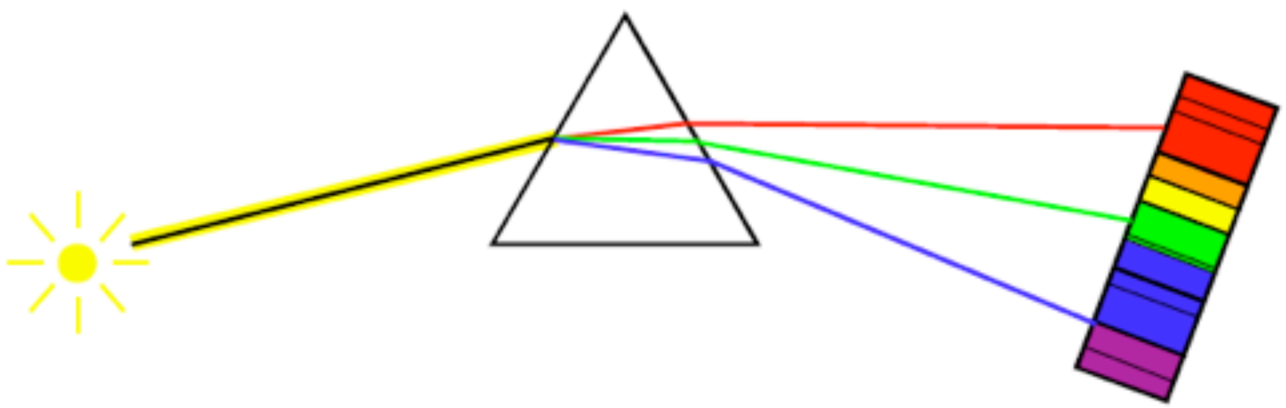
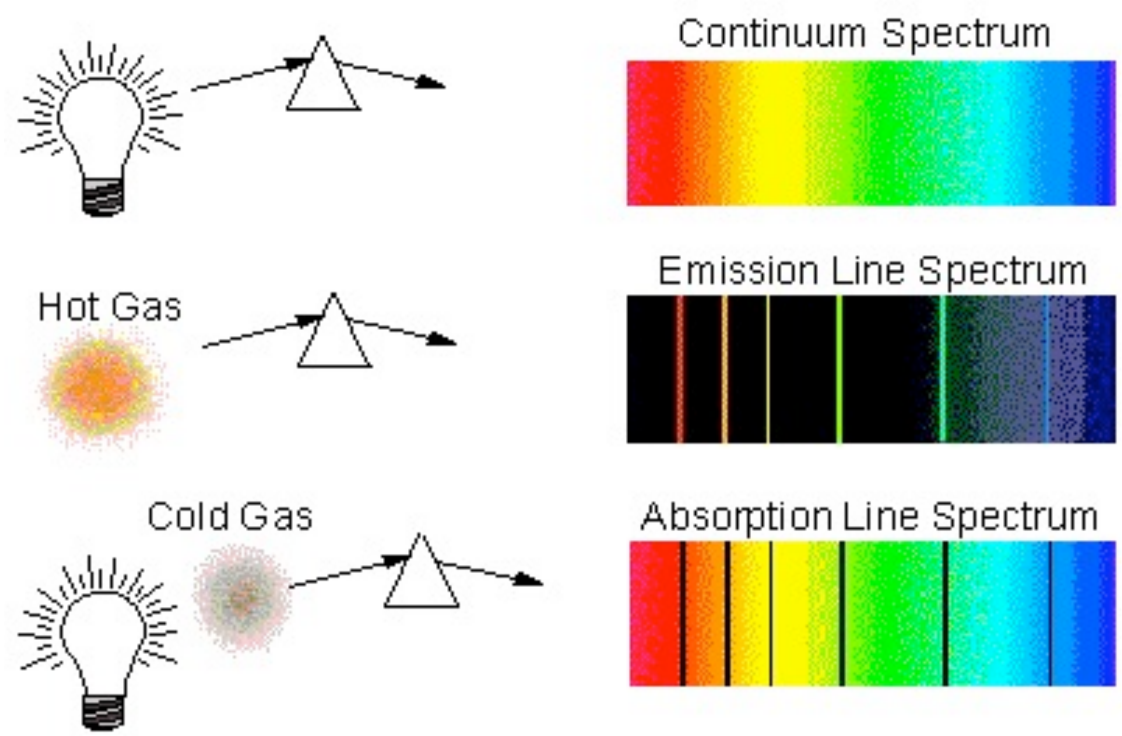
Outline

- Spectral Lines
    - Lyman, Balmer, Paschen Series
- Models of Atoms
    - Rutherford Scattering
- Classical Hydrogen Atom
- Bohr Atom
    - Limitations

