The beginnings of nuclear physics can be traced to Rutherford, who showed that the nucleus is much smaller than the atom, and to Becquerel and Marie and Pierre Curie, who discovered that certain atoms spontaneously emitted radiation.

These emitted radiations were called α, β, and γ rays. α rays were identified as He nuclei, β rays as most likely electrons and γ rays as high energy photons.

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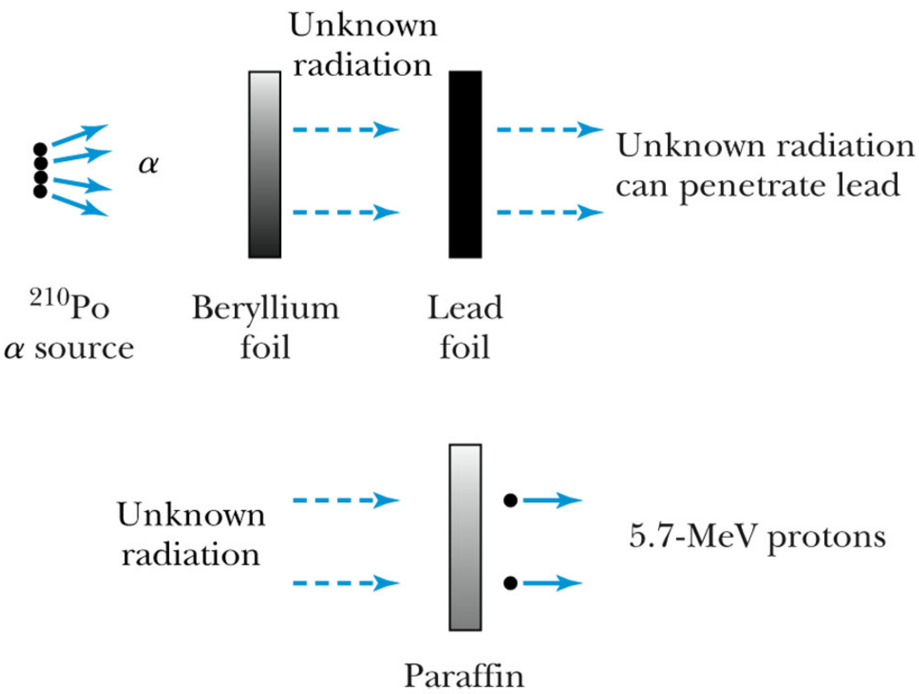
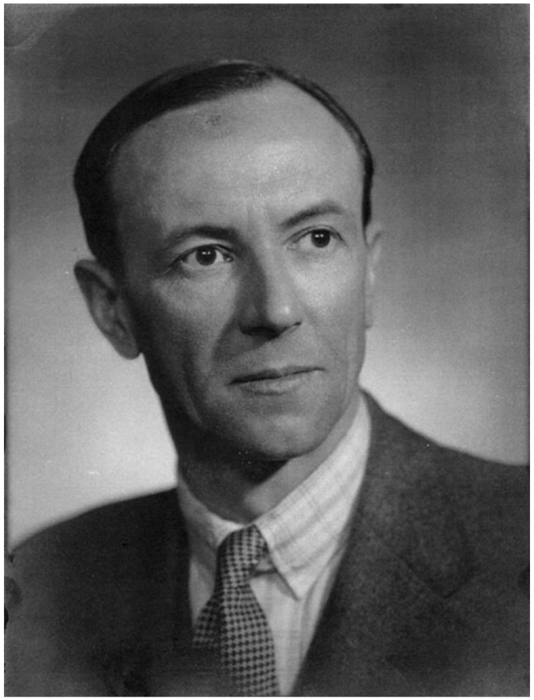
The actual composition of the nucleus was not known, but since electrons were emitted, it was first thought that some electrons were confined inside the nucleus.

There were several reasons to doubt this

1. Confined in such a small well ($r=8\times10^{-15}$ m), they would have energies of order $\pi^2\hbar^2c^2/8m_ec^2r^2 = 1460$ MeV.
2. The nuclear spin of a deuteron, S=1, cannot be from the spins of two protons and one electron.
3. The magnetic moments of nuclei are small compare to μ_B.

Irene Curie and Frederic Joliot showed that radiation from Beryllium bombarded by α particles could penetrate lead foil and ejected 5.7 MeV protons from paraffin.





In 1932, James Chadwick correctly deduced that the new radiation was a beam of particles with the mass of a proton, but with no charge. He called them neutrons and their presence in the nucleus accounts for isotopes.

Table 12.1 Some Nucleon and Electron Properties

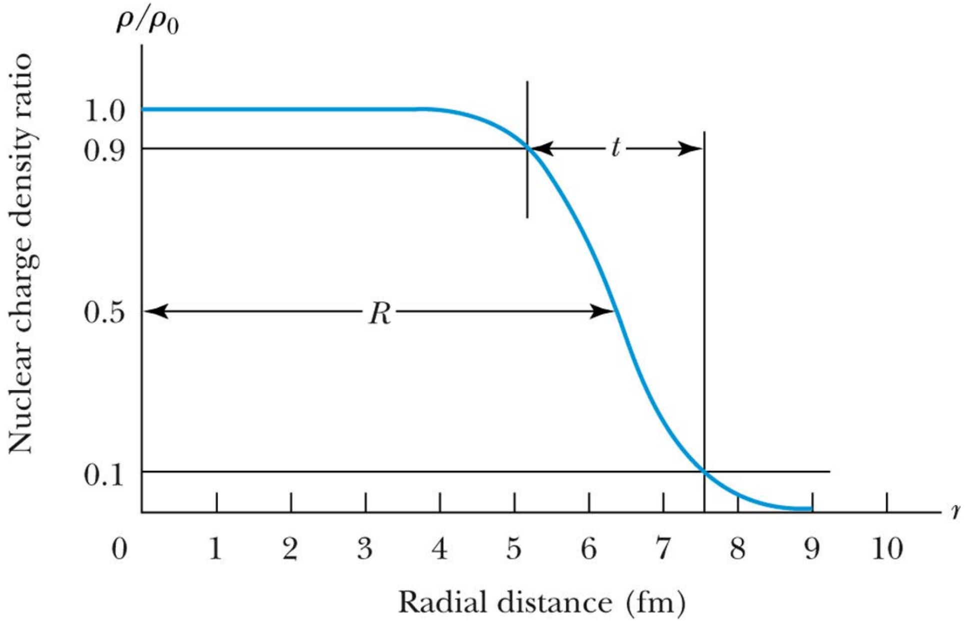
Particle	Symbol	Rest Energy (MeV)	Charge	Mass (u)	Spin	Magnetic Moment
Proton	p	938.272	$+e$	1.0072765	1/2	$2.79\ \mu_N$
Neutron	n	939.566	0	1.0086649	1/2	$-1.91\ \mu_N$
Electron	e	0.51100	$-e$	5.4858×10^{-4}	1/2	$-1.00116\ \mu_B$

With the photon, the particles in the Table were the elementary particles in 1932. The proton and the neutron interact with a new force, the strong nuclear force. This force has a very short range and is much stronger than the Coulomb force that binds electrons to the nucleus.

Nuclei can be thought of as spherical with a radius given by

$$r = r_0 A^{1/3}, \quad r_0 \simeq 1.2\,\mathrm{fm}.$$

The density can be measured by elastic electron scattering. This was first done by Robert Hofstadter, who measured the charge radius of several nuclei.



Like the electron, the proton and the neutron have spin 1/2. They have magnetic moments that are measured in nuclear magnetons

$$\mu_N = \frac{e\hbar}{2m_p}.$$

This measure is much smaller than μ_B since m_p/m_e is about 1837. The measured magnetic moments of the proton and neutron are

$$\mu_p = 2.79\mu_N, \quad \mu_n = -1.91\mu_N.$$

Recall that $\mu_e = -1.00116\mu_B$.

The factors of $2.79\mu_N$ and $-1.91\mu_N$ are quite different from the $-\mu_B$ in the electron case. This is the first indication that the strong force is more complicated than electrodynamics.

Deuterium

The simplest multinucleon nucleus is deuterium, a proton-neutron system bound by the strong force. The deuteron mass is

$$m_d = m_n + m_p - B_d/c^2,$$

where $B_d > 0$ is the binding energy.

To use the <u>atomic</u> masses, add an electron to each side of the previous relation

$$m_d + m_e = m_n + m_p + m_e - B_d/c^2,$$

and realize that $m_d + m_e = M(^2H)$ is the atomic mass of the deuteron if we neglect the small Coulomb binding of the electron. We can then write

$$M(^2H) = m_n + M(^1H) - B_d/c^2.$$

This is done because atomic masses are easier to measure.

