Special Relativity

$$\underbrace{\beta = \frac{v}{c}}_{\text{speed}} \qquad \underbrace{\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}_{\text{Lorentz gamma factor}}$$

$L = \dfrac{L_0}{\gamma}$ } length contraction
L_0: proper length

$T = \gamma T_0$ } time dilation
T_0: proper time

$E_0 = m_0 c^2$

m_0: rest mass
E_0: rent energy

$E = \gamma E_0$: total relativistic energy

$K = (\gamma - 1) E_0$: relativistic kinetic en.

$\vec{p} = \gamma m_0 \vec{u}$: relativistic momentum

$$E^2 = p^2c^2 + m_0^2 c^4 \qquad \frac{p}{E} = \frac{\beta}{c}$$

Relativistic velocity addition
(in the longitudinal direction) :

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}} \qquad \beta = \frac{B + \beta'}{1 + B\beta'}$$

Special Relativity 2.

Relativistic Doppler effect
(in the longitudinal direction):

$$f_{\uparrow} = f_0 \sqrt{\frac{c+v}{c-v}} = \sqrt{\frac{1+\beta}{1-\beta}}$$

$$f_{\downarrow} = f_0 \sqrt{\frac{c-v}{c+v}} = \sqrt{\frac{1-\beta}{1+\beta}}$$

$c = 3 \cdot 10^8 \, m/s$ (299,792,458 m/s)

$c = \lambda \cdot f$

Thermodynamics 1.

Temperature scales:

$$T_K = T_{°C} + 273.15$$

$$\left(T_{°C} = \frac{5}{9}(T_{°F} - 32) \; ; \; T_{°F} = \frac{9}{5} T_{°C} + 32 \right)$$

Thermal expansion:

$$\left.\begin{array}{ll} \Delta L = \alpha \cdot L_0 \cdot \Delta T & : \text{linear} \\ \Delta V = \beta \cdot V_0 \cdot \Delta T & : \text{volumetric} \end{array}\right\} \beta = 3\alpha$$

ΔT is in K or °C NOT in °F

Ideal gas law:

$$\boxed{\begin{array}{c} pV = nRT \\ \text{or} \\ pV = N k_B T \end{array}}$$

$R = 8.31 \ J/(mol\,K)$

$k_B = 1.38 \times 10^{-23} \ J/K$

$N_A = 6.02 \times 10^{23} \ 1/mol$

$nR = N k_B$ therefore $\boxed{R = N_A \cdot k_B}$

n: amount of substance in mol

N: number of atoms or molecules in 1

m: mass of one particle in kg

M: mass of one mole substance or molar mass in kg/mol

$$\boxed{m \cdot N = n \cdot M} \Big\} \text{mass of the system}$$

Thermodynamics 2.

STP: standard temperature and pressure:

$T = 0°C = 273K$

$p = 1atm = 101.3\,kPa = 1.013 \times 10^5\,Pa$

If $n = 1mol$, then $pV = nRT$ gives

$V = 0.0224\,m^3 = 22.4\,l$: molar volume

Kinetic theory:

$$v_{RMS} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{M}}$$ } this formula has always 3 in it, because:

$$\bar{E}_{translation} = 3 \cdot \frac{1}{2} k_BT = \frac{3}{2} k_BT$$

↑ x, y and z directions

But:

$$\bar{E} = \frac{f}{2} k_BT \; ; \; f = \begin{cases} 3 & : \text{monoatomic} \\ 5 & : \text{diatomic or linear} \\ 6 & : \text{polyatomic} \end{cases}$$

$U = E_{th} = N \cdot \bar{E} = \frac{f}{2} N k_BT = \frac{f}{2} nRT$: internal or thermal energy of the gas containing N atoms or molecules.

Thermodynamics 3.

