

Outline

- Blackbody Radiation
- Planck's Formula
- Photoelectric Effect
 - X-Ray Production
- Compton Scattering
- Summary

Blackbody Radiation

As an object gets hot, it radiates energy.

How does the radiation depend on temperature?

What is the distribution among frequencies (or wavelengths)?

$\lambda \downarrow$
as $T \uparrow$

$I(\lambda, T)$ = Intensity as a function of wavelength λ and temperature T

Two experimental facts:

1) $\lambda_{\max} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$

(Wien's displacement law)

2) Total Power (per unit Area)

$$R(T) = \int_0^{\infty} I(\lambda, T) d\lambda$$

given by

$$R(T) = \epsilon \sigma T^4$$

with $\epsilon=1$ for blackbody radiator
(Stefan-Boltzmann law)

$$\sigma = 5.67 \times 10^{-8} \text{ W} / (\text{m}^2 \cdot \text{K}^4)$$



Concept Test

- A blacksmith heats an iron rod which starts to radiate, beginning red, then orange, then yellow, and ultimately white. As the iron becomes hotter, the color changes because
 - Red light has a longer wavelength than blue light ←
 - Red and blue light have the same wavelength
 - Blue light has a longer wavelength than red light

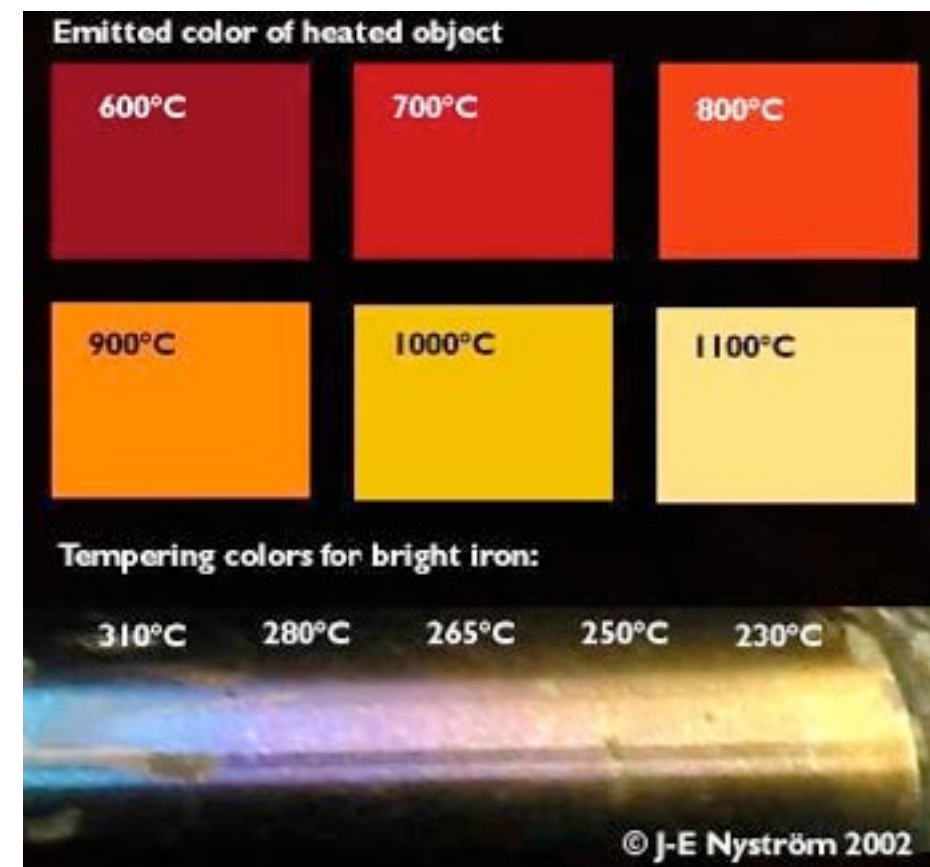
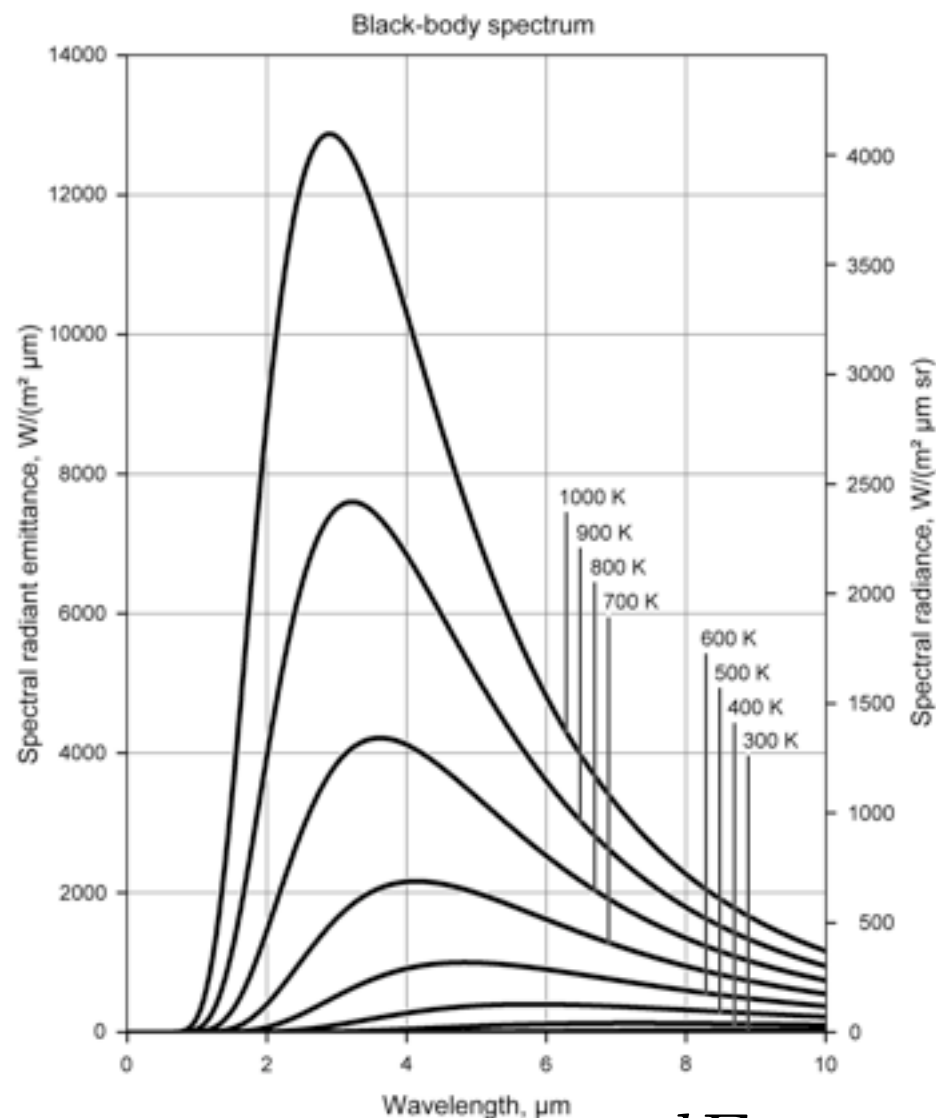