The binding energy is positive since

$$
\begin{aligned}
B_d/c^2 &= m_n + M(^1H) - M(^2H) \\
&= 1.00867\,\mathrm{u} + 1.00783\,\mathrm{u} - 2.01410\,\mathrm{u} \\
&= 0.00239\,\mathrm{u} = 2.224\,\mathrm{Mev/c^2},
\end{aligned}
$$

Where the unit u is defined as 1/12 of the atomic mass of ^{12}C and the conversion is

$$1\,\mathrm{u} = 1.66054 \times 10^{-27}\,\mathrm{kg} = 931.49\,\mathrm{MeV/c^2}.$$

A nucleus X with A nucleons and Z protons is denoted by

$$^A_Z X.$$

atomic mass unit $1\,u \equiv \frac{M(^{12}_{6}C)}{12}$

The binding energy of a general isotope is the difference

$$B({}^{A}_{Z}X)/c^2 = (A - Z)m_n + ZM({}^{1}H) - M({}^{A}_{Z}X).$$

To verify the actual value of the binding energy of the deuteron, we can measure the dissociation energy using photons. The reaction is

$$\gamma + d \rightarrow n + p.$$

In the laboratory frame, the photon and deuteron four-momenta are

$$k^\mu = (\frac{\omega}{c}, 0, 0, \frac{\omega}{c}),\ d^\mu = (M(^2H)c, 0, 0, 0).$$

The minimum photon energy needed to produce and neutron and a hydrogen atom at rest in the center of mass is just m_n+M(^{1}H). Hence, in any frame

$$(n+p)^2 = \left(m_n + M(^1H)\right)^2 c^2.$$

By conservation of energy and momentum, $\hbar$k + d $\rightarrow$ n + p, so

$$\begin{aligned} \left(m_n + M(^1H)\right)^2 c^2 &= (\hbar k + d)^2 \\ &= 2\hbar k\cdot d + d\cdot d = 2\hbar\omega M(^2H) + M^2(^2H)c^2. \end{aligned}$$

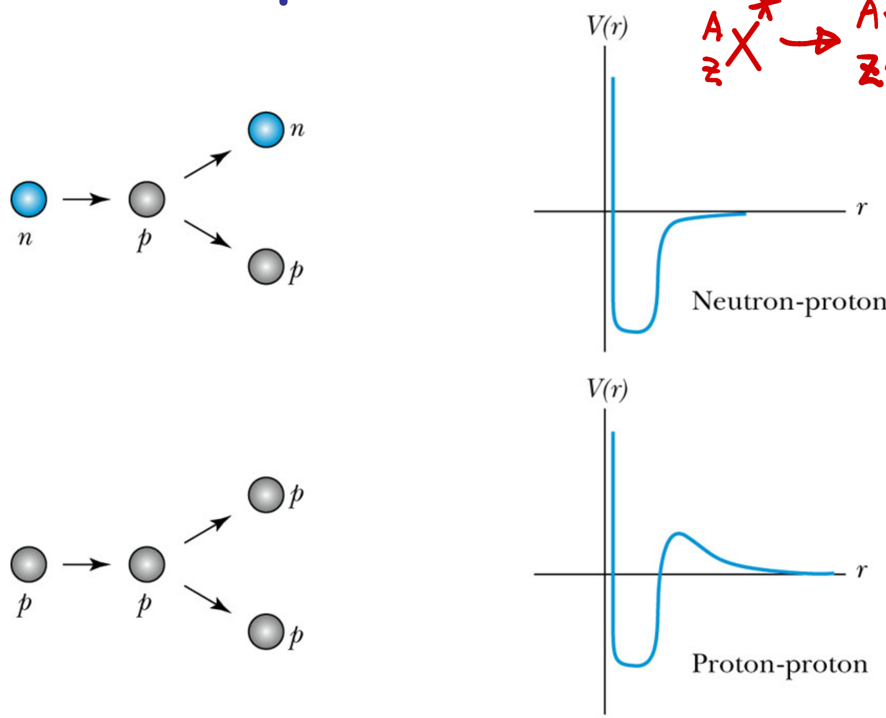
Solving this for $\hbar\omega$ gives

$$\hbar\omega = B\left(1 + \frac{B}{2M(^2H)c^2}\right).$$

The second term in the parenthesis accounts for momentum conservation.

Another indication that the model for deuterium is working is that the spin of this nucleus is 1 and its magnetic moment is $0.86\mu_N$, just about the sum of the proton and neutron magnetic moments.

Since the deuteron has spin 1 and is relatively weakly bound, the nuclear force appears to be sensitive to the nucleon spins. It is also short ranged, falling off rapidly as the separation exceed 3 fm.



(a) (b)

$^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + (n, \gamma)$

This explains why excited nucleus usually decays via emission of neutrons or photons, but not protons.

Measuring the Binding Energy of Deuteron

1. Isotopes (with the same atomic number Z for ${}^{A}_{Z}X$)

	${}^{1}_{1}H$	${}^{2}_{1}H$	${}^{3}_{1}H$
Atom:	Hydrogen	Deuterium	Tritium
Nucleus:	p Proton	p+n Deutron	p+(2n) Triton

${}^{1}_{1}H$ and ${}^{3}_{2}He$ are the only two stable nuclides with more protons (p) than neutrons (n).

$$1\,u \equiv \frac{1}{12}\left(\text{Mass of } {}^{12}_{6}C \text{ atom}\right)$$

Isotopes of hydrogen ($_1$H)

Main isotopes			Decay	
	abundance	**half-life** ($t_{1/2}$)	**mode**	**product**
^{1}H	99.9855%	stable		
^{2}H	0.0145%	stable		
^{3}H	trace	12.33 y	β^-	^{3}He

Standard atomic weight $A_r°$(H)

[1.007 84, 1.008 11][1]

1.0080 ± 0.0002 (abridged)[2]

2. Periodic table atomic mass reports the terrestrial weighted averge of all naturally occurring Carbon isotopes.

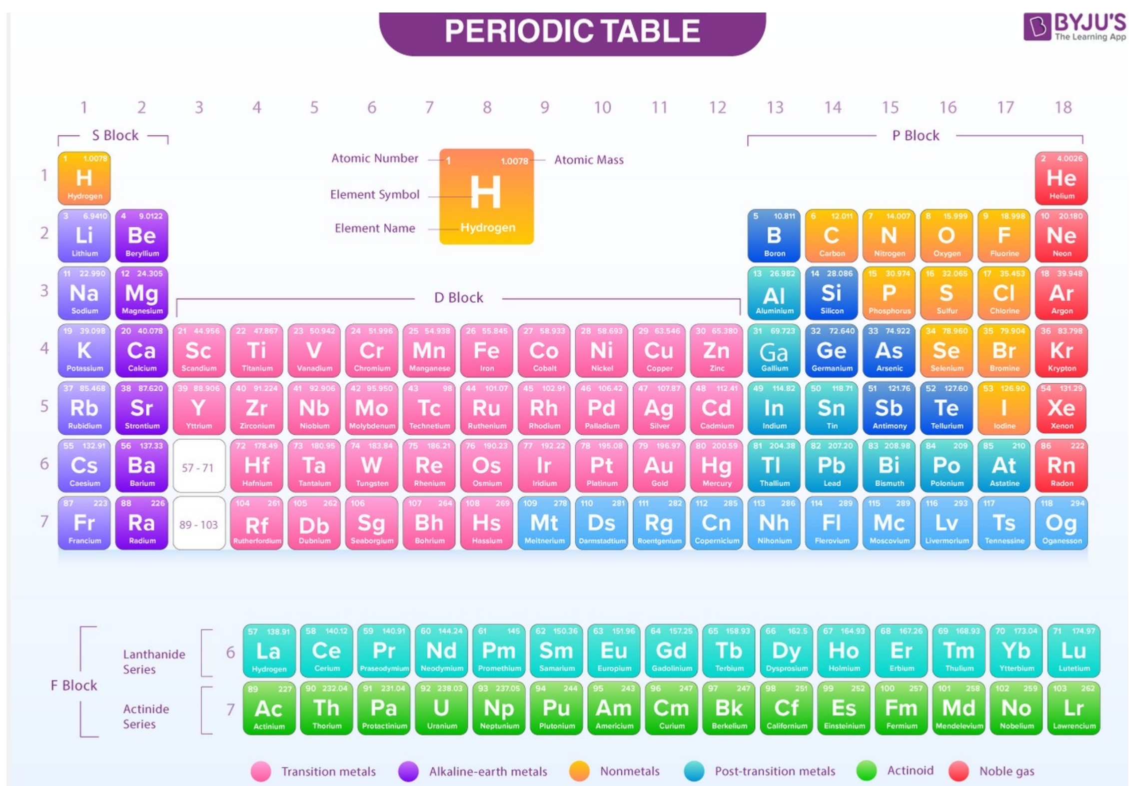
Measuring the ratio $\frac{m}{z} \approx \frac{B^2 r^2}{2V}$ for accelerated ions passing through magnetic field.

	$^{12}_{6}C$	$^{13}_{6}C$
Abundance	98.89%	1.11%
atomic mass	12.0000 U (by definition)	13.00335 U (measured via mass spectrometry)

$(0.9889 \times 12.0000 + 0.0111 \times 13.00335)$

$= 12.0111$ U

$\Rightarrow \; {}_{6}C^{12.011}$



3. Consider the Fusion Process

slow neutron + Proton → Deuteron + Gamma

$(n + p \rightarrow d + \gamma)$

(d: ${}^2_1H$)

By energy Conservation:

$$KE_n + KE_p + m_n c^2 + m_p c^2 = m_d c^2 + KE_d + E_\gamma$$

Since KE_n, KE_p, KE_d $\ll E_b$ ← Deuteron Binding energy $(\sim 2.2\ \text{MeV})$

$KE_n \sim 0.03\,\text{eV}$, $KE_p \sim 1\,\text{eV}$, $KE_d \sim 13\,\text{keV}$

(KE: Kinetic energy)

So, the gamma ray's energy

$$E_\gamma \simeq (m_n + m_p - m_d)c^2 \equiv E_b$$

If including small deuteron recoil energy $(\sim 13\,\text{keV})$, then

$$E_b = E_\gamma + KE_d$$

To consider the matter of nuclear stability, we need to generalize the notion of binding energy. If a given nucleus X could possibly decay into nuclei R and S, the binding energy B is defined as

$$B = \left(M(R) + M(S) - M({}^{A}_{Z}X)\right) c^2.$$

If B>0, X is stable under this decay and if B<0 X may energetically decay into nuclei R and S. This decay will occur unless there are other constraints preventing it.

