The Hydrogen atom 1.

Coulomb potential:

$$V(r) = -k\,\frac{e^2}{r} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r}$$

Schrödinger equation

$$-\frac{\hbar^2}{2m}\cdot\underbrace{\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)}_{\text{Laplace operator : }\Delta} + V\psi = E\psi$$

$$-\frac{\hbar^2}{2m}\cdot\Delta\psi + V\psi = E\psi$$

In Cartesian coordinates:

Nabla operator : $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

Laplace operator: $\Delta = \nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$

Laplace operator in spherical coordinates:

$$\Delta = \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\,\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial}{\partial\theta}\right)$$

$$+\frac{1}{r^2\sin^2\theta}\cdot\frac{\partial^2}{\partial\varphi^2}$$

coordinates : (r, θ, φ)

The Hydrogen atom 2.

Separation of variables:

$$\Psi(r,\Theta,\varphi) = R(r)\cdot f(\Theta)\cdot g(\varphi)$$

Radial equation:

$$\frac{1}{r^2}\cdot\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{2\mu}{\hbar^2}\left[E-V-\frac{\hbar^2}{2\mu}\cdot\frac{\ell(\ell+1)}{r^2}\right]R=0$$

associated Laguerre equation

Polar or angular equation:

$$\frac{1}{\sin\Theta}\cdot\frac{d}{d\Theta}\left(\sin\Theta\cdot\frac{df}{d\Theta}\right)+\left[\ell(\ell+1)-\frac{m_\ell^2}{\sin^2\Theta}\right]f=0$$

associated Legendre equation

Azimuthal equation:

$$\frac{d^2g}{d\varphi^2} = -m_\ell^2\cdot g$$

Quantum numbers:

principal : n ; $n=1,2,3,\ldots$

orbital angular momentum: ℓ
$\ell = 0, 1, 2, \ldots (n-1)$;
magnetic: m_ℓ

$m_\ell = -\ell, \ldots -1, 0, +1, \ldots +\ell$; $-\ell \le m_\ell \le \ell$

The Hydrogen atom 3.

$$\Psi(r,\theta,\varphi) = R(r)\cdot f(\theta)\cdot g(\varphi) =$$

$$= R_{nl}(r)\cdot Y_{lm_l}(\theta,\varphi)$$

$R_{nl}(r)$: radial wave functions

$n=1,\ l=0$: $R_{10}(r) = \frac{2}{\sqrt{a_0^3}}\cdot e^{-\frac{r}{a_0}}$

$n=2,\ l=0$: $R_{20}(r) = \frac{1}{\sqrt{(2a_0)^3}}\cdot\left(2-\frac{r}{a_0}\right)\cdot e^{-\frac{r}{2a_0}}$

$n=2,\ l=1$: $R_{21}(r) = \frac{1}{\sqrt{3(2a_0)^3}}\cdot\frac{r}{a_0}\cdot e^{-\frac{r}{2a_0}}$