Heat:

$Q = C \cdot \Delta T$; C: heat capacity

$Q = c \cdot m \cdot \Delta T$; c: specific heat

$c = \frac{C}{m}$

$1\,\text{cal} = 4.1868\,\text{J}$ $\quad$ $1\,\text{Cal} = 1\,\text{kcal} = 4186.8\,\text{J}$

Phase transitions or transformations:

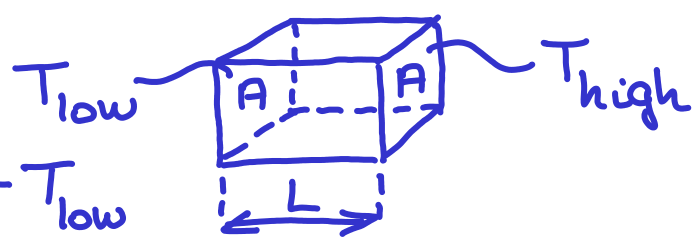
solid-liquid: $Q = L_F \cdot m$; L_F = heat of fusion

liquid-gas: $Q = L_V \cdot m$; L_V = heat of vaporization

Heat conduction:

$Q = k \cdot \frac{A}{L} \cdot \Delta T \cdot t$;

$P = Q/t$; $\Delta T = T_{high} - T_{low}$



Heat radiation:

$P = \sigma \varepsilon A T^4$; $\sigma = 5.67 \times 10^{-8}\,\text{W}/(\text{m}^2\text{K}^4)$

$\lambda_{max} \cdot T = b$; $b = 2.898 \times 10^{-3}\,\text{m} \cdot \text{K}$

black-body: $a = 1$ and $\varepsilon = 1$

(Heat convection: way too complicated
Hot fluid rises.)

Thermodynamics 4.

First Law: $\Delta U = Q + W$ $\left.\right\}$ $W_{gas} = -W$, $Q_{gas} = -Q$

W is the work done ON the system (gas).
Q is the heat transferred To the system (gas).

Second Law: $\frac{Q}{T} \leq \Delta S$: the entropy of a closed system cannot decrease.

Molar heats for ideal gases:

$Q = C_v \cdot n \Delta T$: at constant volume

$Q = C_p \cdot n \Delta T$: at constant pressure

$C_v = \frac{f}{2} R$; $C_p = \frac{f+2}{2} R = C_v + R$

Ideal Gas processes:

→ isothermal: constant T $\longleftrightarrow$ $\Delta U = 0$

$p_1 V_1 = p_2 V_2$

→ isobaric: constant p

$V_1 / T_1 = V_2 / T_2$

→ isochoric: constant V $\longleftrightarrow$ $W = 0$

$p_1 / T_1 = p_2 / T_2$

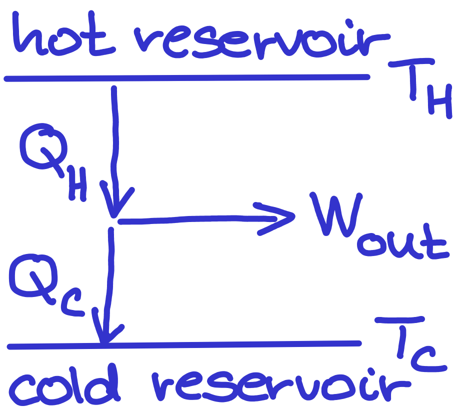
→ adiabatic: $Q = 0$ $\longleftrightarrow$ constant S

$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$; $\gamma = C_p / C_v = \frac{f+2}{f}$

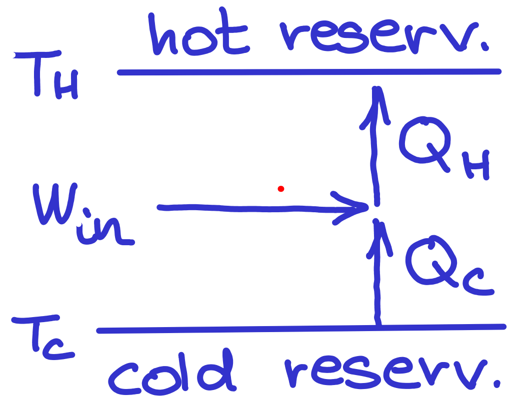
(isocaloric, iso-entropic or isentropic)

Thermodynamics 5.

Heat engines



refrigerators
heat pump



$$\boxed{Q_H = W + Q_c}$$

conservation of energy

The efficiency:

Engine:

$$\eta = \frac{W_{out}}{Q_H} = \frac{T_H - T_c}{T_H}$$

$$0 \le \eta < 1$$

Carnot

The coefficient of performance:

Refrigerator:

$$\kappa = \frac{Q_c}{W_{in}} = \frac{T_c}{T_H - T_c}$$

Heat pump:

$$\kappa = \frac{Q_H}{W_{in}} = \frac{T_H}{T_H - T_c}$$

Quantum Physics 1.