Example: ^{8}Be

The nuclear binding energy of ^{8}Be is

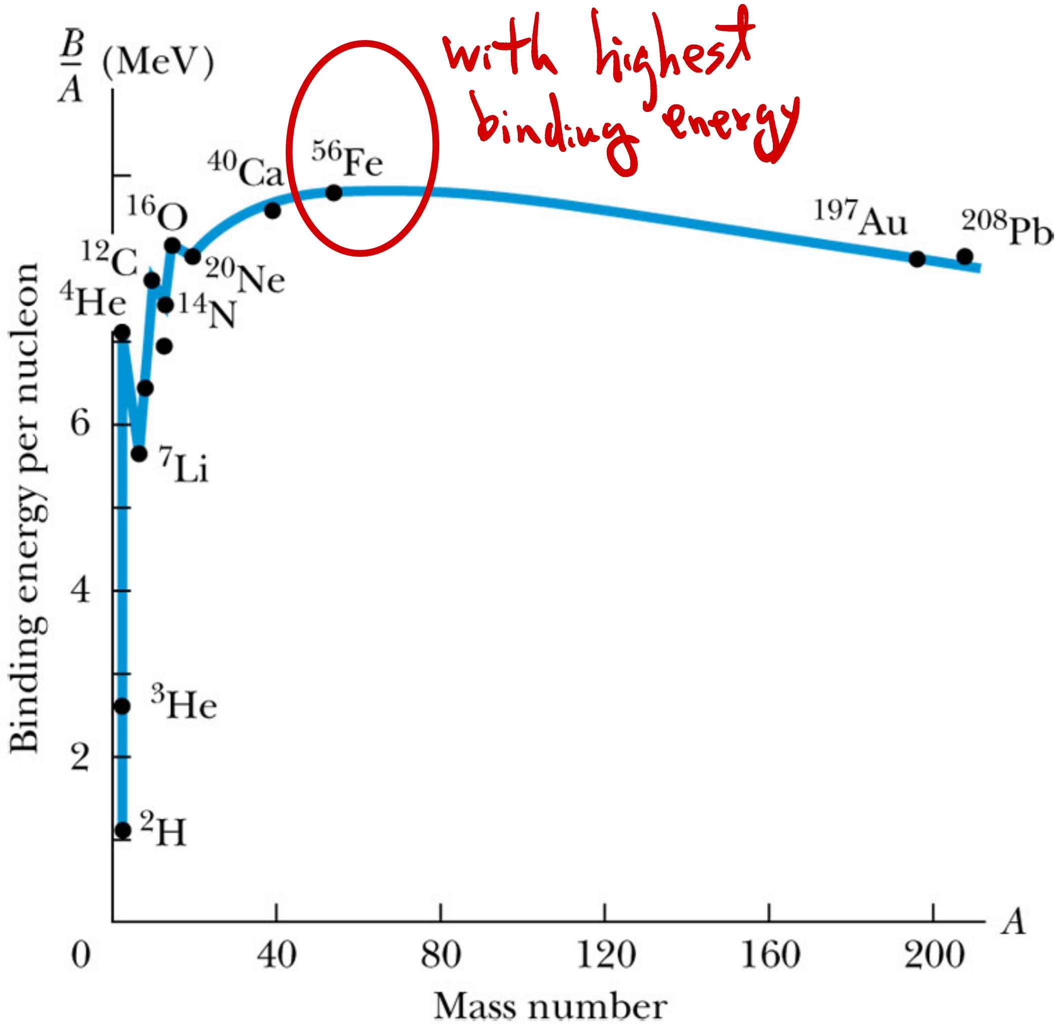
$$B(^8Be) = \left(4m_n + 4M(^1H) - M(^8Be)\right)c^2 = 56.5\,\mathrm{MeV},$$

so ^{8}Be is stable to dissociation into all its parts. However, if we look at its stability with respect to decay into $\alpha + \alpha$

$$B(^8Be \to 2\alpha) = \left(2M(^4He) - M(^8Be)\right)c^2 = -0.093\,\mathrm{MeV},$$

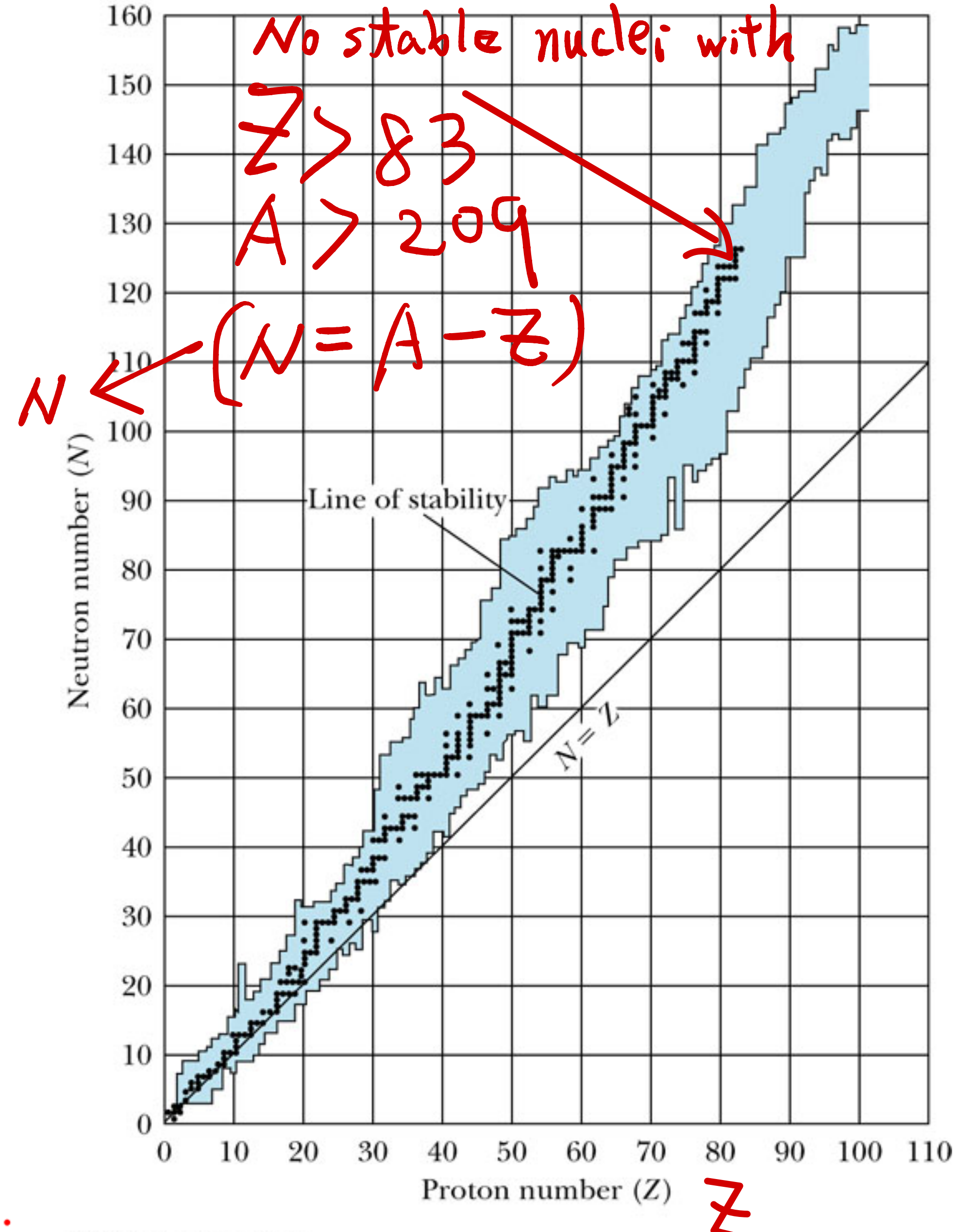
so decay into 2α occurs and the decay lifetime is about .1 fs. This is why the Sun is mostly hydrogen and helium.

A plot of the binding energy per nucleon peaks at about 8.8 MeV for iron.