Spectral Lines



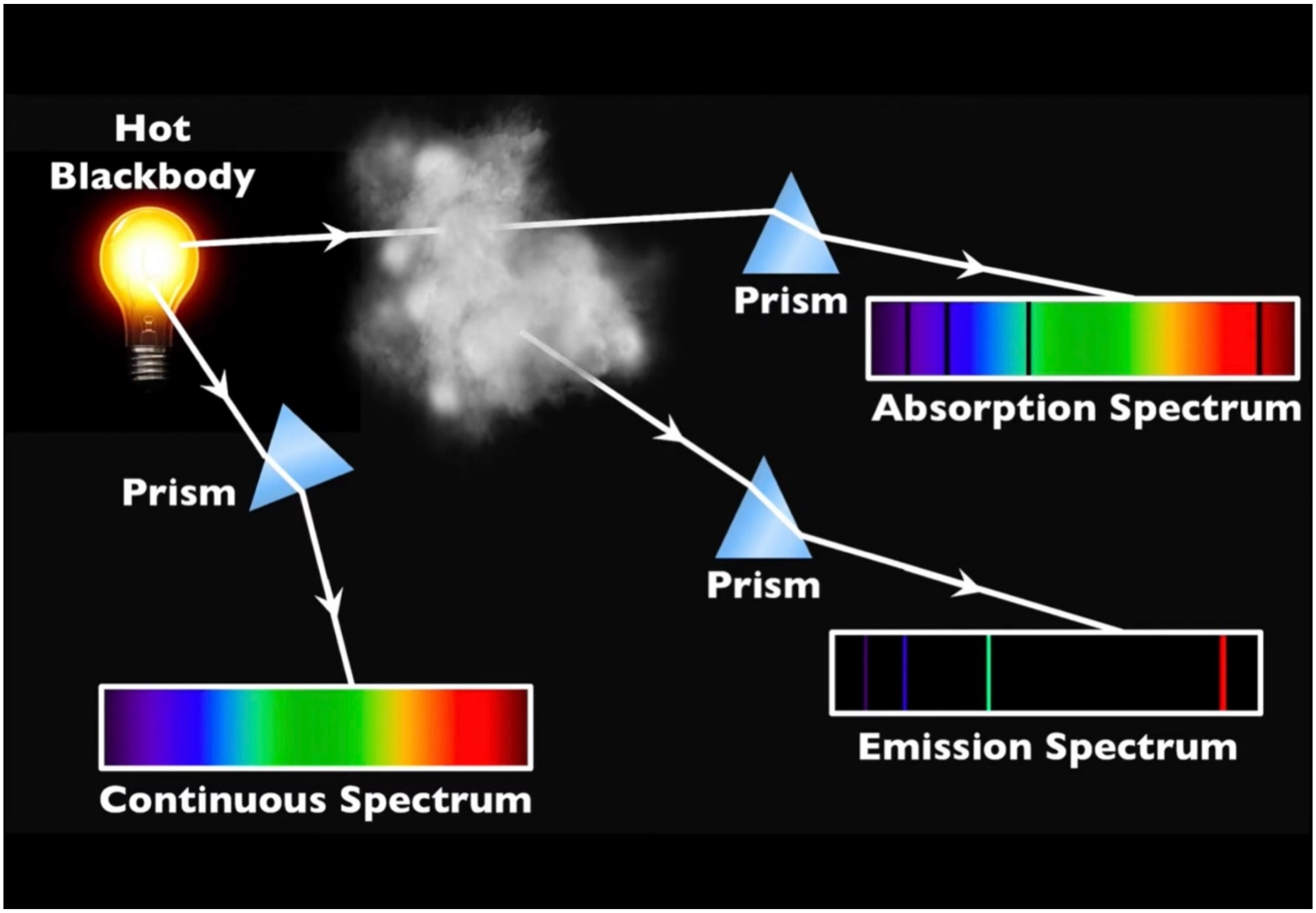


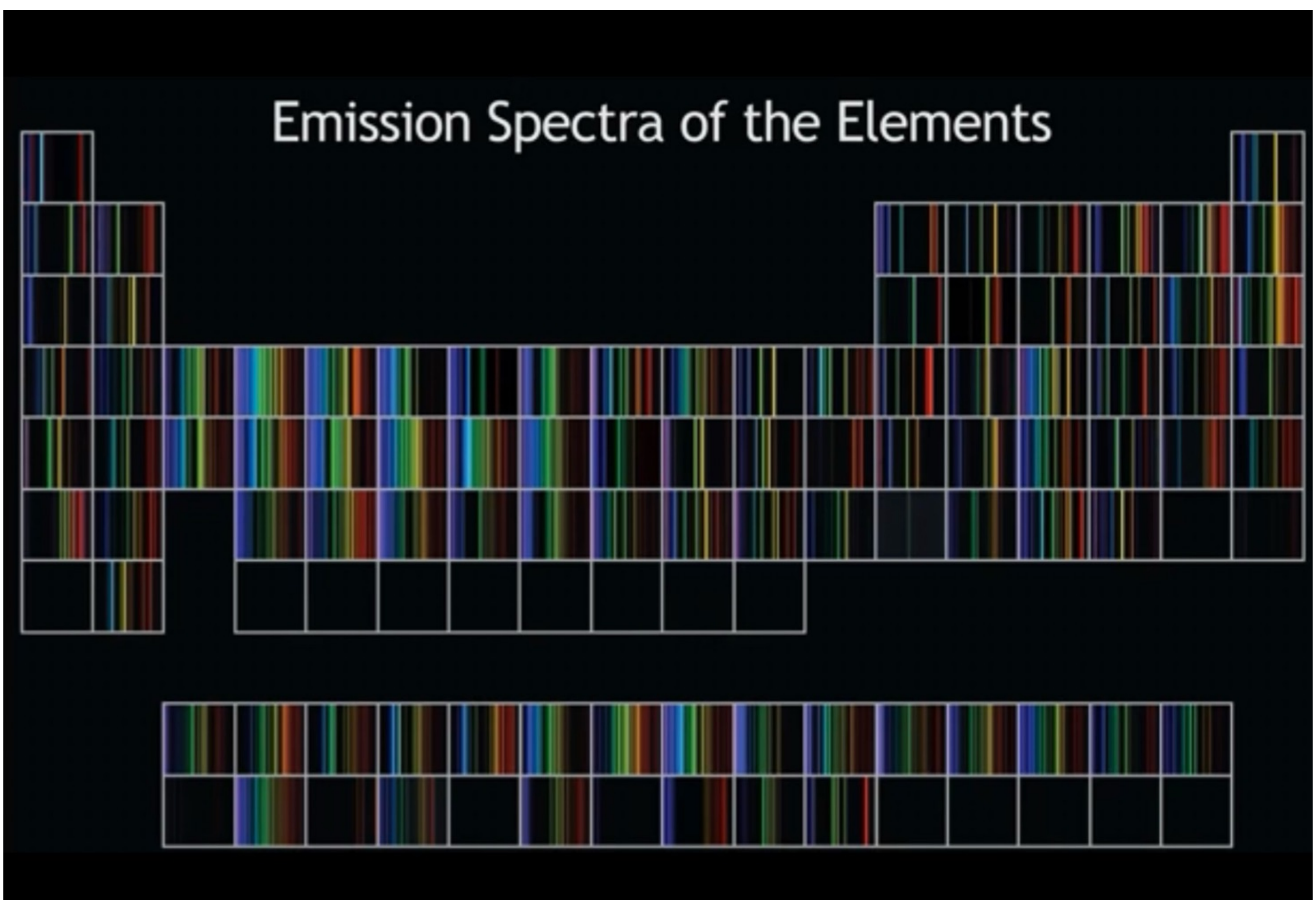
- 1814-1824: Von Fraunhofer discovered absorption lines in sun.

- 1850's: Kirchhoff discovered characteristic emission lines of elements

- 1859: Kirchoff and Bunsen discovered new elements, Cesium and Rubidium, by first observing their spectral lines.

Image: http://www.astro-assist.com


Hot
Blackbody
Prism
Prism
Prism
Absorption Spectrum
Emission Spectrum
Continuous Spectrum


Emission Spectra of the Elements

Concept Test

- A bright star shines through a dark gaseous nebula. The spectrum of this star will consist primarily of
  - Bright Lines
  - Dark Lines ← Star = “light bulb”
  - Neither

Balmer Series

- 1885: Balmer found a formula for the wavelengths of the spectral lines in Hydrogen:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{4} - \frac{1}{n^2} \right)$$

where $n=3,4,5,\ldots$

The constant R_H is Rydberg's constant:

$$R_H = 1.096776 \times 10^7 \text{ m}^{-1}$$

Rydberg Equation

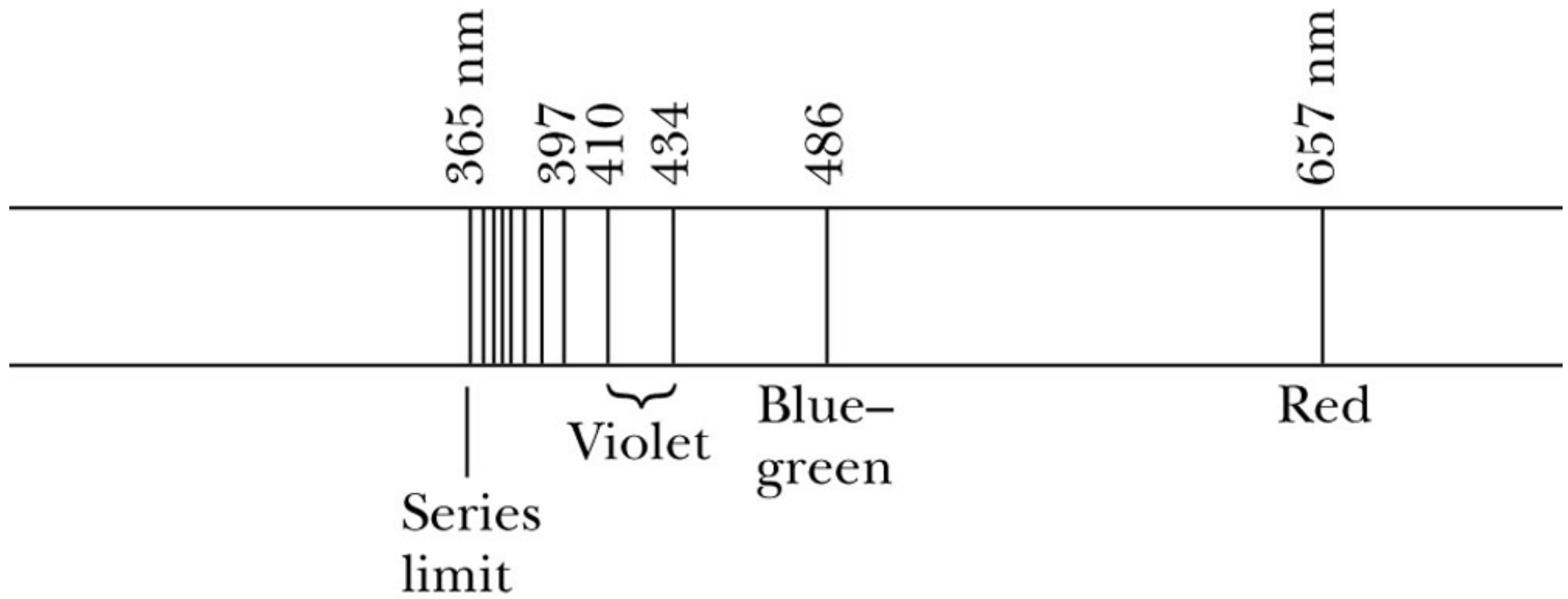
Table 3.2 Hydrogen Series of Spectral Lines