Image: <http://www.saunalahti.fi>

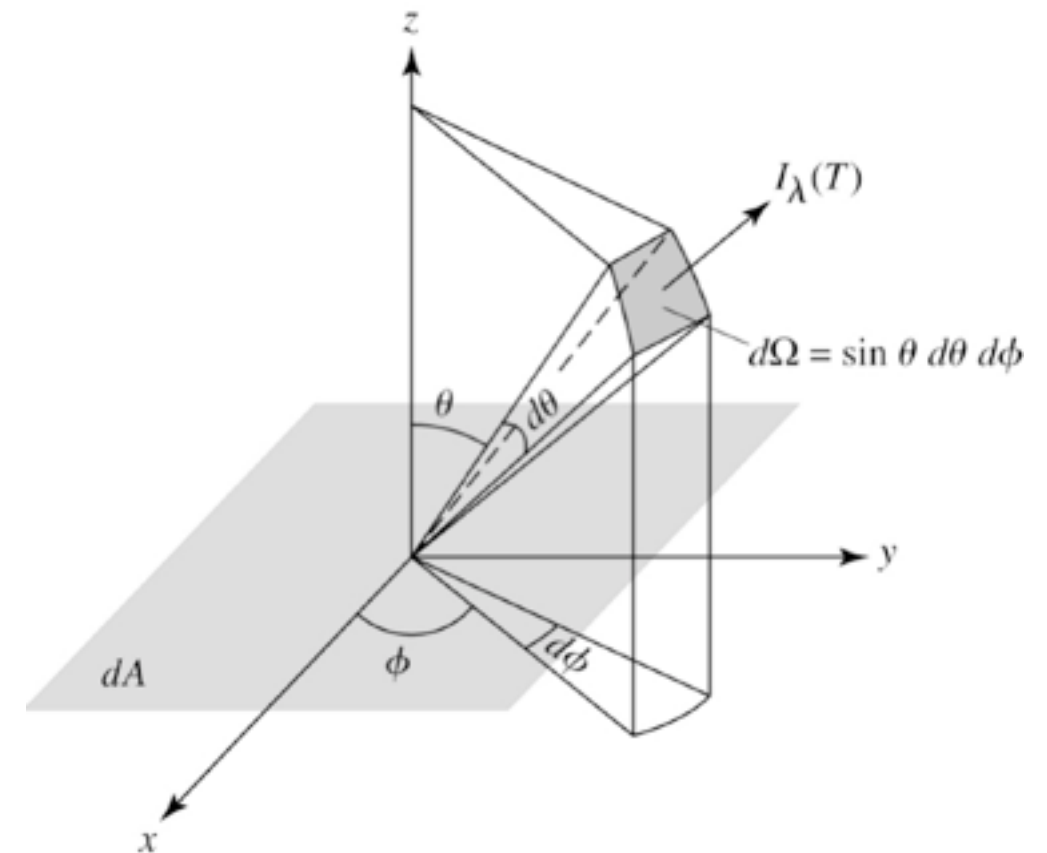
Blackbody Spectrum

$I(\lambda, T)$



$$I(\lambda, T) = \frac{dE}{d\Omega dA d\lambda dt}$$

$$\neq \frac{2\pi ckT}{\lambda^4}$$



Lord Rayleigh and Sir James Jeans tried to explain the distribution using equipartition of energy: the average energy in each mode of oscillation of the radiation is $kT/2$.

Planck's Formula

Each oscillation mode can not absorb just any amount of energy. Each mode can only absorb energy in packets of fixed size.

It described the data perfectly!

The size of each energy packet (now termed **a quantum**) is proportional to the frequency of the mode:

$$E = h \nu$$

$$E \propto \text{Frequency!}$$

The proportionality factor (Planck's constant) is

$$h = 6.63 \times 10^{-34} \text{ J s.}$$

The above formula was first presented at a meeting of the German Physical Society on Dec. 14, 1900.

The birthday of Quantum Physics!

Light comes
in discrete
quanta! Photons



Max Planck

1848-1947

Nobel Prize 1918

$$I(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Planck's Constant

The energy in each frequency mode ν can only come in integer multiples of some fundamental unit (quanta):

$$E = h \nu$$

with

$$\begin{aligned} h &= 6.6261 \times 10^{-34} \text{ J} \cdot \text{s} \\ &= 4.1357 \times 10^{-15} \text{ eV} \cdot \text{s} \end{aligned}$$

$$1 \text{ V} = 1 \text{ J/C}$$

$$e \approx 1.6 \times 10^{-19} \text{ C}$$

$$1 \text{ eV} \approx 1.6 \times 10^{-19} \text{ J}$$

In a cathode ray tube, the change in potential energy of the electron is eV , where V is the voltage difference between cathode and anode.

Thus, eV is the Kinetic Energy of the electron when it hits the anode.