A plot of N vs Z shows that for A>40 stable nuclei prefer more neutrons than protons. This is a result of the Coulomb repulsion between the protons. This energy is

$$\Delta E_{\text{Coul}} = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\epsilon_0 R}$$
$$= 0.72\frac{Z(Z-1)}{A^{1/3}}\,\text{MeV}.$$



Considerations such as Coulomb energies, surface effects, neutron-proton asymmetry and pairing led Bohr and von Weizsaecker to propose the liquid drop model that expressed the binding energy as

$$B({}^A_Z X) = a_V A - a_A A^{2/3} - \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\epsilon_0 R} - a_S \frac{(N-Z)^2}{A} + \delta.$$

The parameters are determined by fitting.

$$\begin{aligned} a_V &= 14\,\text{MeV},\ a_A = 13\,\text{MeV},\ a_s = 19\,\text{MeV}, \\ \delta &= \begin{cases} +22\,\text{MeV}\ A^{-3/4} & \text{for even} - \text{even} \\ 0 & \text{for even} - \text{odd},\ \text{odd} - \text{even} \\ -22\,\text{MeV}\ A^{-3/4} & \text{for odd} - \text{odd} \end{cases} \end{aligned}$$

Radioactivity

A sample of N atoms of an element that decays spontaneously is characterized by its activity R, defined as

$$R = -\frac{dN}{dt}.$$

R is measured in Becquerels (Bq), where 1 Bq = 1 decay/s. The traditional unit of activity was the Curie (Ci), which is Ci=3.7×10^{10} decays/s=3.7×10^{10} Bq. If R is proportional to N, R = λN, we can write

$$\frac{dN(t)}{dt} = -\lambda N(t).$$

This can be solved by rearranging and integrating to get

$$\begin{aligned} \frac{dN(t)}{N(t)} &= -\lambda dt \\ \ln N(t) &= -\lambda t + \text{const.} \\ N(t) &= N(0)e^{-\lambda t}. \end{aligned}$$

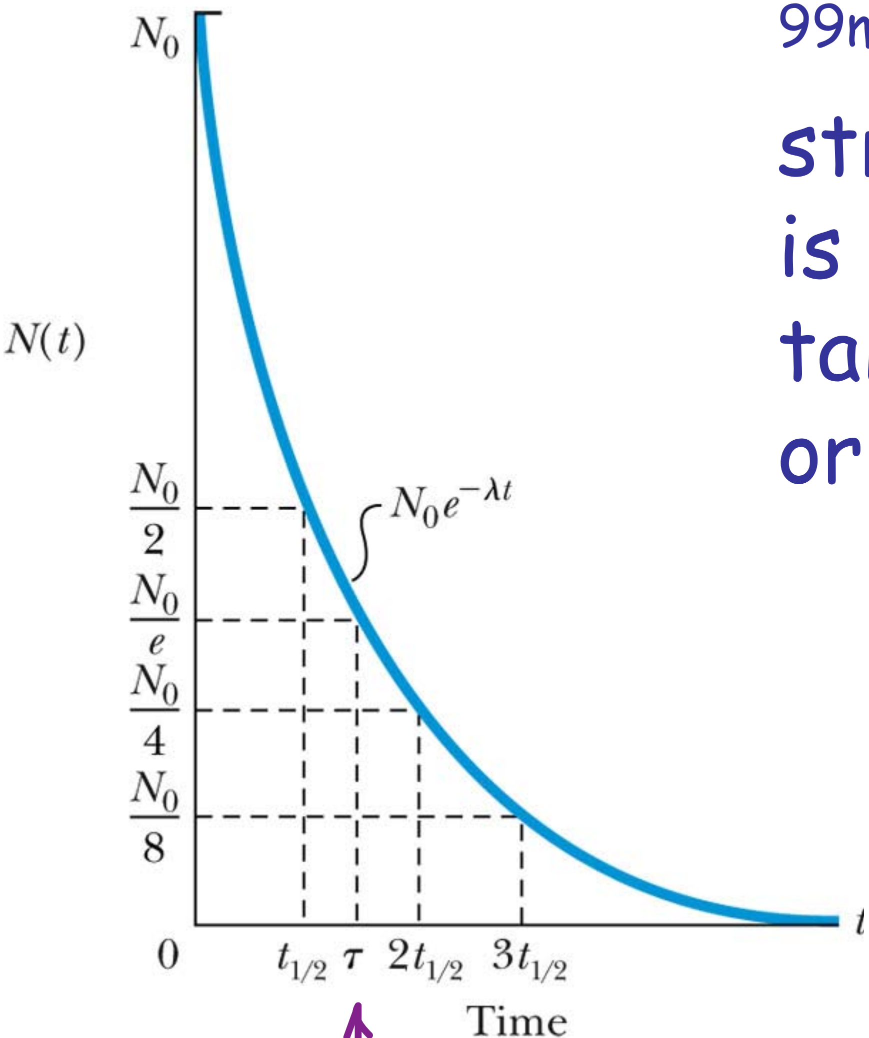
life-time

$\tau \equiv \frac{1}{\lambda}$

This is the radioactive decay law, and the half-life can be found from

$$\begin{aligned} N(t_{1/2}) &= N(0)/2 = N(0)e^{-\lambda t_{1/2}} \\ \ln(1/2) &= -0.693 = -\lambda t_{1/2} \\ t_{1/2} &= 0.693/\lambda. \end{aligned}$$

An isotope of Technetium, ^{99m}Tc, is used in cardiac stress tests. Its half life is 6.01 hr. How long will it take for 99% of the original dose to decay?



$$\begin{aligned} 0.01N(0) &= N(0)e^{-0.693\,T/6.01\,\mathrm{hr}} \\ \ln(0.01) &= -.115\,T/\mathrm{hr} \\ T &= 40\,\mathrm{hr}. \end{aligned}$$

$\tau = \frac{1}{\lambda}$

α, β and γ Decay

For unstable nuclei, we introduce the Q value of the decay as

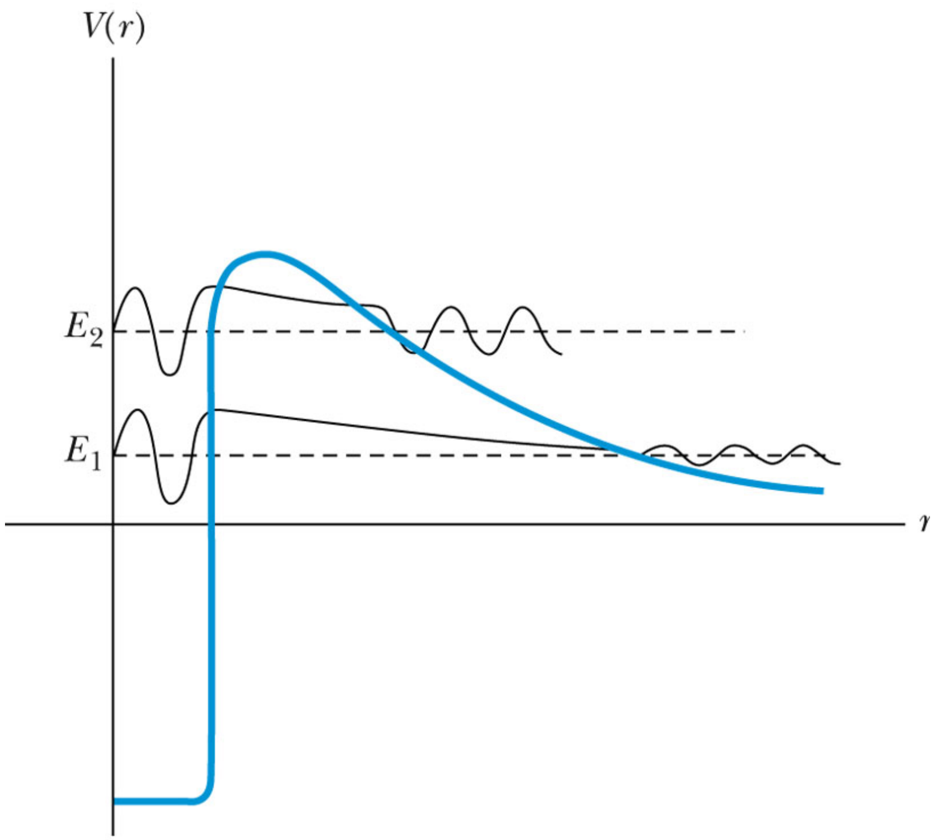
$$M(^A_Z X) = M_D + M_y + Q/c^2,$$

where D is the *daughter* nucleus (taken as the heavest decay product), y is some decay product and Q is the total kinetic energy of the decay products. The Q value,

$$Q = \left(M(^A_Z X) - M_D - M_y\right) c^2,$$

is positive for spontaneous decay.

α decay occurs when a He nucleus tunnels out of the nuclear potential. For ^{238}U, the tunneling probability is about 10^{-40} so the α particle must strike the potential 10^{40} times.