Black-body radiation:

Wien's displacement law:

$$\lambda_{max} \cdot T = b$$

$\hookrightarrow b = 2.898 \cdot 10^{-3}$ m·K

Stefan-Boltzmann law:

$$P = \sigma \varepsilon A T^4 \; ; \quad 0 \leq \varepsilon \leq 1 \; : \text{emissivity}$$

$\varepsilon = 1$ for black body

$\hookrightarrow \sigma = 5.6705 \cdot 10^{-8} \dfrac{W}{m^2 K^4}$

Planck's radiation curve:

$$I_\lambda = \frac{2\pi c^2 h}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

$$I_f = \frac{2\pi h}{c^2} \cdot \frac{f^3}{e^{\frac{hf}{k_B T}} - 1}$$

Key assumption: energy quantum:

$$\Delta E = h \cdot f$$

$\hookrightarrow h = 6.6261 \cdot 10^{-34}$ Js

Planck constant

Quantum Physics 2.

Rydberg formula for the H-atom:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{k^2}\right) \qquad n = 1, 2, 3, 4, \dots \quad k > n$$

$\hookrightarrow R_H = 1.096776 \cdot 10^7 \ 1/m$

Einstein's formula for the photo-electric effect:

$$hf = \varphi + eV_0 \qquad \varphi\text{: work function}$$

$$hf = \varphi + \frac{1}{2}mv^2 \qquad V_0\text{: stopping pot.}$$

$\frac{1}{2}mv^2$: kinetic energy of the photo-electron, small, only a couple of eV.

X-ray production by bremsstrahlung:

$$eV_0 = h \cdot f_{max} = \frac{hc}{\lambda_{min}} \qquad (c = \lambda \cdot f)$$

Compton-effect:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\Theta);$$

Energy conservation:

$$hf + m_e c^2 = hf' + E_e \ ; \ f = \frac{c}{\lambda}$$

$$E_e = K + m_e c^2$$

Compton wavelength:

$$\lambda_c = \frac{h}{m_e c} = \frac{hc}{m_e c^2} = \frac{1240\,eVnm}{511{,}000\,eV} = 2.4263\,pm$$

Quantum Physics 3.

Photon:

$$\left.\begin{array}{l}\left.\begin{array}{l} E = hf \\ c = \lambda f \end{array}\right\} \Rightarrow E = \dfrac{hc}{\lambda} \\ E^2 = p^2c^2 \Rightarrow E = pc \Rightarrow p = \dfrac{E}{c} \end{array}\right\} \Rightarrow p = \frac{h}{\lambda}$$

De Broglie: electron wave/matter wave:

$\lambda = \dfrac{h}{p}$: de Broglie wavelength

Bragg's law:

$$n \cdot \lambda = 2d \cdot \sin\theta \qquad n = 1, 2, 3, 4, \ldots$$

Bohr model:

$$E_{n,Z} = -\frac{Z^2}{n^2} \cdot E_0 \; ; \quad E_0 = 13.6\,\text{eV}$$

$$r_{n,Z} = \frac{n^2}{Z} \cdot a_0 \; ; \quad a_0 = 0.529\,\text{Å}$$

$$\beta_{n,Z} = \frac{Z}{n} \cdot \alpha \; ; \quad \alpha \cong \frac{1}{137}$$

E_0 : ionization energy of the Hydrogen atom

a_0 : Bohr radius

α : fine structure constant

Wave mechanics 1.

Schrödinger equation (time indep.):

$$-\frac{\hbar^2}{2m}\cdot\frac{d^2\Psi(x)}{dx^2}+V(x)\cdot\Psi(x)=E\cdot\Psi(x)$$

$E\in\mathbb{R}$: energy of the system

Normalization:

$$1=\int_{-\infty}^{\infty}\varrho(x)\,dx=\int_{-\infty}^{\infty}|\Psi|^2\,dx$$

Probability density: $\varrho(x)=|\Psi(x)|^2$

Heisenberg Uncertainty Principle:

$$\Delta x\cdot\Delta p_x\geq\hbar/2$$

$$\Delta y\cdot\Delta p_y\geq\hbar/2$$

$$\Delta z\cdot\Delta p_z\geq\hbar/2$$

$$\Delta t\cdot\Delta E\geq\hbar/2$$

$$\hbar=\frac{h}{2\pi}$$

It holds for Conjugated parameters.