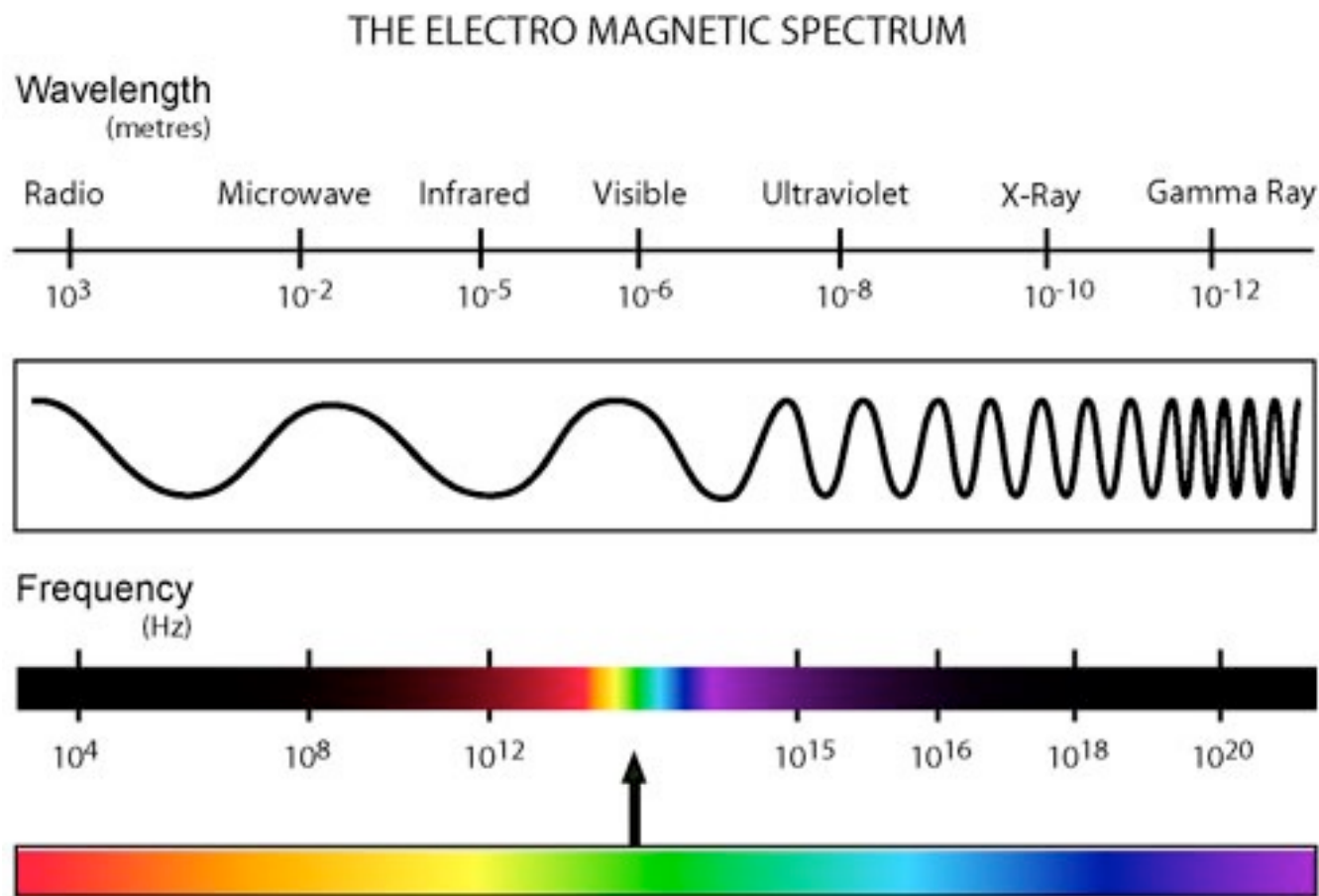
If all of this energy is converted into a single quantum of X-ray radiation (called a **photon**), then the photon would have a frequency given by

$$\nu = eV/h, \quad \lambda = c/\nu$$

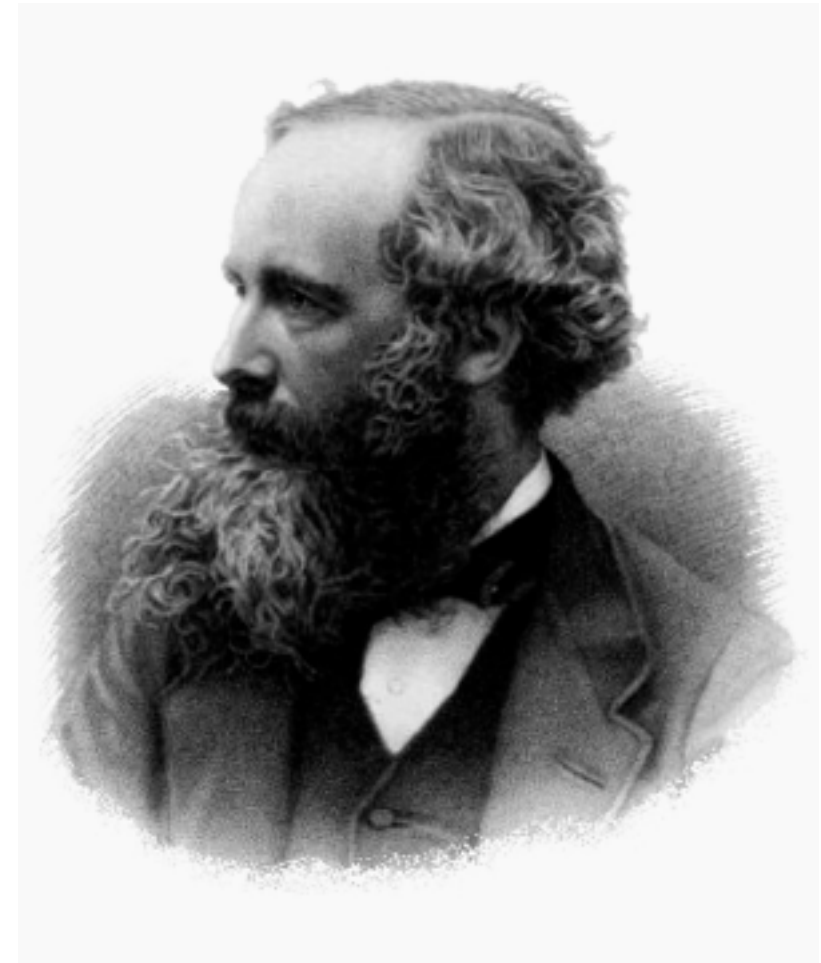
This is the maximum frequency X-ray that could be emitted in the process.

For example, $2000 \text{ V} \rightarrow \lambda = 0.621 \text{ nm}$

Light Quanta?