Discoverer (year)	Wavelength	n	k
Lyman (1916)	Ultraviolet	1	>1
Balmer (1885)	Visible, ultraviolet	2	>2
Paschen (1908)	Infrared	3	>3
Brackett (1922)	Infrared	4	>4
Pfund (1924)	Infrared	5	>5

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{k^2}\right) \qquad R_H = 1.096776\times 10^7 \text{ m}^{-1}$$

Image: Thornton and Rex

Interpretation (Bohr)

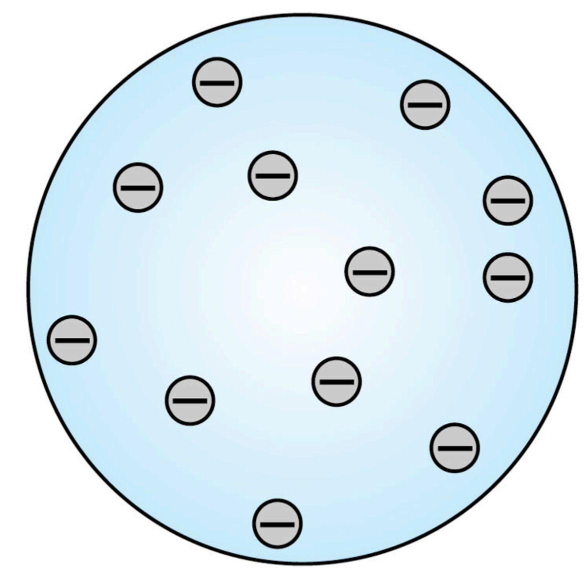


- Discrete spectral lines
  - E=hν (Planck)
  - Discrete Energies!
- Atom interacting with light
- Hence, discrete atomic “states”
- *Quantization*

Image: Thornton and Rex

Models of Atoms

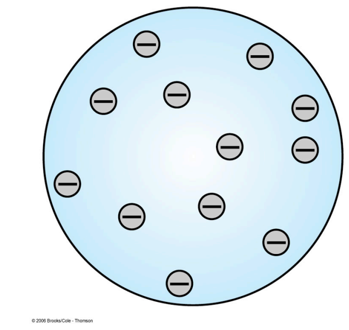
- Electrons + some positive charge must reside inside atom: but how?
- Pre-history: Thomson's Plum-Pudding
  - Electrons embedded in uniform + background ("pudding")



- Heat atoms, electrons vibrate, create E&M radiation (light)

Image: Thornton and Rex

Concept Test



- α particles are the nucleii of Helium atoms, have a charge of +2 and a mass of approximately 8000 times m_e. If the α particlesscatter off of a "plum pudding" atom, a continuous distribution of positive charge with a few (light) electrons embedded in it, we expect:
  - All of the α particles to be absorbed
  - All of the α particles to bounce backward
  - None of the α particles to bounce backward

Image: Thornton and Rex

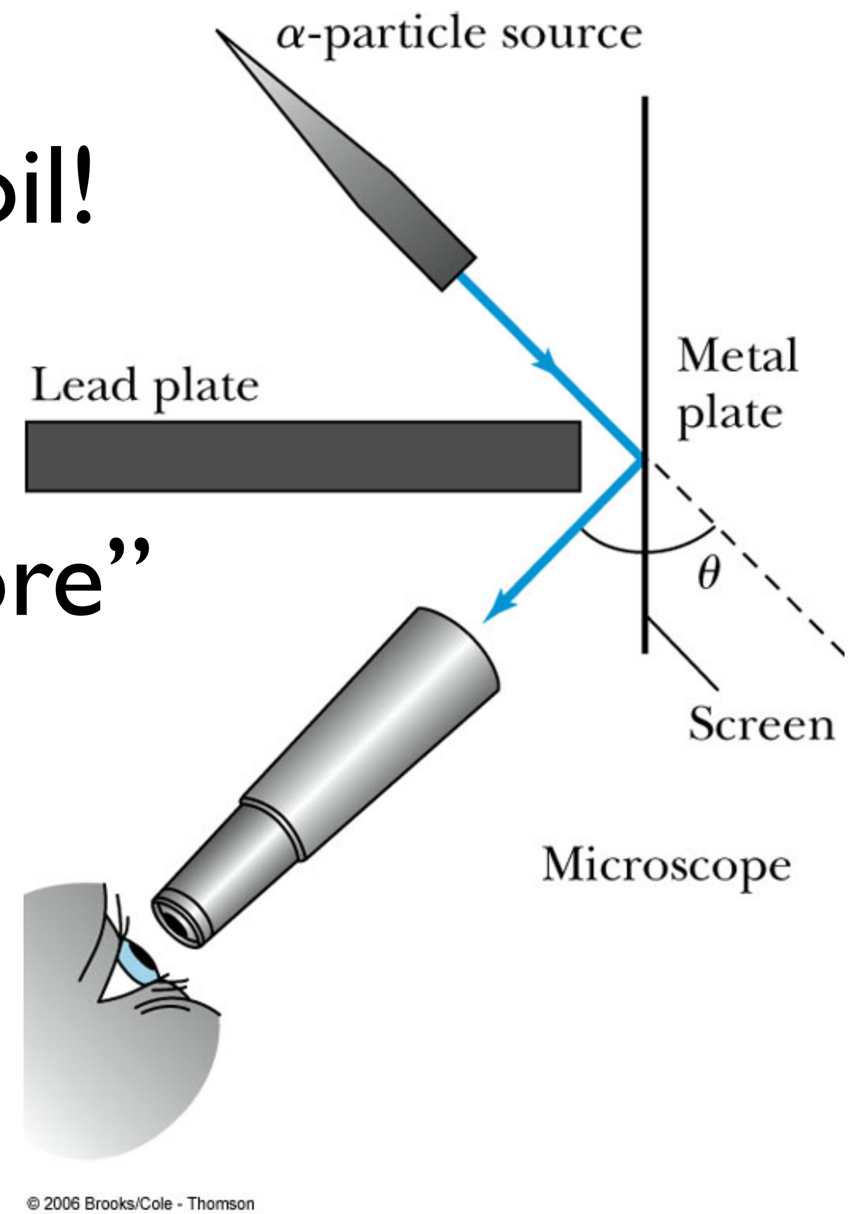
Geiger and Marsden (1909)

- α rays (modern: He nucleii)

"back-scatter" off of a thin gold foil!

- Rutherford (1911)
  - Atoms have a "hard charged core" of size $\sim 10^{-14}$m!