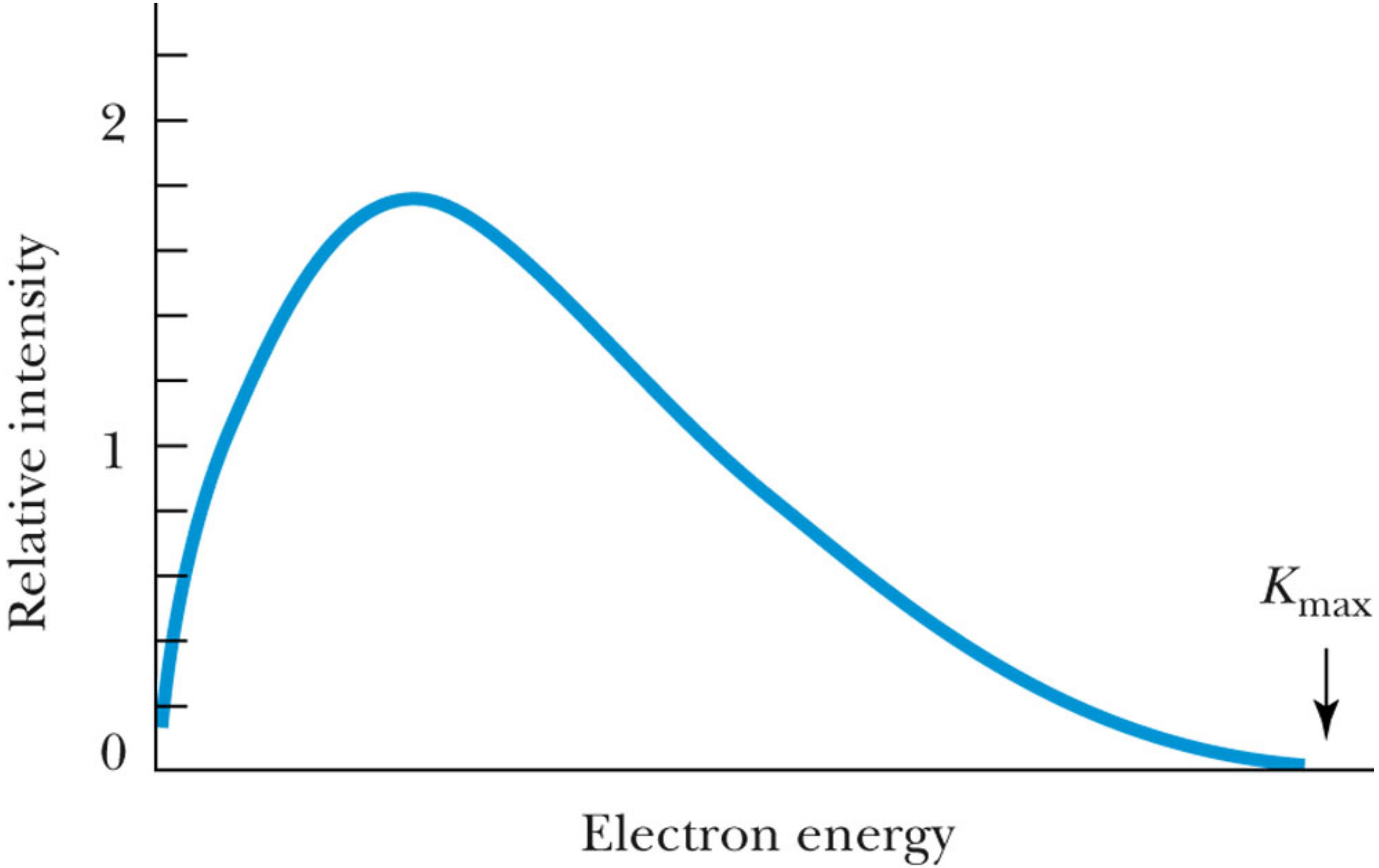
The transversal time is about 10^{-20}s, so the lifetime is around 10^{20}s. This is a reasonable estimate of the ^{238}U lifetime.

β decay

Some nuclei decay by β^- (electron) emission. In this case, a neutron in the nucleus changes into a proton and an electron. For example

$$^{14}_{6}C \rightarrow ^{14}_{7}N + \beta^- .$$

The problem with this assumption is that, because of momentum conservation, the β will have a definite energy. Experiments show quite a different result.


Relative intensity
2
1
0
K_{max}
Electron energy

This was a puzzle as was the question of angular momentum conservation -- there were too few particles with spin 1/2. In 1930, Wolfgang Pauli hypothesized that a massless neutral spin 1/2 particle accompanied the β particle.

We could then write

$$
\begin{aligned}
n &\rightarrow p + e^- + \bar{\nu} \\
{}^{14}_{6}C &\rightarrow {}^{14}_{7}N + \beta^- + \bar{\nu}.
\end{aligned}
$$

or, in general

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}D + \beta^- + \bar{\nu}. $$

The Q value for β⁻ decay can be obtained by adding Z electrons to each side of this equation.

$$\begin{aligned} m_X + Zm_e &= m_D + (Z+1)m_e + Q/c^2 \\ Q &= \left(M({}^A_Z X) - M({}^{A}_{Z+1} D)\right)c^2. \end{aligned}$$

Some nuclei, like ^{14}O can decay by β^+ emission.

$$^{14}O \rightarrow {}^{14}N + \beta^+ + \nu.$$

Here adding 8 electrons gives Q as

$$Q = \left(M({}^A_Z X) - M({}^{A}_{Z-1} D) - 2m_e\right)c^2.$$

$$Q = \left(\sum_i m_i - \sum_f m_f\right)c^2$$

Beta decay

$\boxed{\beta^-}$: ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}D + \beta^- + \underbrace{\bar{\nu}_e}_{\text{anti-neutrino}}$

$$Q = \left[M({}^{A}_{Z}X) - M({}^{A}_{Z+1}D) \right] c^2$$

example : $n \rightarrow p + \underset{e^-}{\beta^-} + \bar{\nu}_e$ $\quad \tau = 885\,s$, $T_{1/2} = 613\,s$

$${}^{14}_{6}C \rightarrow {}^{14}_{7}N + \beta^- + \bar{\nu}_e$$

$\boxed{\beta^+}$: ${}^{A}_{Z}X_N \rightarrow {}^{A}_{Z-1}D + \beta^+ + \nu_e$

$$Q = \left[M({}^{A}_{Z}X_N) - M({}^{A}_{Z-1}D) - 2m_e \right] c^2$$

$$p \rightarrow n + e^+ + \nu_e$$

$\boxed{EC}$: electron capture

$${}^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}D + \nu_e$$

$$Q = \left[M({}^{A}_{Z}X) - M({}^{A}_{Z-1}D) \right] c^2$$

$$p + e^- \rightarrow n + \nu_e$$

How could electron capture process occur?

⇒ For the inner shell (such as 1S) electron, its wavefunction (probability amplitude) is not zero at r=0. [$R_{n0}(r) \neq 0$]

Thus, it can be captured by the neucleus.

Table 7.1 Hydrogen Atom Radial Wave Functions

n	ℓ	$R_{n\ell}(r)$
1	0	$\frac{2}{(a_0)^{3/2}} e^{-r/a_0}$
2	0	$\left(2 - \frac{r}{a_0}\right) \frac{e^{-r/2a_0}}{(2a_0)^{3/2}}$
2	1	$\frac{r}{a_0} \frac{e^{-r/2a_0}}{\sqrt{3}(2a_0)^{3/2}}$
3	0	$\frac{1}{(a_0)^{3/2}} \frac{2}{81\sqrt{3}} \left(27 - 18\frac{r}{a_0} + 2\frac{r^2}{a_0^2}\right) e^{-r/3a_0}$
3	1	$\frac{1}{(a_0)^{3/2}} \frac{4}{81\sqrt{6}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0}$
3	2	$\frac{1}{(a_0)^{3/2}} \frac{4}{81\sqrt{30}} \frac{r^2}{a_0^2} e^{-r/3a_0}$

$R_{n0}(r) \neq 0$

$${}^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z}D + \nu_e$$

$$(p + e^- \rightarrow n + \nu_e)$$

γ Decay

Like atoms, nuclei have excited state with the same A and Z. These states are called *isomeric states.* They decay by emitting photons, called gamma rays. The γ energy is just the energy difference between the excited state and the ground state. For this transition $^{A}X^{*} \rightarrow {}^{A}X + \gamma$

$$\Delta E = E^{*} - E = \hbar\omega + \frac{\hbar^2\omega^2}{2M_X c^2}$$

$$\Delta E = \hbar\omega\left(1 + \frac{\hbar\omega}{2M_X c^2}\right).$$

$\frac{p_x^2}{2M_x}$

with $p_x = \frac{\hbar\omega}{c}$

Natural Radioactivity

While the light elements H, He, Li were synthesized within minutes of the Big Bang, the heavier elements were all made by stellar processes and dispersed by supernovae.

Many of these elements have short lifetimes and can only be created artificially. Some, however have long lifetimes and can still be found on Earth.