Classically:
EM Radiation = Waves
Energy $\propto$ (Amplitude)²



James Clerk Maxwell
1831-1879

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho_e$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

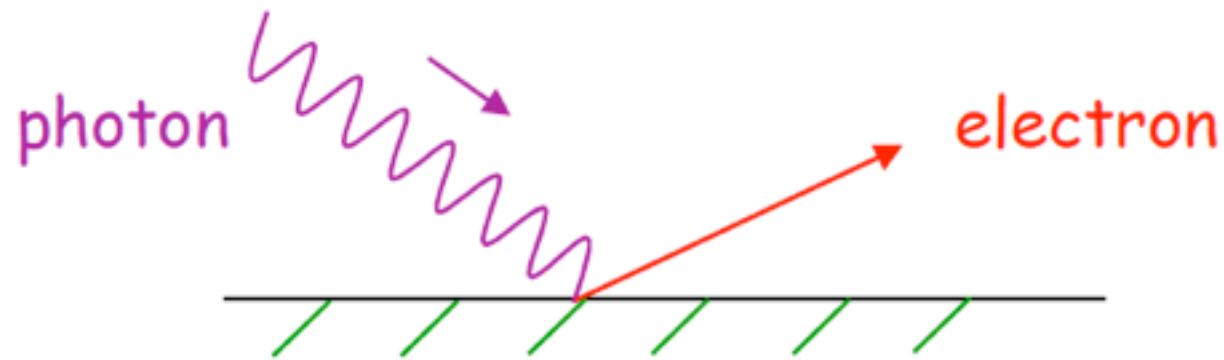
$$-\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + 4\pi\vec{j}_e$$

Photoelectric Effect I

1887 - Heinrich Hertz

Visible or UV light on metal surface may release electrons



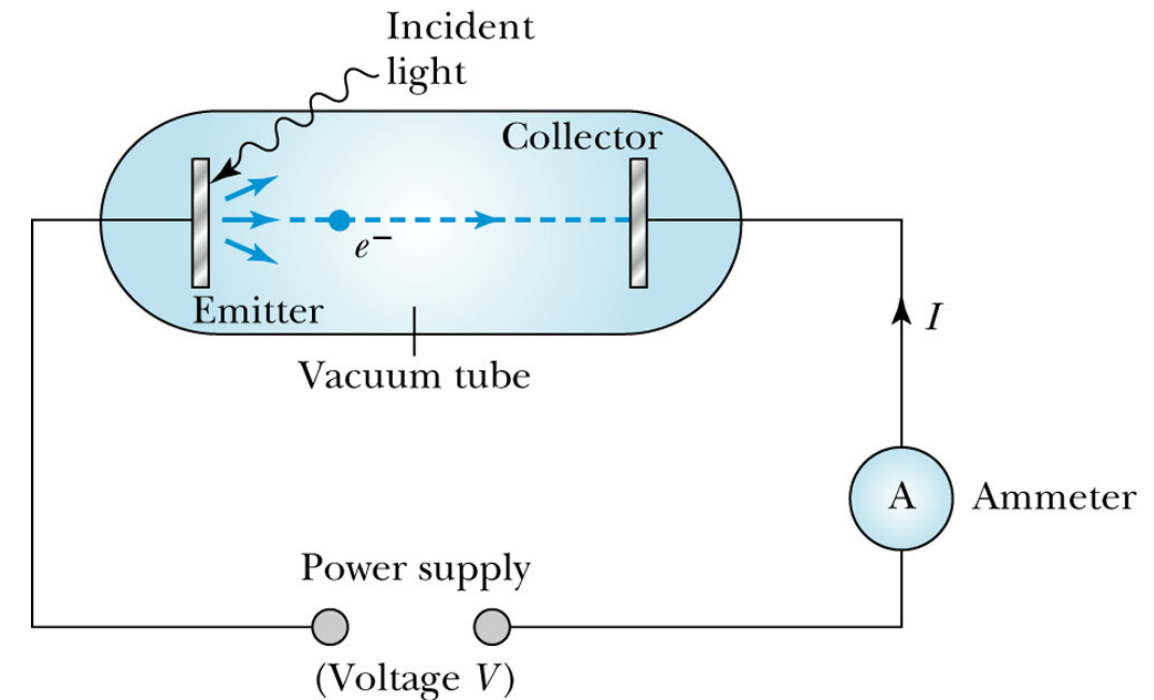
Classical theory says energy of electrons should increase with intensity of light.

However, this was not the case.

Classically:

EM Radiation = Waves

Energy $\propto$ (Amplitude)²



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Phillip Lenard
1862-1947
Nobel Prize 1905

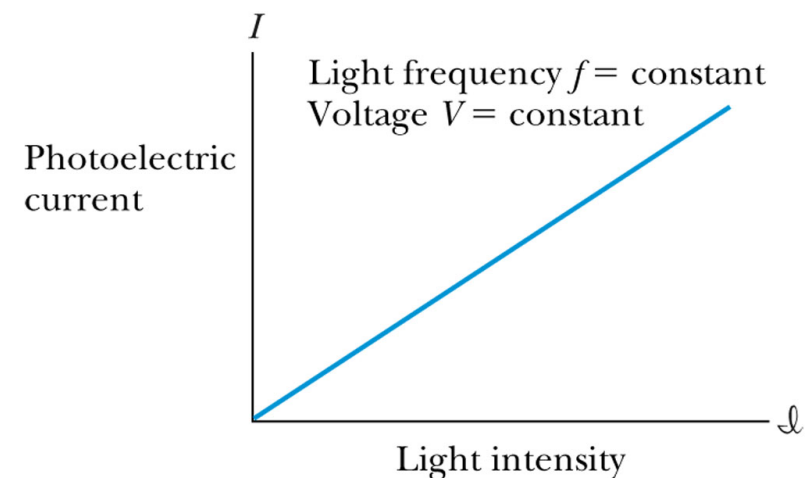
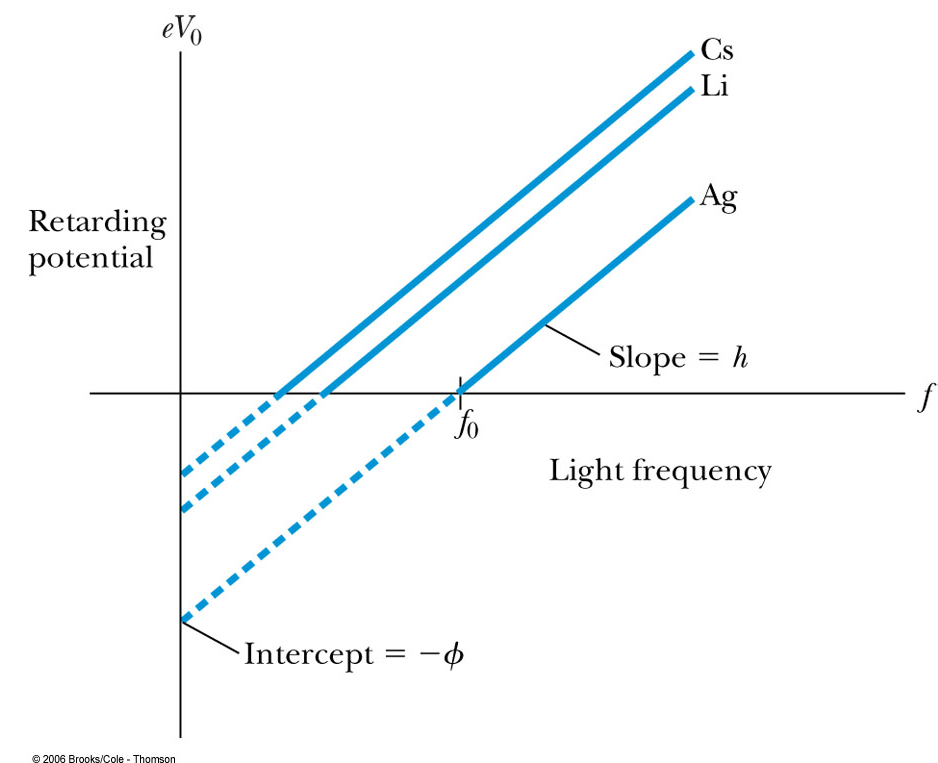
Image: Thornton and Rex