Ernest Rutherford
1871-1937
Nobel Prize 1908

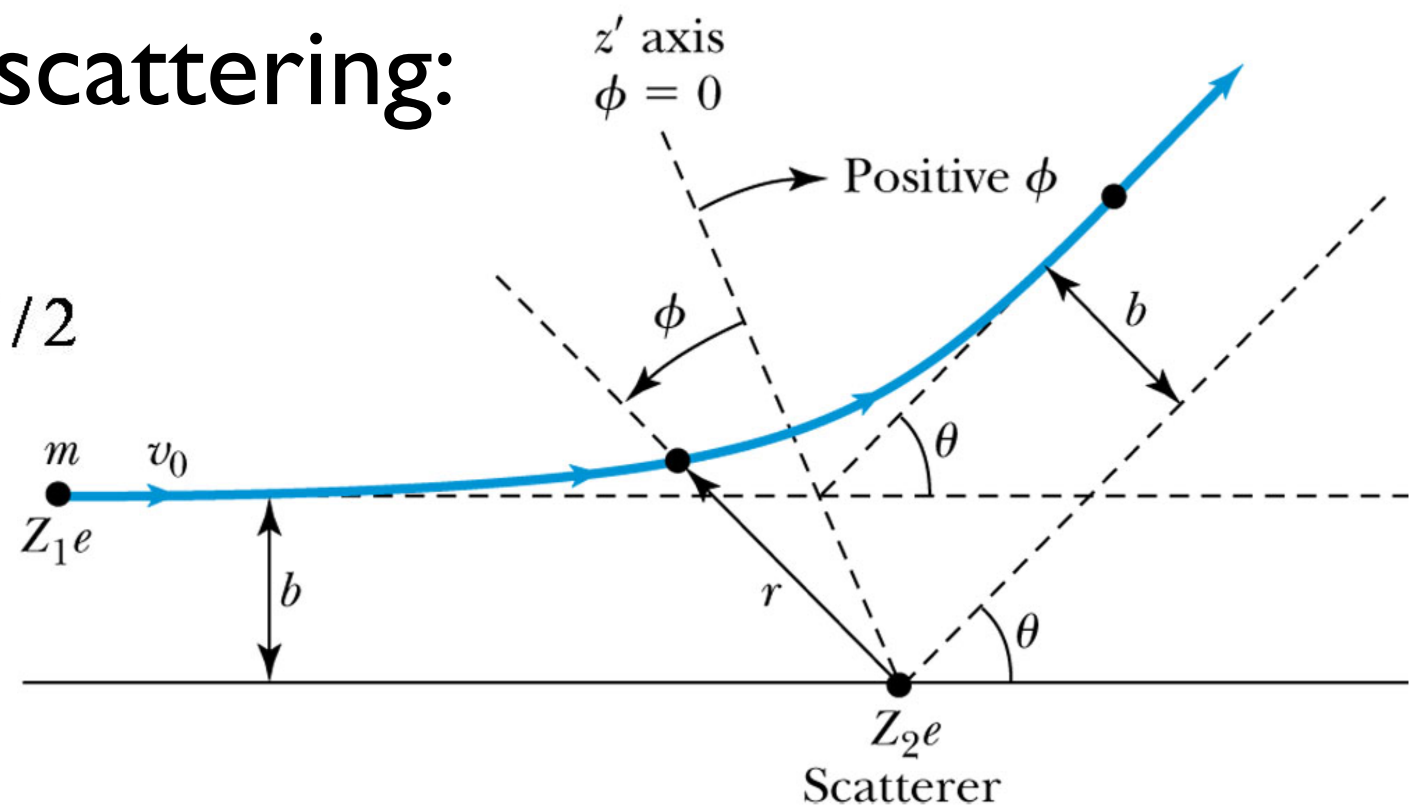


Images: Thornton and Rex
http://wikipedia.org

Scattering Experiments

- We study the properties of atoms (and smaller objects) using *scattering experiments*.
- Two parameters: impact parameter (b) and scattering angle (θ)
- For Coulomb scattering:

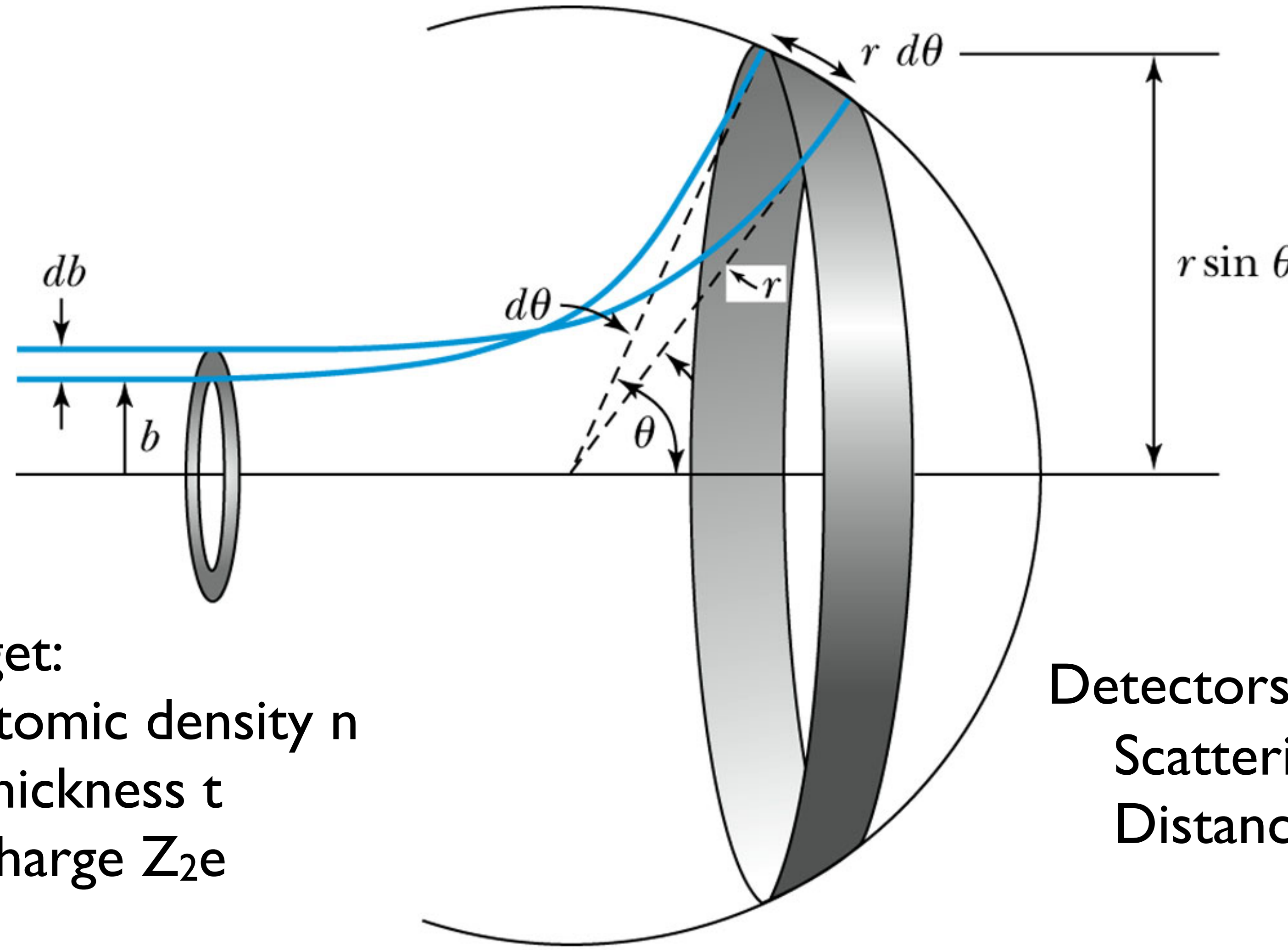
$$b = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 K}\cot\frac{\theta}{2} \qquad \text{where } K = m{v_0}^2/2$$



Images: Thornton and Rex

Rutherford Scattering Formula

N_i incoming particles
- Charge Z_1e
- Kinetic Energy K



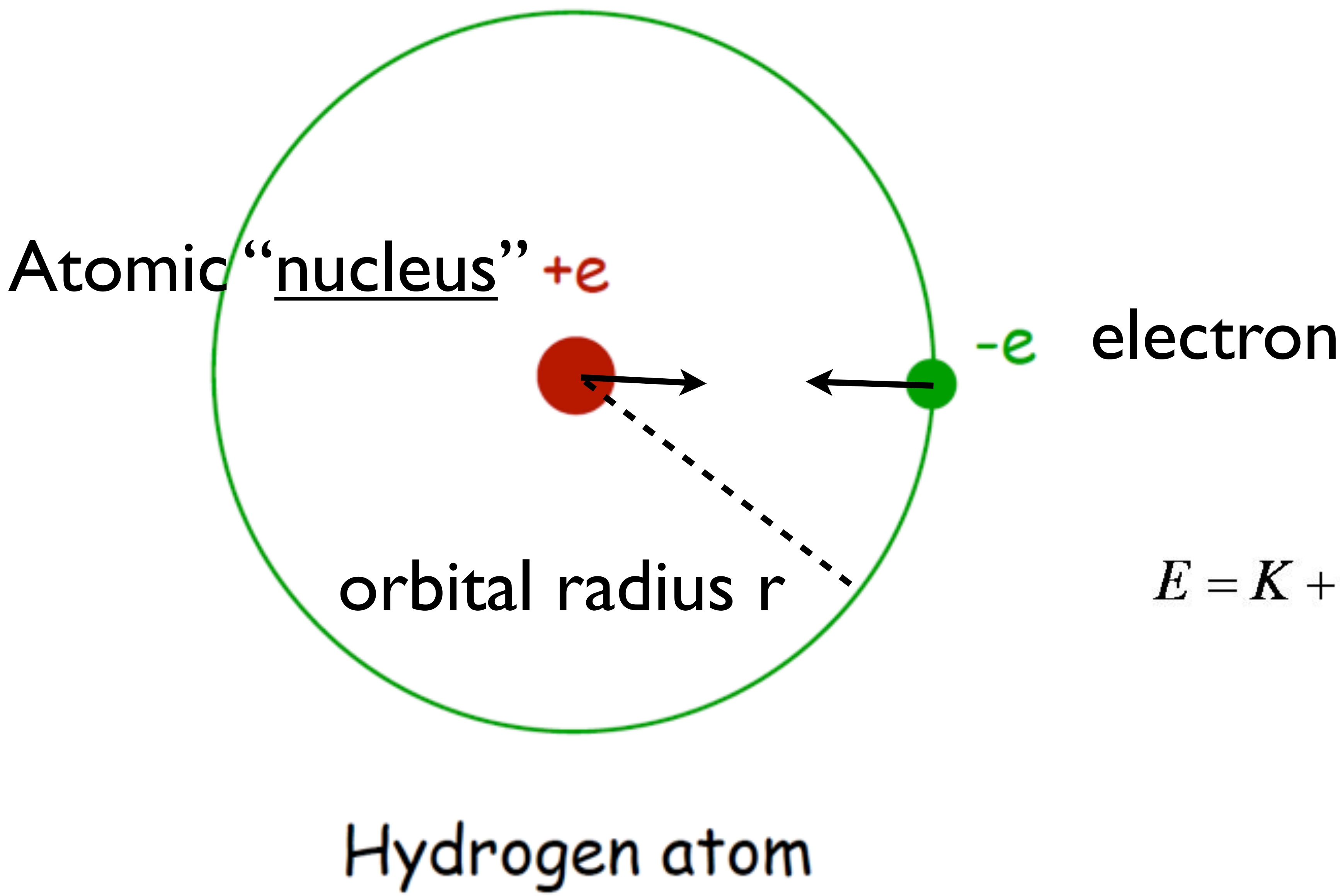
Target:
- atomic density n
- thickness t
- charge Z_2e