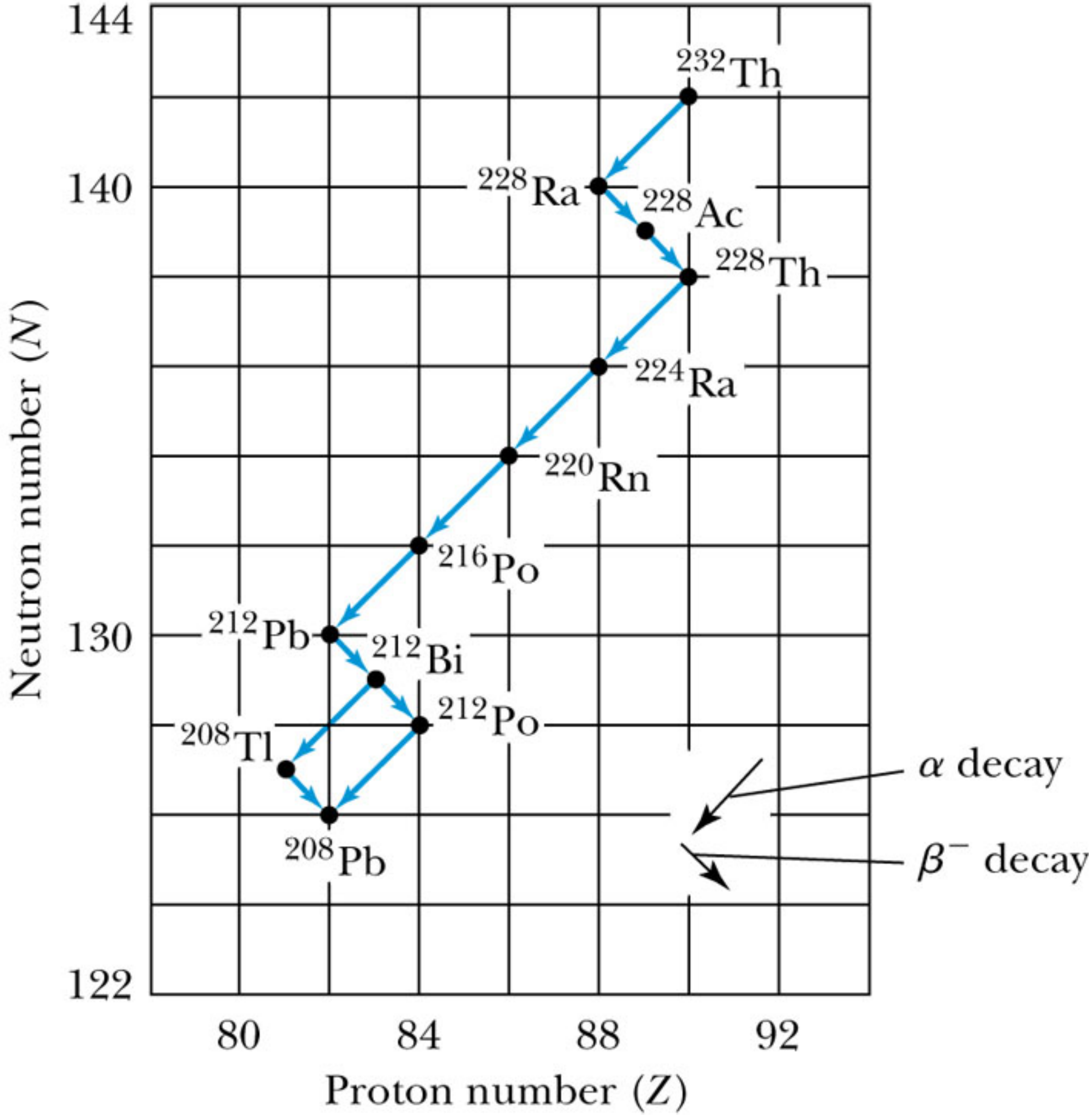
Table 12.2 Some Naturally Occurring Radioactive Nuclides

Nuclide	$t_{1/2}$ (y)	Natural Abundance
$^{40}_{19}K$	1.28×10^9	0.01%
$^{87}_{37}Rb$	4.8×10^{10}	27.8%
$^{113}_{48}Cd$	9×10^{15}	12.2%
$^{115}_{49}In$	4.4×10^{14}	95.7%
$^{128}_{52}Te$	7.7×10^{24}	31.7%
$^{130}_{52}Te$	2.7×10^{21}	33.8%
$^{138}_{57}La$	1.1×10^{11}	0.09%
$^{144}_{60}Nd$	2.3×10^{15}	23.8%
$^{147}_{62}Sm$	1.1×10^{11}	15.0%
$^{148}_{62}Sm$	7×10^{15}	11.3%

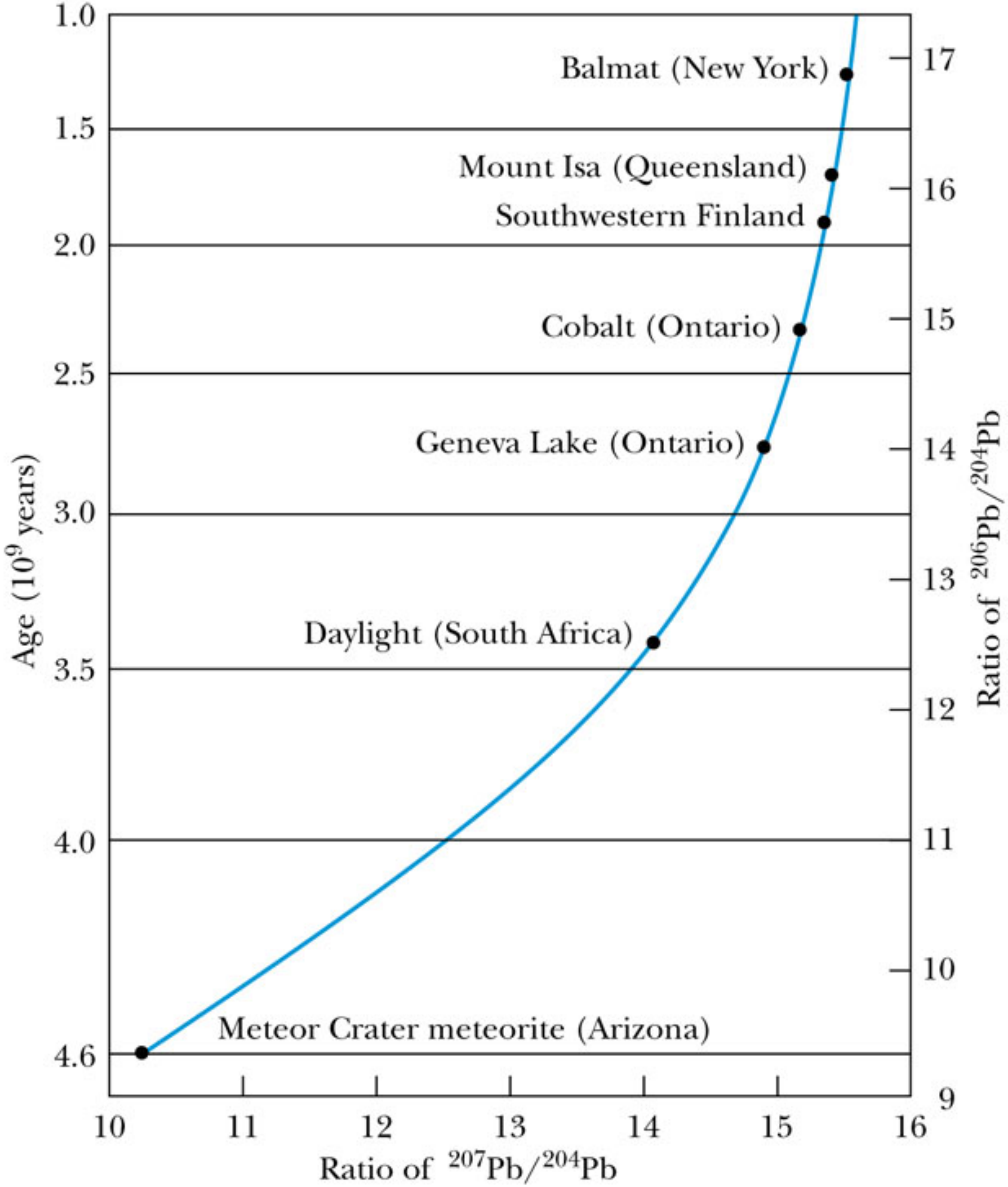
There are three radioactive series that eventually decay to lead isotopes.

Table 12.3 The Four Radioactive Series

Mass Numbers	Series Name	Parent	$t_{1/2}$ (y)	End Product
$4n$	Thorium	$^{232}_{90}Th$	1.40×10^{10}	$^{208}_{82}Pb$
$4n + 1$	Neptunium	$^{237}_{93}Np$	2.14×10^{6}	$^{209}_{83}Bi$
$4n + 2$	Uranium	$^{238}_{92}U$	4.47×10^{9}	$^{206}_{82}Pb$
$4n + 3$	Actinium	$^{235}_{92}U$	7.04×10^{8}	$^{207}_{82}Pb$


144
140
130
122
Neutron number (N)
232Th
228Ra
228Ac
228Th
224Ra
220Rn
216Po
212Pb
212Bi
212Po
208Tl
208Pb
α decay
β⁻ decay
80
84
88
92
Proton number (Z)

^{204}Pb is essentially stable and is not created by any decay chain. ^{206}Pb is also stable, but is being enriched by the decay of ^{238}U. ^{207}Pb is also stable and is being enriched by ^{235}U decay. Its decay is relatively rapid compared to ^{238}U and thus comparing the ratios of $^{206}Pb/^{204}Pb$ to $^{207}Pb/^{204}Pb$ can be used to date the Earth's formation.



Balmat (New York)
Mount Isa (Queensland)
Southwestern Finland
Cobalt (Ontario)
Geneva Lake (Ontario)
Daylight (South Africa)
Meteor Crater meteorite (Arizona)
Age (10^9 years)
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.6
Ratio of ^{206}Pb/^{204}Pb
17
16
15
14
13
12
11
10
9
10
11
12
13
14
15
16
Ratio of ^{207}Pb/^{204}Pb

The carbon isotope ^{14}C is formed by cosmic ray neutrons colliding with ^{14}N via

$$n + {}^{14}_{7}N \rightarrow {}^{14}_{6}C + p.$$

This isotope of carbon has a half life of 5730 years and is present in any living material in the ratio $^{14}C/^{12}C = 1.2 \times 10^{-12}$. When an object dies, the ^{12}C remains, but the ^{14}C decays. This is the basis of a dating technique proposed by 1960 Nobel Prize winner Willard Libby.