Photoelectric Effect II

Experiments (Lenard) showed:

- 1) KE of photoelectrons depends only on ν (frequency) of light, independent of I (its intensity).
- 2) # of photoelectrons is proportional to I
- 3) For a given metal, there is a minimum ν , below which no photoelectrons are emitted.
- 4) The photoelectrons are emitted instantaneously, independent of I .

Classical theory could not explain these observations.



Concept Test

- In the photoelectric effect, if a fixed *frequency* of light shines on a metal, as the intensity of the light is increased
 - the number of electrons ejected increases 
 - the energy of the ejected electrons increases
 - both the number and energy of the ejected electrons increases

Photoelectric Effect III

1905 - Einstein explained:

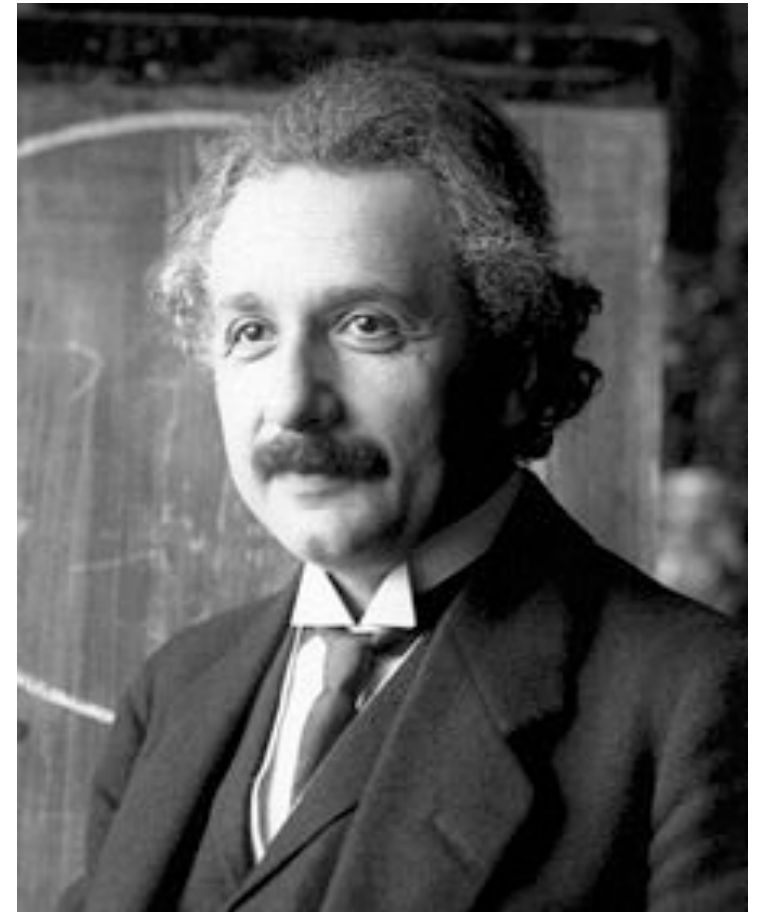
Electromagnetic radiation transferred in discrete bundles of energy ("photons").
The energy of each photon is

$$E = h \nu$$

The KE of an emitted photoelectron is

$$KE = h \nu - \phi$$

- $h\nu$ is Energy of the incident photon
- ϕ is the binding energy of electron to the metal surface (the work function).