Detectors:
- Scattering angle θ
- Distance r

$$N(\theta) = \frac{N_i n t}{16}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{Z_1^{\,2} Z_2^{\,2}}{r^2 K^2 \sin^4(\theta/2)}$$

Images: Thornton and Rex

Rutherford's Atomic Model



Bound by Coulomb Force

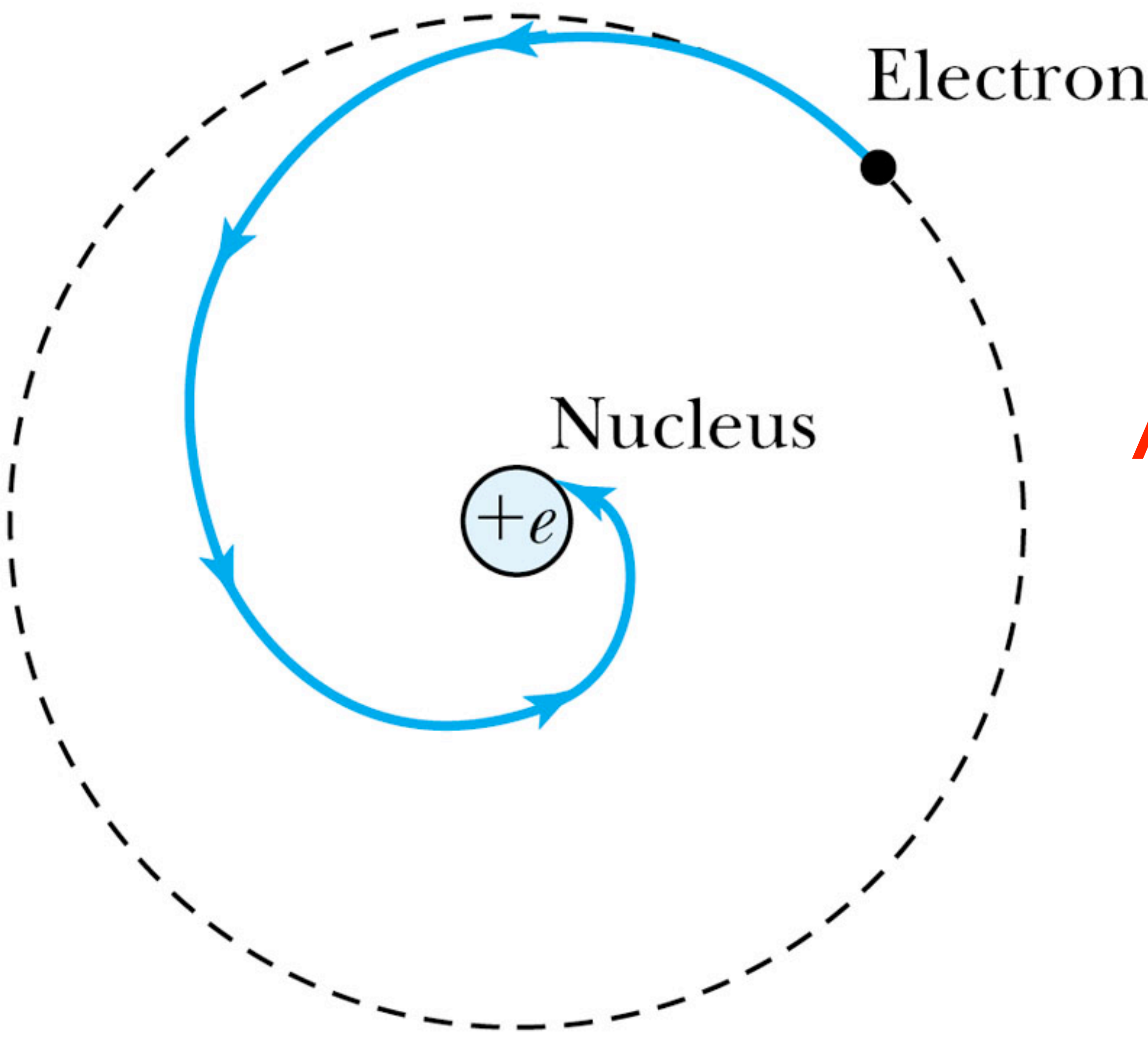
$$\vec{F}_e = \frac{-1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\hat{e}_r = \frac{mv^2}{r}$$

$$E = K + V = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = \frac{-e^2}{8\pi\varepsilon_0 r}$$

nucleus: 10^{-14} m

Images: Thornton and Rex

Problem with the Classical Model



Accelerating electrons should radiate energy - and crash into the nucleus!

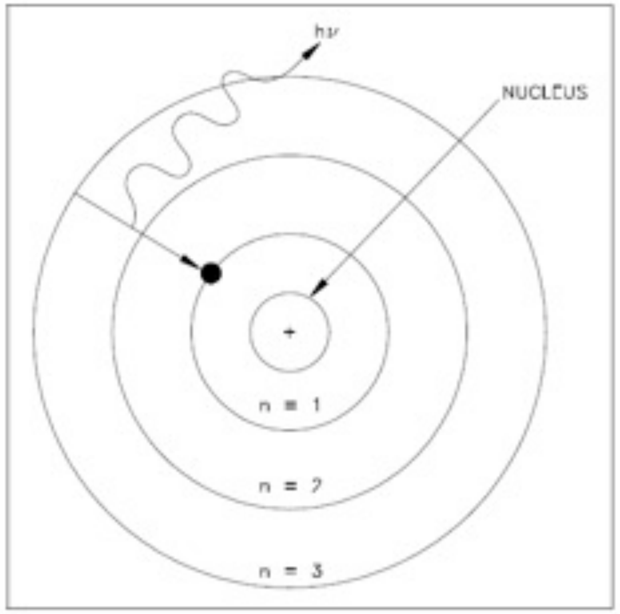
Images: Thornton and Rex

Bohr Model

- Electrons can exist only in particular (quantized) "stationary states".
- Spectral lines correspond to energy *differences* between stationary states, when electrons "jump" between states.
- Angular momentum of electron quantized, equal to an integer multiple of $h/2\pi$!