Nuclear Interactions

The first laboratory experiment that resulted in a nuclear reaction was

$$\alpha + {}^{14}_{7}N \rightarrow p + {}^{17}_{8}O,$$

performed by Rutherford in 1919 using 7.7 MeV α particles from polonium decay. Note that the number of nucleons and the total charge are conserved in the reaction.

Reactions of this type are written

$${}^{14}_{7}N(\alpha, p){}^{17}_{8}O.$$

$\alpha = {}^{4}_{2}He$

$p = {}^{1}_{1}H$

Conservation laws in $^{14}_{7}N(\alpha, p)^{17}_{8}O$

① Notation: $^{A}_{Z}X$ Contains Z protons (p) and $N (\equiv A-Z)$ neutrons (n)

A is the total number of nucleons (including p and n)

② Total electric charge is conserved.

③ Total number of nucleons is conserved. (Baryon number)

④ Mass is not conserved. $(m_n > m_p)$

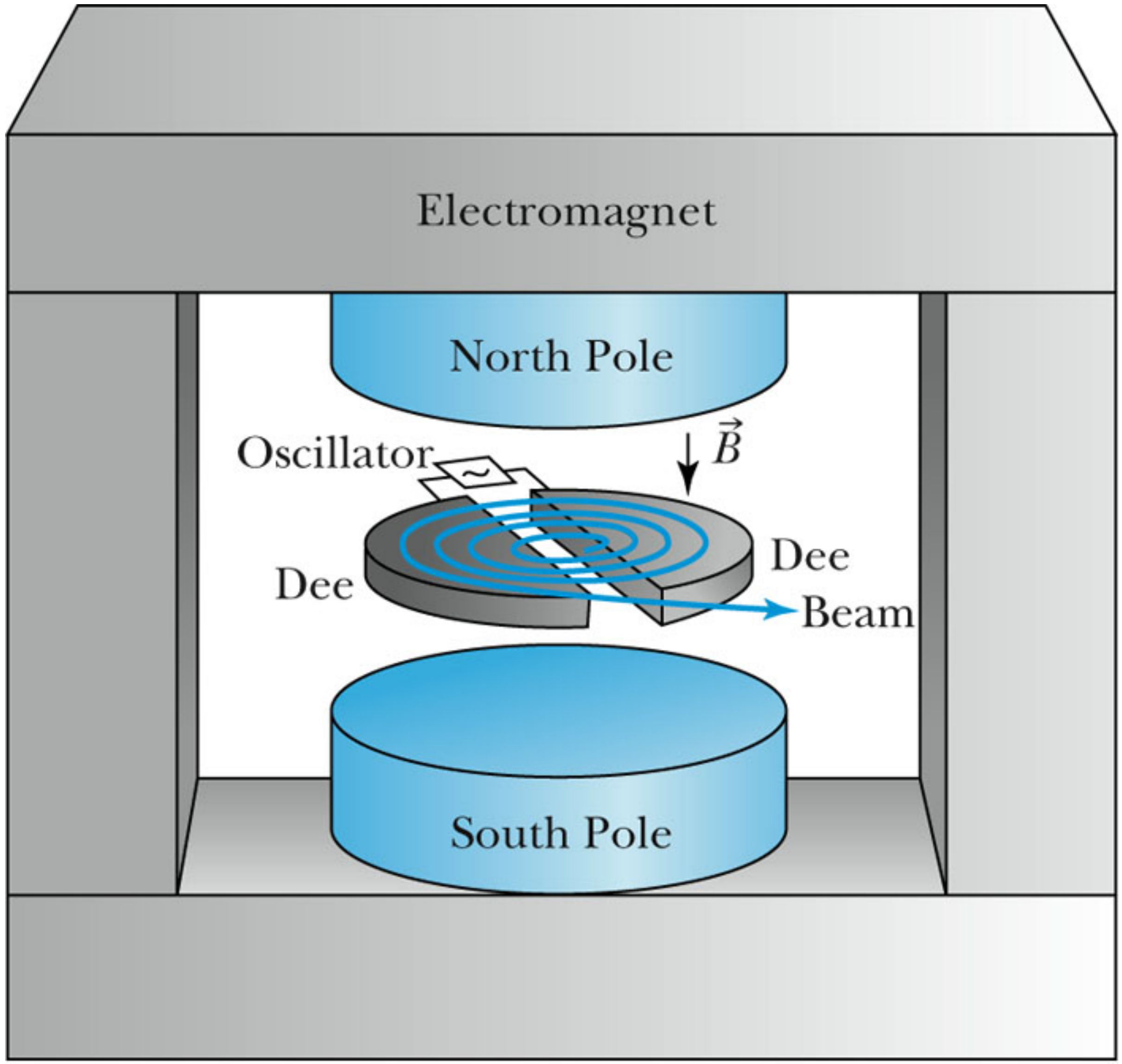
$\Rightarrow$ Nuclear binding energy arised from converting the mass difference to energy

$$E_b = (\Delta M) c^2$$

Particle accelerators are now used to initiate nuclear reactions. They evolved as:

1. Cockcroft-Walton accelerators.
2. Van de Graaff electrostatic accelerators.
3. E. O. Lawrence and M. S. Livingston's cyclotron.

The cyclotron has been the workhorse of low energy nuclear physics to the present day. MSU's cyclotron laboratory is a leader in nuclear research.


Electromagnet
North Pole
Oscillator
$\vec{B}$
Dee
Dee
Beam
South Pole

(a)

Typically, reactions with two particles in the exit channel are studied. There are, however reactions like

$$\alpha + ^{20}_{10}Ne \rightarrow n + p + ^{22}_{11}Na$$

are also allowed. The objective is to study the *differential cross section* for a particular process. The total probability for a reaction to occur in a target of area *A* is related to the total cross section by

$$P = \underline{ntA}\sigma_T/A,$$

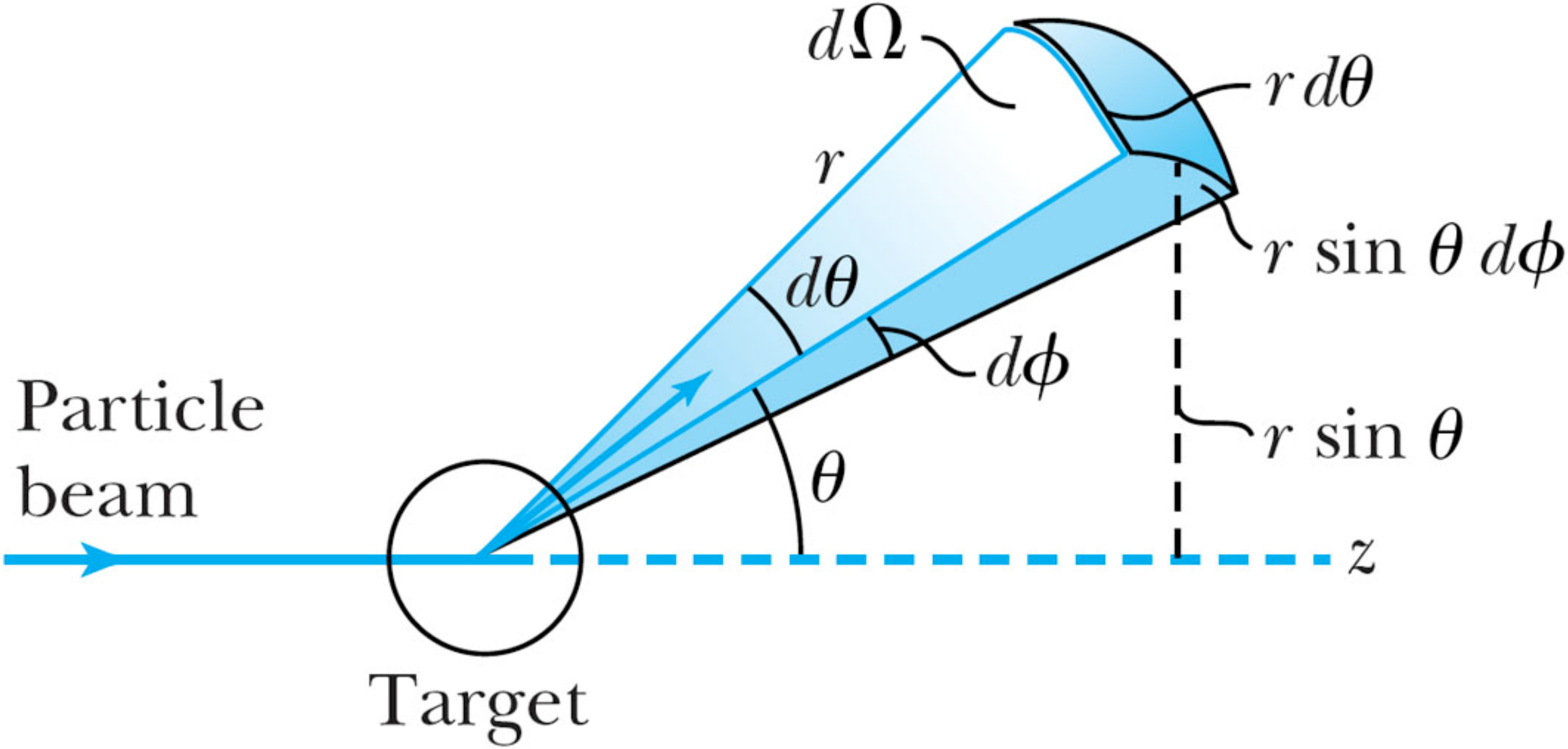
where n is the number of targets/volume and t is the target thickness.