Albert Einstein

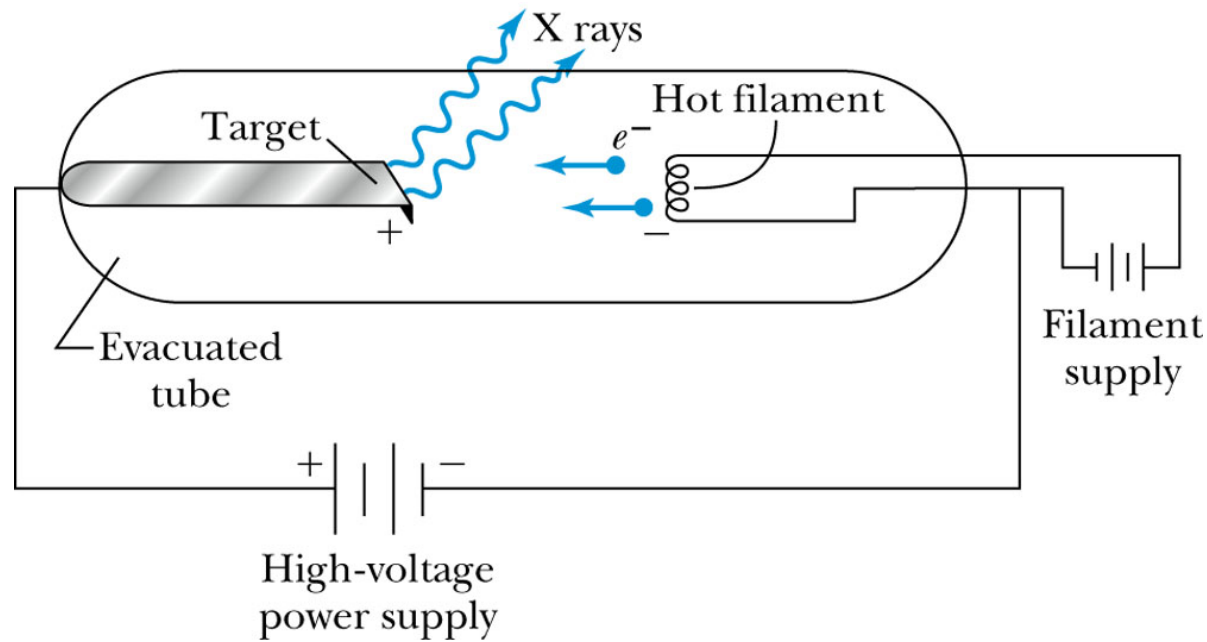
1879-1955

Nobel Prize 1921

Einstein extended Planck's hypothesis
to explain the photoelectric effect!

X-Ray Production

Inverse photoelectric effect:



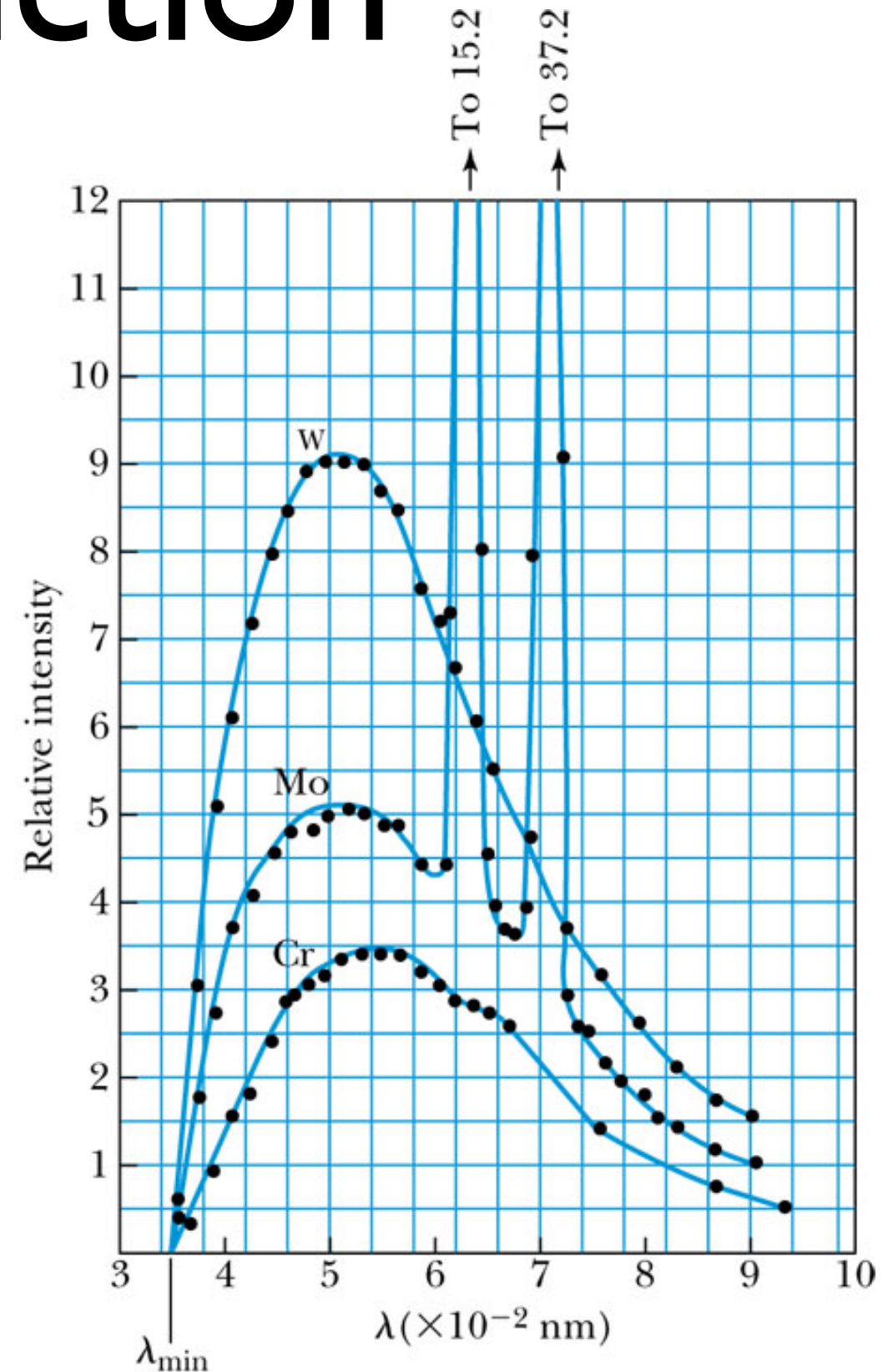
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$$eV_0 = hf_{\max} = \frac{hc}{\lambda_{\min}}$$

Duane-Hunt limit

$$\lambda_{\min} = \frac{hc}{eV_0} = \frac{1.240 \times 10^{-6} \text{ V} \cdot \text{m}}{V_0}$$

Maximum frequency
related to maximum energy ✓



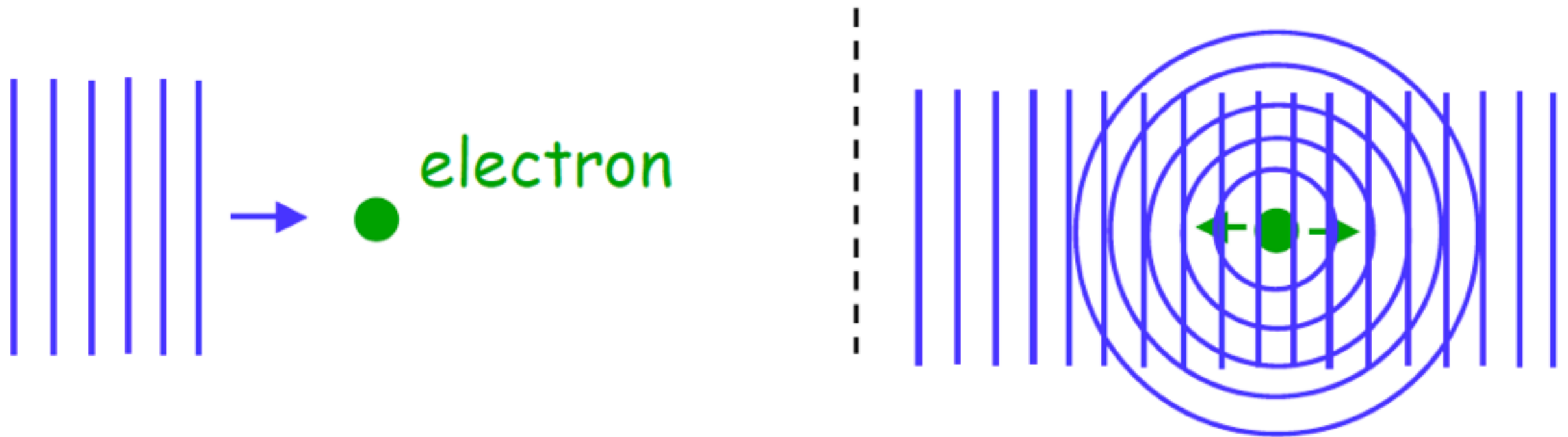
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Images: Thornton and Rex