$\hbar = h/(2\pi)$

$\hbar = 1.05 \times 10^{-34}\ \mathrm{J \cdot s}$

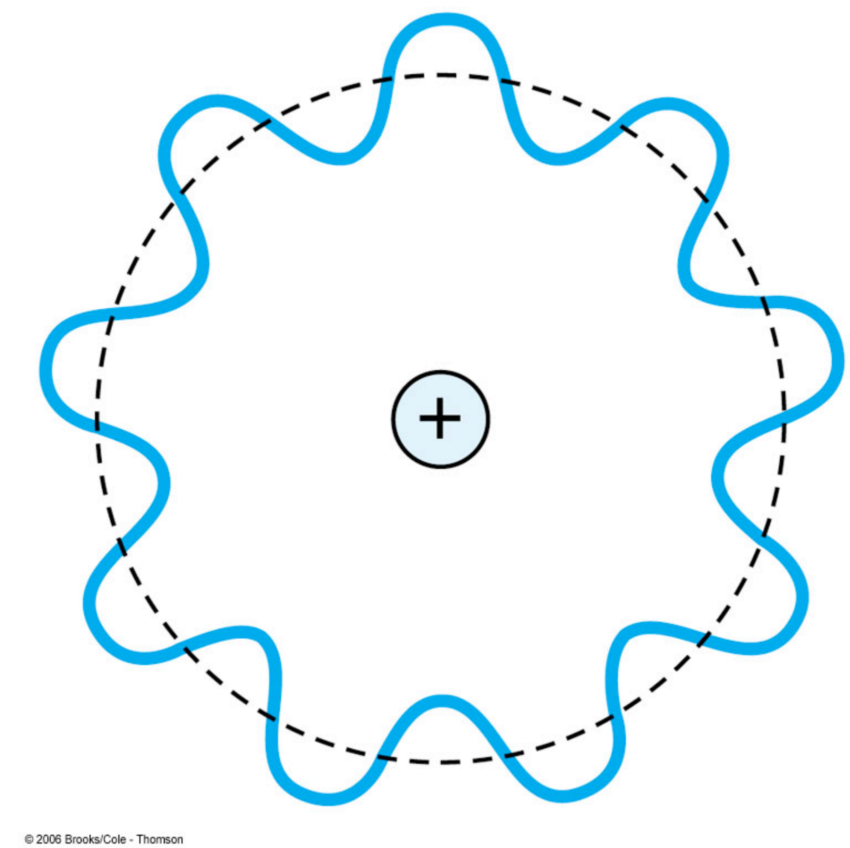


Niels Bohr
1885-1962
Nobel Prize 1922

Images: knowledgepublications.com

de Broglie & Bohr

- Bohr: $L = mvr = nh/2\pi$
- de Broglie: $\lambda = h/p$
- $p = mv$, $pr = nh/2\pi$
- Ergo: $n\lambda = 2\pi r$



- "Stationary State"=orbit with an integral number of electron de Broglie wavelengths!
- "Stationary States" = standing electron waves!

Image: Thornton and Rex

Bohr Limitations

- Nucleus is not infinitely heavy
  - $m_e \rightarrow \mu$=reduced mass
- Many electron atoms?
  - Not all electrons in n=1 state! Why?
- No systematic way forward
  - What about other systems?
  - How do quantum systems evolve?

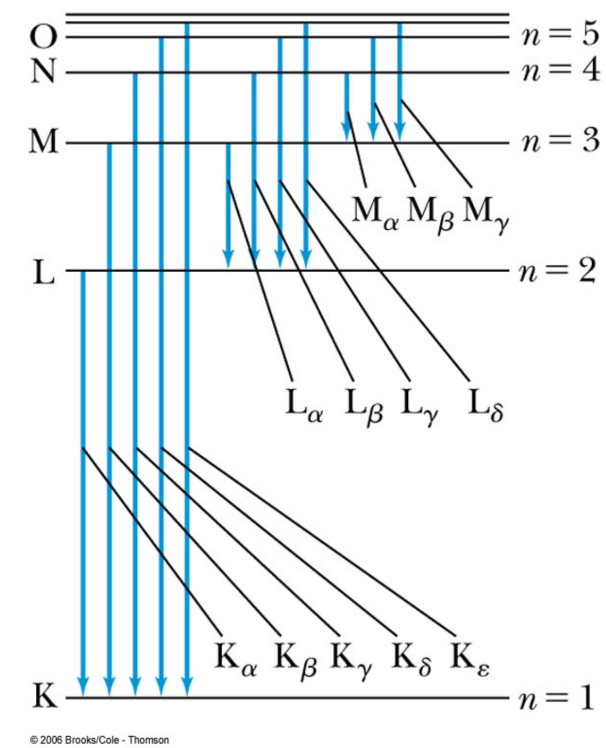


Image: Thornton and Rex

Summary, so far

- Bohr’s model of the atom
  - Builds on Rutherford’s “planetary” model.
  - “Stationery” states = “standing electron waves”.
  - Spectral lines = photon emission via electron “jumps” to different levels.

Outline

- Bohr Model Numbers
  - Bohr Shell Hypothesis
  - X-ray Spectra and Atomic Number
- Waves vs. Particles
  - Fourier Series/Transforms
  - Complex Exponentials
- Born's Interpretation of Ψ

Bohr Model

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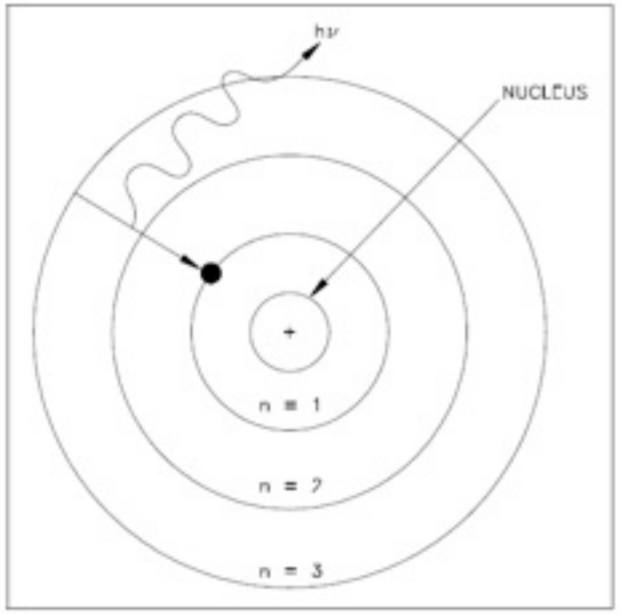


Niels Bohr
1885-1962
Nobel Prize 1922

Images: knowledgepublications.com
wikipedia.org

Bohr Radius (H atom)

Coulomb Force $$\frac{e^2}{4\pi\epsilon_0\, r_n^2} = \frac{m_e v^2}{r_n} = \frac{L^2}{m_e r_n^3}$$ Centripetal Force

Bohr Quantization $$L = n\hbar$$

$$r_n = \frac{4\pi\epsilon_0 n^2 \hbar^2}{m_e e^2} = a_0 n^2$$

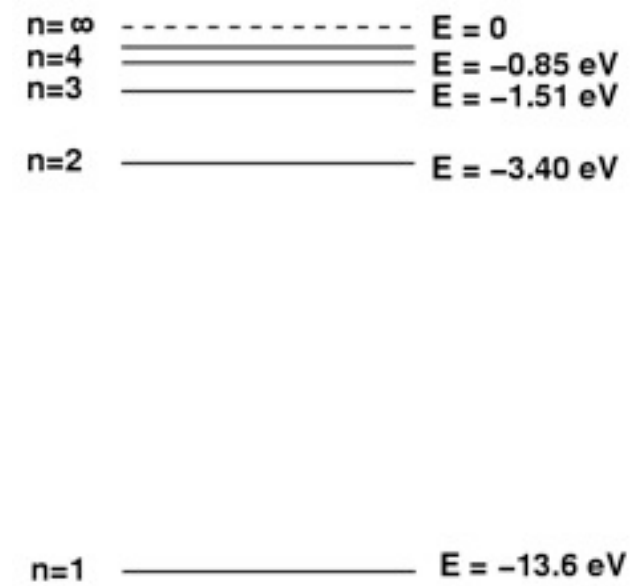
nucleus: 10^{-14} m (Rutherford)

Bohr Radius $$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} = 0.53 \times 10^{-10}\,\mathrm{m} = 0.53\mathring{A}$$

$$\frac{v}{c} = \frac{e^2}{4\pi\epsilon_0 c\hbar} \cdot \frac{1}{n} \equiv \frac{\alpha}{n} \qquad \alpha = \frac{e^2}{4\pi\epsilon_0 c\hbar} \approx \frac{1}{137}$$