For a particular target, ntA can be calculated from

$$ntA = \frac{\rho N_A N_M t A}{M_g},$$

where ρ is the target density, N_A is Avogadro's number, N_M is the number of atoms/molecule and M_g is the gram-molecular weight.

The actual scattering produces particles in all directions θ,ϕ, creating a differential distribution $d\sigma/d\Omega$.

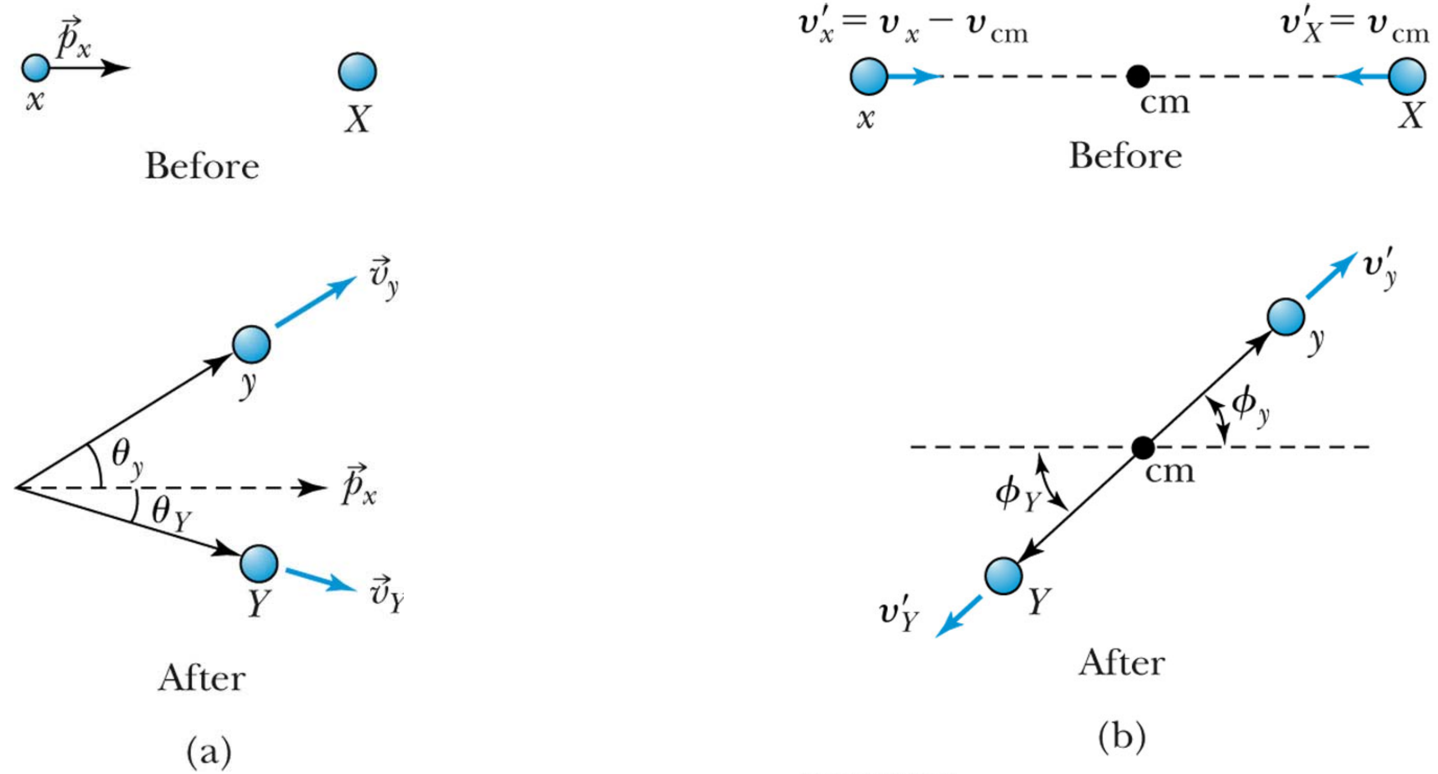


$$d\Omega = \frac{(r \sin \theta)\,(r\,d\theta)\,(d\phi)}{r^2} = \sin \theta \, d\theta \, d\phi$$

The total cross section is then

$$\sigma_T = \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \frac{d\sigma}{d\Omega}.$$

Reaction Kinematics



From conservation of energy, the Q value is

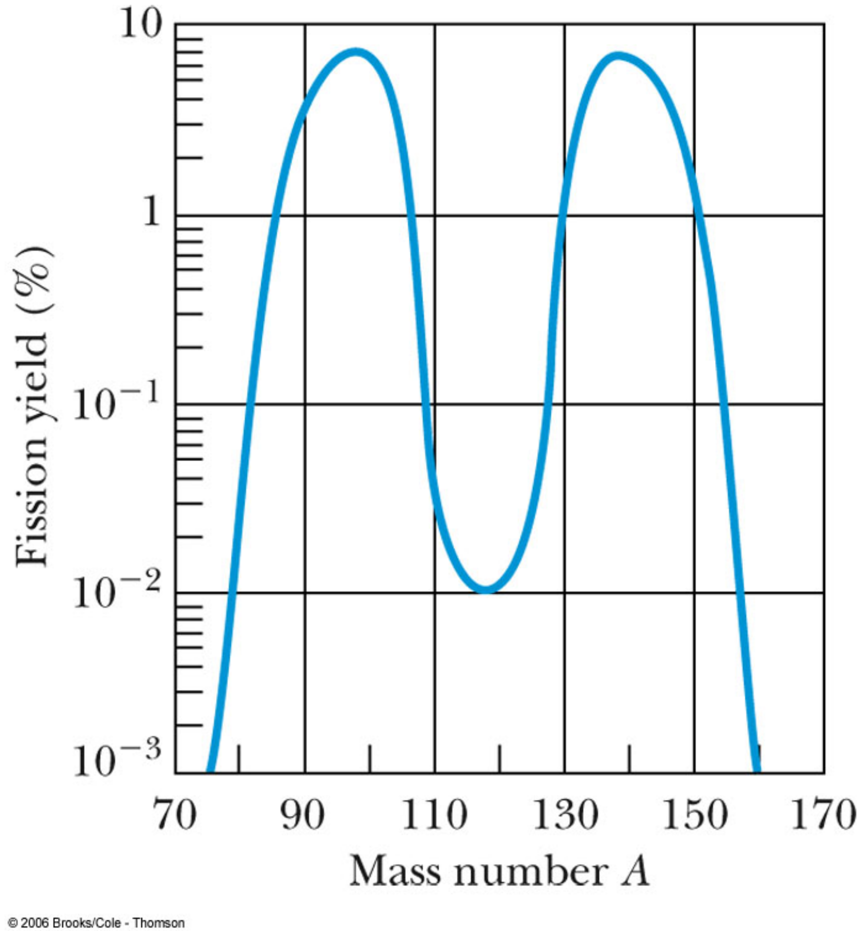
$$Q = (M_x + M_X - M_y - M_Y) = K_y + K_Y - K_x.$$

If $Q < 0$, the reaction will not occur unless K_x is large enough. Because momentum must be conserved, this energy must be available in the center of mass, where the total momentum is zero. Using $v_y = v_Y = 0$ at threshold, the minimum K_x is

$$K_{\mathrm{th}} = -Q\left(\frac{M_x + M_X}{M_X}\right).$$

Fission

Heavy elements, A>240, can decay spontaneously into lighter nuclei, but it is more common that low energy neutron collisions cause the breakup of these heavy nuclei.



$$n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U^* \rightarrow {}^{90}_{40}Zr + {}^{134}_{52}Te + 3n.\ (Q = 185\text{MeV})$$

There are many combinations that work, as illustrated in the figure.

A controlled use of the neutrons released when ^{235}U fissions can be used to boil water and drive turbines that generate electricity.

Table 13.2 Daily Fuel Requirements for 1000-MWe Power Plant

Material	Amount	
Coal	8×10^6 kg	(1 trainload/day)
Oil	40,000 barrels (6400 m^3)	(1 tanker/week)
Natural gas	2.5×10^6 ft^3 (7.1×10^4 m^3)	
Uranium	3 kg	

Fusion

$$Q \equiv \left(\sum_i m_i - \sum_{m_f} m_f\right) c^2$$

While fission works by having less tightly bound nuclei decay into more tightly bound nuclei, fusion combines less tightly bound light nuclei into more tightly bound heavier nuclei. In the Sun, the reactions are

$$
\begin{aligned}
{}^1H + {}^1H &\rightarrow {}^2H + \beta^+ + \nu, & Q_1 &= 0.42\,\text{MeV} \quad (\times 2)\\
{}^2H + {}^1H &\rightarrow {}^3H + \gamma, & Q_2 &= 5.49\,\text{MeV} \quad (\times 2)\\
{}^3H + {}^3H &\rightarrow {}^4He + {}^1H + {}^1H, & Q_3 &= 12.86\,\text{MeV}.
\end{aligned}
$$

This cycle uses up 4 protons and generates 24.7 MeV of energy.

$(2Q_1 + 2Q_2 + Q_3)$