Compton Scattering I

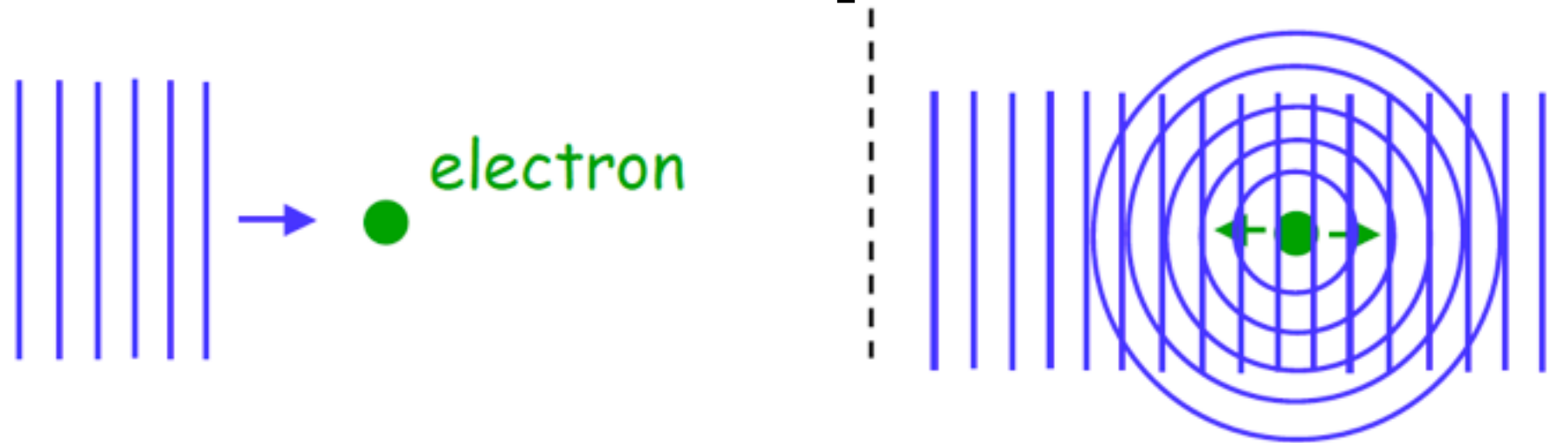
Classical Picture of Scattering of Light:

Wave picture:



Light scatters off all electrons in metal, and scattered frequency same as incident frequency

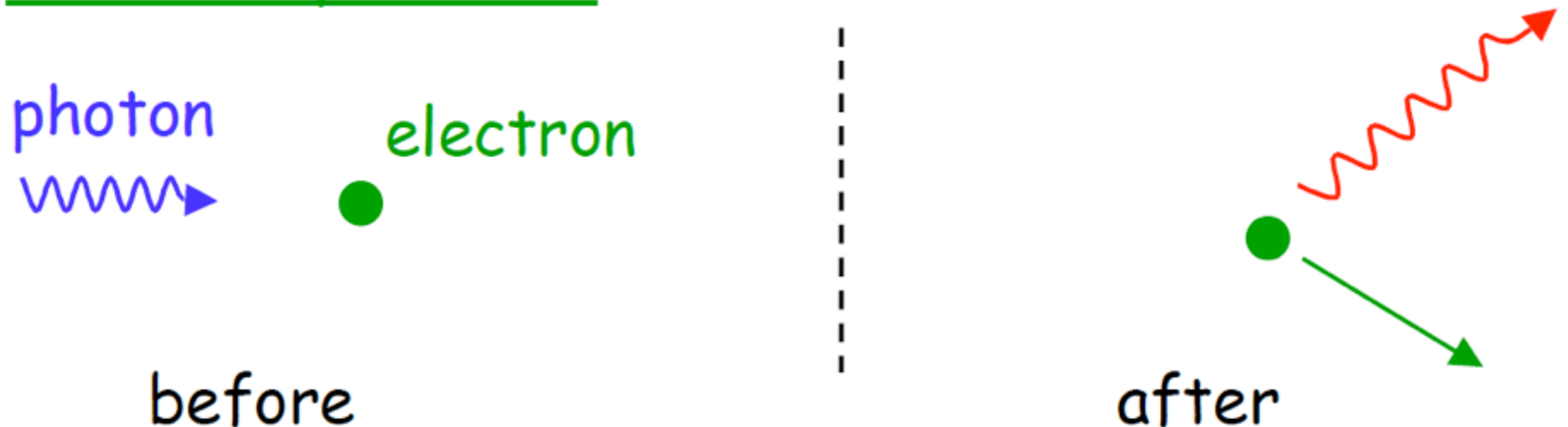
Concept Test



- Light is a transverse wave: the direction of the oscillating E and B fields is perpendicular to the direction of propagation. What is wrong with the diagram above?
- Light doesn't have to be blue
- The electron is moving in the wrong direction ←
- The scattered light is not a spherical wave
- Electrons aren't green