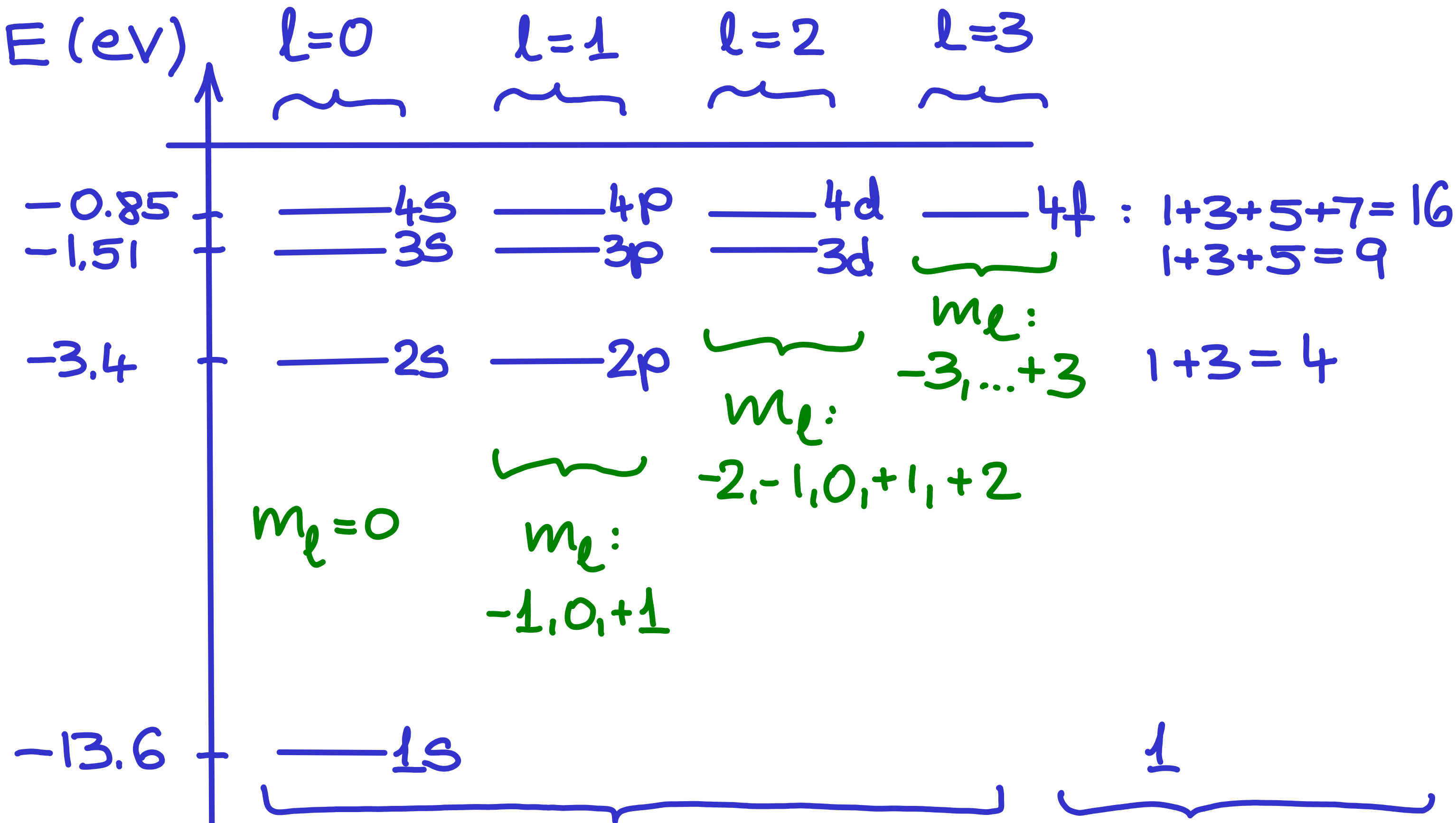
$Y_{lm_l}(\theta,\varphi)$: spherical harmonics

$l=0,\ m_l=0$: $Y_{00} = \frac{1}{\sqrt{4\pi}}$

$l=1,\ m_l=0$: $Y_{10} = \sqrt{\frac{3}{4\pi}}\cdot\cos\theta$

$l=1,\ m_l=\pm 1$: $Y_{1\pm1} = \mp\sqrt{\frac{3}{8\pi}}\cdot\sin\theta\cdot e^{\pm i\varphi}$

The Hydrogen atom 4.



Energy: $E_n = -\frac{E_0}{n^2}$

$E_0 = 13.6\,\text{eV}$

degeneracy: n^2

The number of orbitals corresponding to a given principle quantum number n is n^2

$$\sum_{l=0}^{n-1}(2l+1) = n^2$$

l = 0, 1, 2, 3, 4, 5 . . .

s p d f g h . . .

(label)