Wave mechanics 2.

Infinite square well

One dimension:

$$\Psi_n(x) = \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{n\pi x}{L}\right) \; ; \quad n = 1, 2, 3, \ldots$$

$$E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2} = n^2 \cdot \frac{h^2}{8mL^2} = n^2 \cdot E_1$$

Three dimensions:

$$E_{n_1, n_2, n_3} = \frac{h^2}{8m}\left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2}\right)$$

Simple Harmonic Oscillator:

$$V(x) = \frac{1}{2} k x^2 \qquad (k: \text{spring constant})$$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega = \left(n + \frac{1}{2}\right) hf; \quad n = 0, 1, 2, \ldots$$

$$E_0 = \frac{1}{2}\hbar\omega : \text{zero point energy}$$

Atom's orbitals

Shell n	subshell	ℓ	m_ℓ	# of orbitals	# of $\bar{e}$
K $n=1$	1s	0	0	1	2
L $n=2$	2s 2p	0 1	0 -1, 0, 1	1 3] 4	8
M $n=3$	3s 3p 3d	0 1 2	0 -1, 0, 1 -2, -1, 0, 1, 2	1 3 5] 9	18
N $n=4$	4s 4p 4d 4f	0 1 2 3	0 -1, 0, 1 -2, -1, 0, 1, 2 -3, -2, -1, 0, 1, 2, 3	1 3 5 7] 16	32
O $n=5$	5s 5p 5d 5f 5g	0 1 2 3 4	0 -1, 0, 1 -2, -1, 0, 1, 2 -3, -2, -1, 0, 1, 2, 3 -4, -3, -2, -1, 0, 1, 2, 3, 4	1 3 5 7 9] 25	50

$n = 1, 2, 3, \ldots$ (K, L, M, N, O, P...)

$\ell = 0, 1, 2, \ldots (n-1)$ (s, p, d, f, g, h...)

$m_\ell = -\ell, \ldots, 0, \ldots \ell$: # : $(2\ell+1)$

$m_s = \pm \frac{1}{2}$

$L = |\vec{L}| = \sqrt{\ell(\ell+1)} \cdot \hbar$

Shell n has n different subshells, n^2 orbits and $2n^2$ electrons.

Spectroscopic symbols

$$n \, {}^{M}L_{J}$$

n: principal quantum number (often dropped)

L: orbital angular momentum (S, P, D, F, G, H)

M: multiplicity of the state
$M = 2S + 1$; S: spin q.n.

J: total angular momentum quantum number

Examples:

Hydrogen ground state:

$\boxed{\uparrow}$ $1s^1$: $1\,{}^{2}S_{1/2}$ or ${}^{2}S_{1/2}$

Helium ground state:

$\boxed{\uparrow\downarrow}$ $1s^2$: $1\,{}^{1}S_{0}$ or ${}^{1}S_{0}$

Nuclear physics

Notation: $^{A}_{Z}X_N$; $A = Z + N$

$\alpha, \beta^+, \beta^-, \gamma, p, n, d, t$

Nuclear radius: $R = r_0 \cdot A^{1/3}$; $r_0 = 1.2\,fm$

Binding energy:

$$B(^{A}_{Z}X_N) = [Z \cdot M(^{1}H) + N \cdot m_N - M(^{A}_{Z}X_N)]\,c^2$$

$$1\,AMU = 1u = \frac{1}{12} M(^{12}_{6}C_6)$$

$$1\,AMU = 931.494061 \;\; MeV/c^2$$

Radioactivity:

$$N = N_0 \cdot e^{-\lambda t} \qquad R = R_0 \cdot e^{-\lambda t}$$

$$N = N_0 \cdot e^{-t/\tau} \qquad R = R_0 \cdot e^{-t/\tau}$$

$$N = N_0 \cdot 2^{-t/T_{1/2}} \qquad R = R_0 \cdot 2^{-t/T_{1/2}}$$

$$R = \lambda \cdot N \qquad R_0 = \lambda \cdot N_0$$

$$\lambda = \frac{1}{\tau} \;;\; T_{1/2} = \ln 2 \cdot \tau \;;\; \ln 2 = 0.6931$$

Q-value: initial minus final

mass-energy $\quad Q = \left(\sum_i m_i - \sum_f m_f\right) c^2$