Bohr H Energy Levels

$$E_n = -\frac{e^2}{8\pi\varepsilon_0 r_n} = -\frac{e^2}{8\pi\varepsilon_0 a_0 n^2} \equiv -\frac{E_0}{n^2}$$



$E_0 \approx 13.6$ eV

Transition from state n to state m:

$$\begin{aligned} E_{k\to n} &= E_k - E_n \\ &= \frac{hc}{\lambda} \\ &= E_0\left(\frac{1}{n^2} - \frac{1}{k^2}\right) \end{aligned}$$

$$\begin{aligned} \frac{1}{\lambda} &= \frac{E_0}{hc}\left(\frac{1}{n^2} - \frac{1}{k^2}\right) \\ &= R_\infty\left(\frac{1}{n^2} - \frac{1}{k^2}\right) \\ R_\infty &= \frac{13.6\,\mathrm{eV}\cdot 1.602\times 10^{-19}\,\mathrm{J/eV}}{1.986\times 10^{-25}\,\mathrm{J\,m}} \\ &= 1.097\times 10^{7}\,\mathrm{m} \end{aligned}$$

Bohr model reproduces Rydberg formula!

Image: www.physicsforums.com

Reduced Mass Correction (Replacing m_e by μ_e)

The electron and hydrogen nucleus actually revolve about their mutual center of mass as shown in Figure 4.17. This is a two-body problem, and our previous analysis should be in terms of r_e and r_M instead of just r. A straightforward analysis derived from classical mechanics shows that this two-body problem can be reduced to an equivalent one-body problem in which the motion of a particle of mass μ_e moves in a central force field around the center of mass. The only change required in the results of Section 4.4 is to replace the electron mass m_e by its **reduced mass** μ_e where

$$\frac{1}{\mu_e} = \frac{1}{m_e} + \frac{1}{M} \Rightarrow$$

$$\boxed{\mu_e = \frac{m_e M}{m_e + M} = \frac{m_e}{1 + \frac{m_e}{M}}} \tag{4.36}$$

and M is the mass of the nucleus (see Problem 53). In the case of the hydrogen atom, M is the proton mass, and the correction for the hydrogen atom is $\mu_e = 0.999456\, m_e$. This difference can be measured experimentally. The Rydberg constant for infinite nuclear mass, R_∞, defined in Equation (4.29), should be replaced by R, where

$$R = \frac{\mu_e}{m_e} R_\infty = \frac{1}{1 + \frac{m_e}{M}} R_\infty = \frac{\mu_e e^4}{4\pi c \hbar^3 (4\pi\epsilon_0)^2} \tag{4.37}$$

The Rydberg constant for hydrogen is $R_H = 1.096776 \times 10^7\ \mathrm{m}^{-1}$.

Concept Test

- Let's apply the Bohr model to carbon, whose nucleus has Z=+6. The number of electrons in the neutral atom is therefore
  - 4
  - 5
  - 6 ← Are all six electrons in the lowest energy stationary state?
  - 7

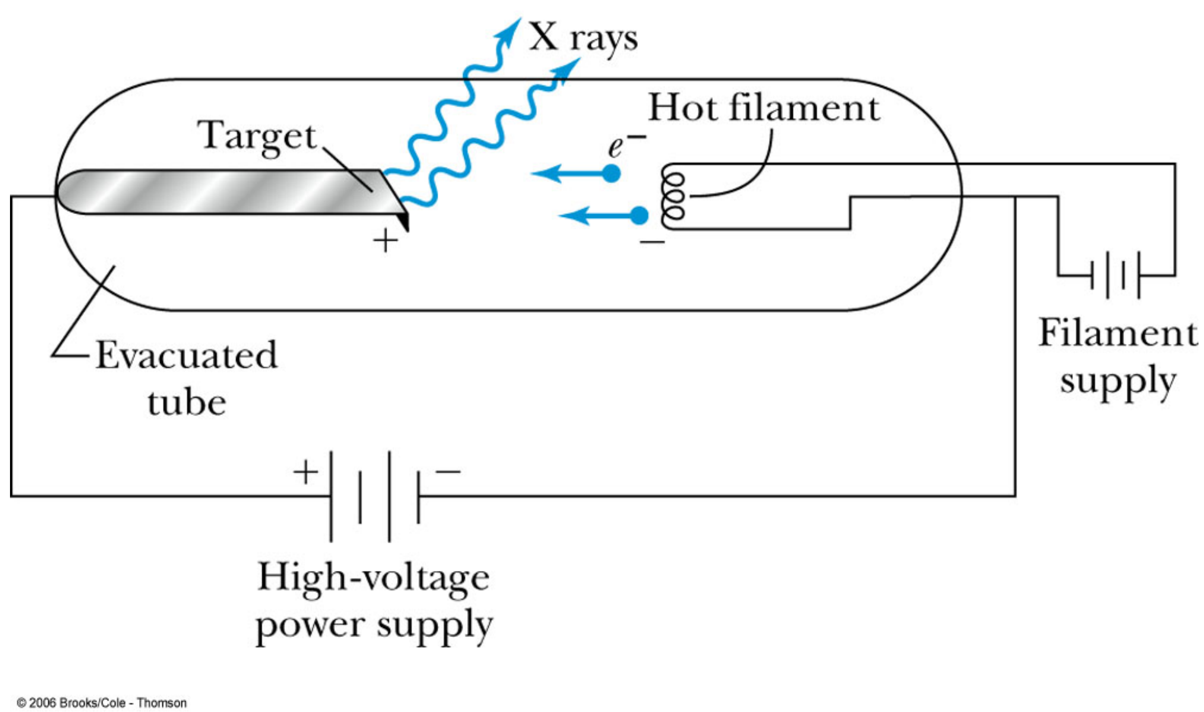
Bohr Shell Hypothesis

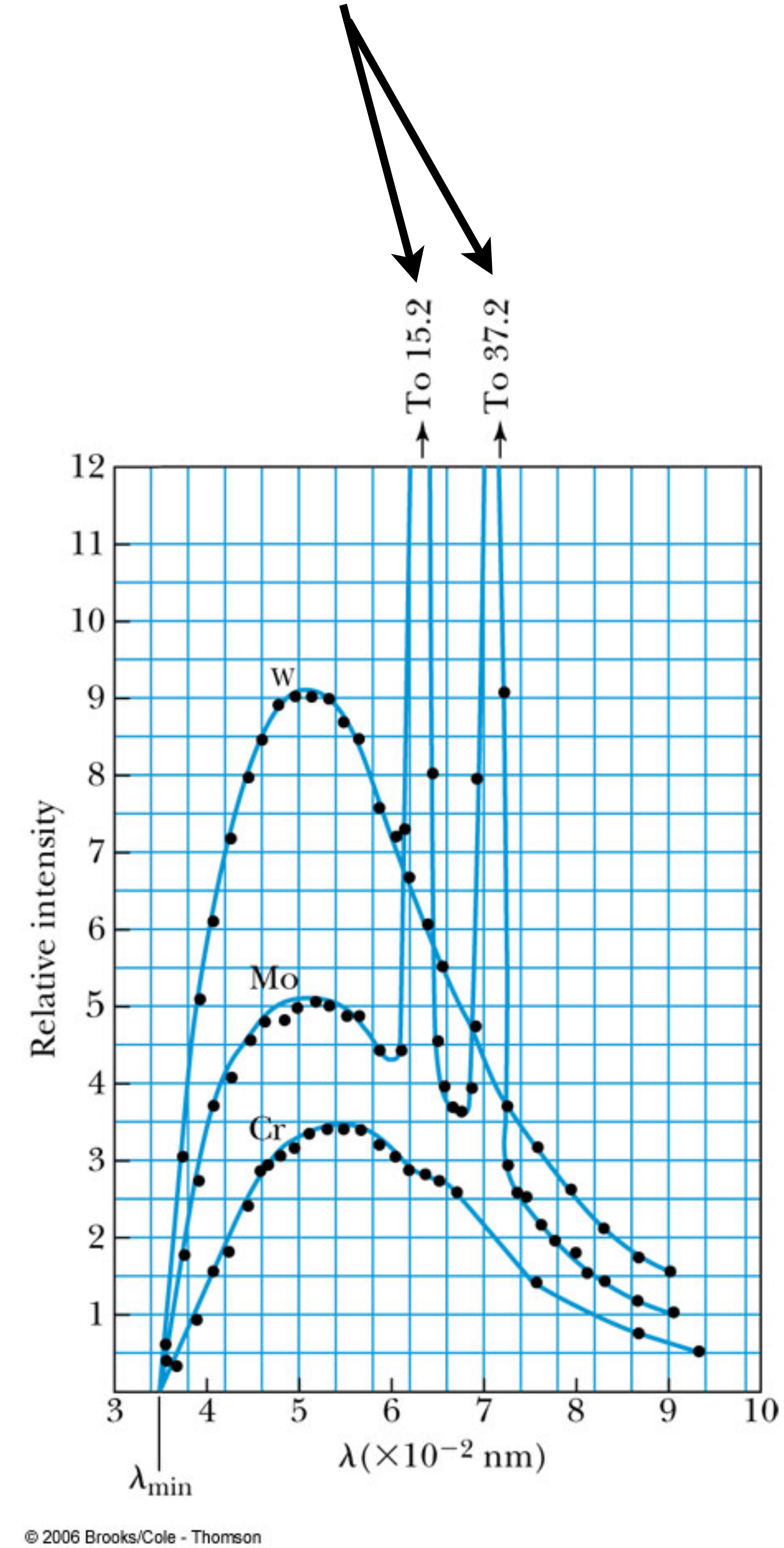
$$\frac{1}{\lambda_{k\rightarrow n}} = Z^2 R \left(\frac{1}{n^2} - \frac{1}{k^2} \right)$$

