cycle.

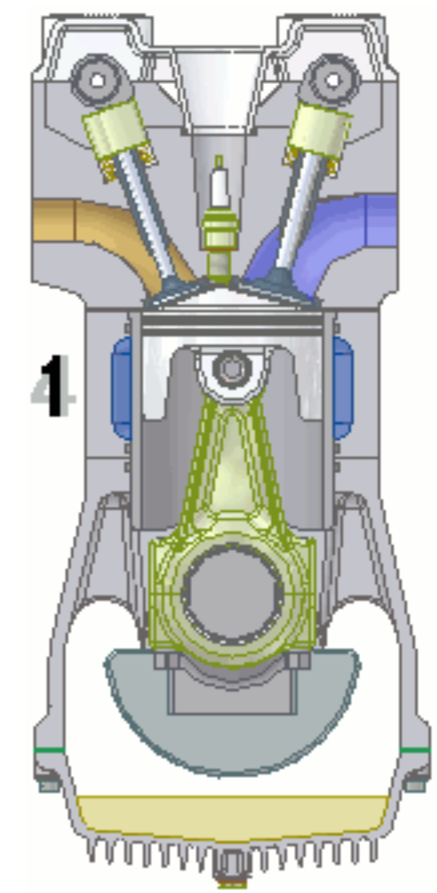
B. Any other fully reversible cycle is exactly as efficient as a Carnot cycle.

C. A Carnot refrigerator is the best possible refrigerator.

D. The exact efficiency of a given Carnot engine depends on whether its 'working gas' is ideal.

Otto Cycle



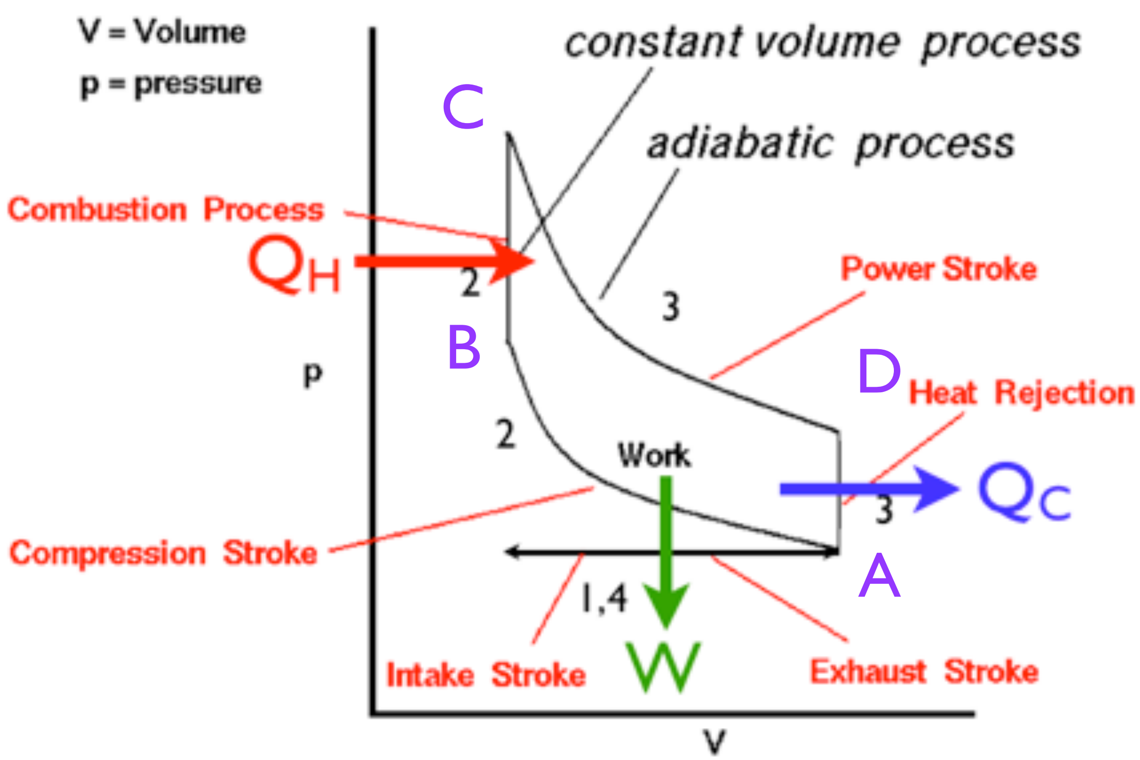


$$W = Q_H - Q_C$$

$$\eta = \frac{W}{Q_H}$$

Images: http://www.grc.nasa.gov & http://en.wikipedia.org

Ideal Gas Otto Cycle



n moles of an ideal gas

$$\boxed{W = Q_H - Q_C}$$

$$Q_H = nC_V(T_C - T_B) \quad Q_C = nC_V(T_D - T_A)$$

$$\eta = \frac{W}{Q_H} = 1 - \frac{Q_H}{Q_C} = 1 - \frac{T_D - T_A}{T_C - T_B}$$

$$\boxed{V_B = V_C \quad \& \quad V_A = V_D} \quad \boxed{T_B V_B^{\gamma-1} = T_A V_A^{\gamma-1} \;\&\; T_C V_B^{\gamma-1} = T_D V_A^{\gamma-1}}$$

$$\therefore \quad \frac{T_D - T_A}{T_C - T_B} = \left(\frac{V_B}{V_A}\right)^{\gamma-1} = \frac{T_A}{T_B} = \frac{T_D}{T_C}$$

Hottest: T_C

Coldest: T_A

$$\boxed{\eta = 1 - \left(\frac{V_B}{V_A}\right)^{\gamma-1} = 1 - \frac{T_A}{T_B} = 1 - \frac{T_D}{T_C}}$$ $<\eta_{Carnot}$!

Image: http://www.grc.nasa.gov

Summary

- Heat Engines transform heat into work
    - $\Delta U=0$ over entire cycle, $\Delta Q=\Delta W$
- Analyze a Cycle: step by step!
- 2nd Law in Kelvin Form
    - Input heat cannot be completely converted to work, $\eta<1$ always!
- The Carnot Cycle
    - Carnot Efficiency
    - No engine can be more efficient than Carnot!

$$\eta_{Carnot} = 1 - \frac{T_C}{T_H}$$