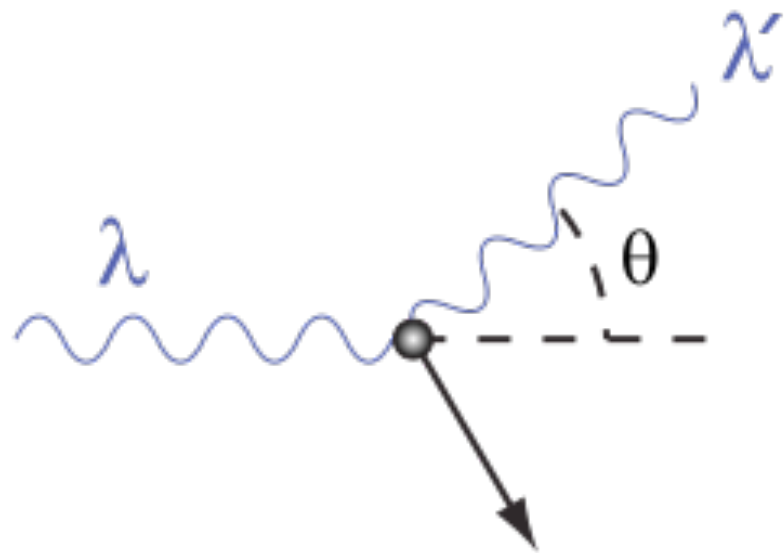
Compton Scattering II

Quantum picture:



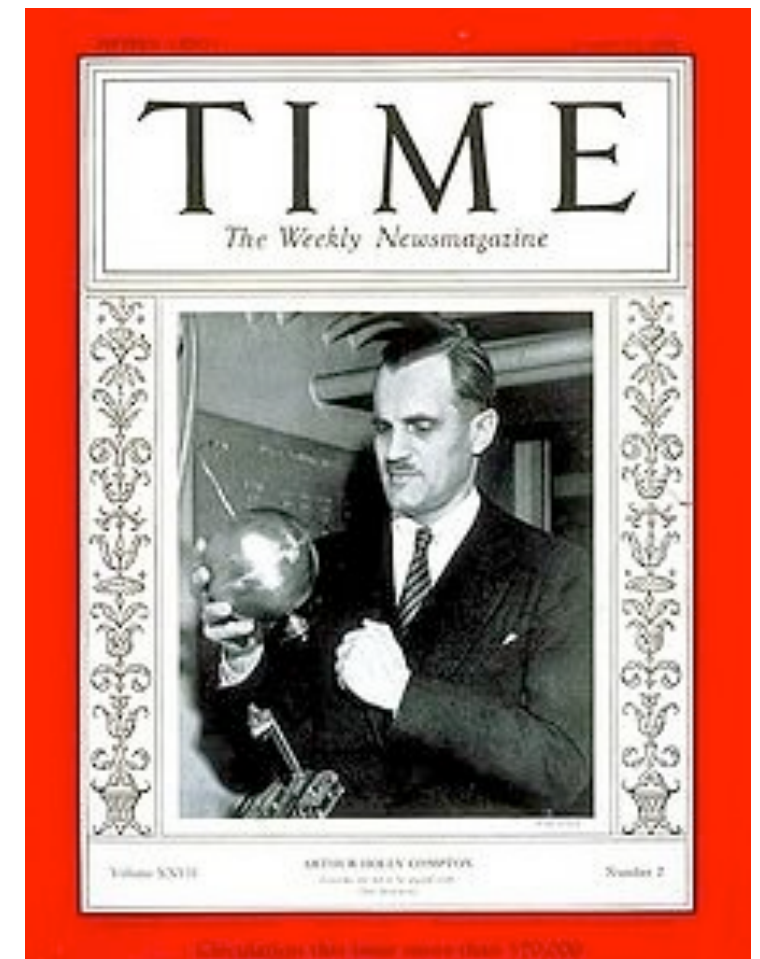
A quantum of light scatters off an electron, and can change energy and hence frequency

Compton Formula



$$\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$$

$$\frac{1}{E_\gamma} - \frac{1}{E'_\gamma} = \frac{1}{m_e c^2} (1 - \cos \theta)$$



Arthur Holly Compton
1892-1962
Nobel Prize 1927

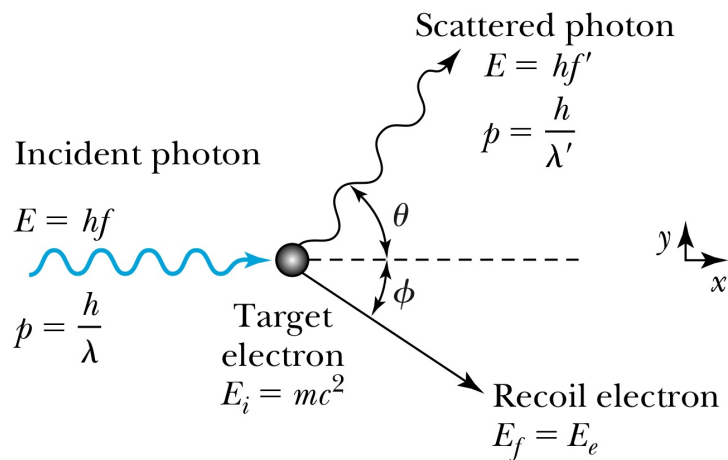


Table 3.4 Results of Compton Scattering

Energy or Momentum	Initial System	Final System
Photon energy	hf	hf'
Photon momentum in x direction (p_x)	$\frac{h}{\lambda}$	$\frac{h}{\lambda'} \cos \theta$
Photon momentum in y direction (p_y)	0	$\frac{h}{\lambda'} \sin \theta$
Electron energy	mc^2	$E_e = mc^2 + \text{K.E.}$
Electron momentum in x direction (p_x)	0	$p_e \cos \phi$
Electron momentum in y direction (p_y)	0	$-p_e \sin \phi$

In the final system the electron's total energy is related to its momentum by Equation (2.70):

$$E_e^2 = (mc^2)^2 + p_e^2 c^2 \quad (3.39)$$

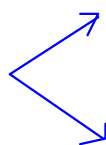
We can write the conservation laws now, initial = final, as

$$\text{Energy} \quad hf + mc^2 = hf' + E_e \quad (3.40a)$$

$$p_x \quad \frac{h}{\lambda} = \frac{h}{\lambda'} \cos \theta + p_e \cos \phi \quad (3.40b)$$

$$p_y \quad \frac{h}{\lambda'} \sin \theta = p_e \sin \phi \quad (3.40c)$$

momentum
conservation



Summary

- Planck's formula, $E=h\nu$, explains
 - Blackbody Spectrum
 - Photoelectric Effect
 - X-ray Production
 - Compton Scattering

Light behaves both like a particle and a wave!

Light “particle” - Photon