- Bohr model generalizes to any single-electron atom: $e^2 \rightarrow Ze^2$
- Bohr model yields many quantized energy levels for any atom, depending on n.
- Bohr asserted that any given shell could only hold a certain number of electrons - after it was filled, electrons must occupy the next available level. Why? (We will see!)

X-Ray Spectra Peaks:

Inverse photoelectric effect:



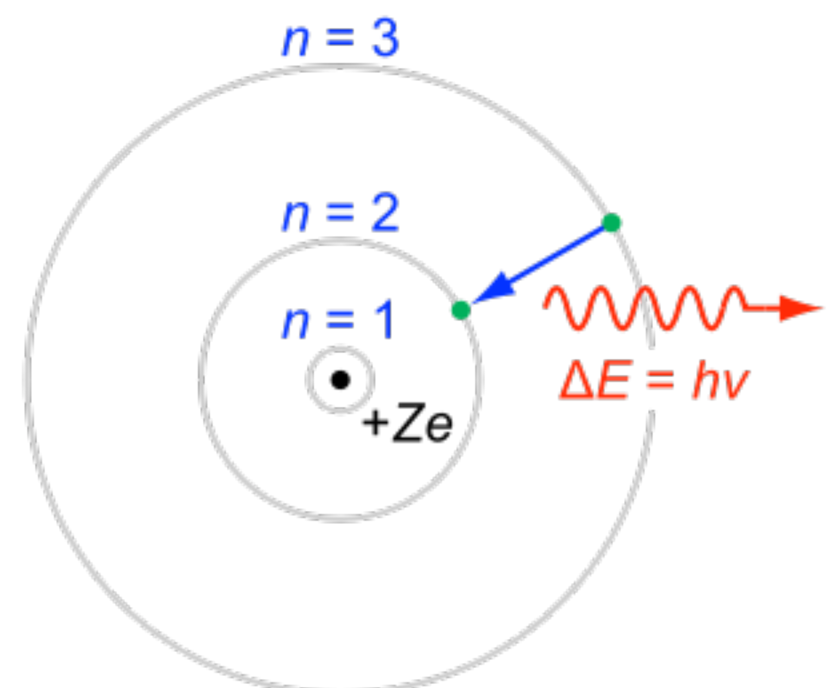


Can be explained in terms of shell hypothesis...

Images: Thornton and Rex

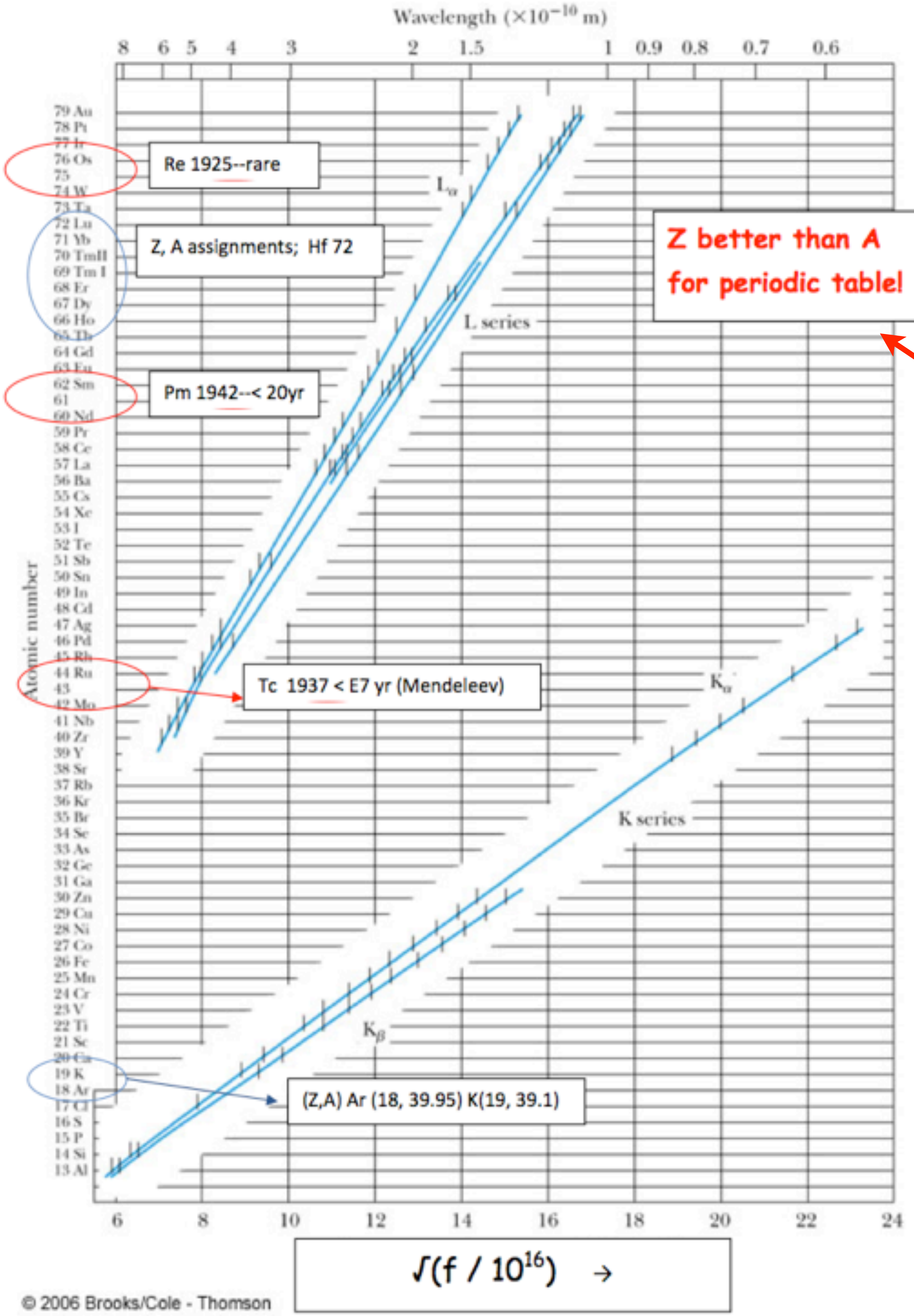
X-ray Peaks II

- X-rays excite electrons from n=1 "shell" (K-shell).
- An electron from an upper shell cascades down to take its place
- Since $E_n \propto Z^2$, we expect square root of peak frequencies to be linear in Z!



Images http://wikipedia.org

Moseley Plot (1913)



Corrected Mendeleev!

Henry Moseley
1887-1915

Images: Thornton and Rex