How to slow your aging

Proper time : $T_0 = 16.1$ yrs

Dilated time : $T = 27.7$ yrs

Time dilation: $T = \gamma T_0$

$$T = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot T_0$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{T_0}{T}$$

$$1 - \frac{v^2}{c^2} = \frac{T_0^2}{T^2}$$

$$1 - \frac{T_0^2}{T^2} = \frac{v^2}{c^2}$$

$$c^2\left(1 - \frac{T_0^2}{T^2}\right) = v^2$$

$$c \cdot \sqrt{1 - \frac{T_0^2}{T^2}} = v$$

$$v = c \cdot \sqrt{1 - \frac{16.1^2}{27.7^2}} = 0.814c$$

$$v = 2.44 \cdot 10^8 \text{ m/s}$$

Relativistic roast beef

How fast does a $L_0 = 1600\,m$ long spaceship move, if we see is only $L = 1050\,m$ long?

Lorentz contraction:

$$L = \frac{L_0}{\gamma} = \sqrt{1-\beta^2} \cdot L_0$$

$$\left(\frac{L}{L_0}\right)^2 = 1 - \beta^2$$

$$\beta^2 = 1 - \left(\frac{L}{L_0}\right)^2$$

$$\beta = \sqrt{1 - \left(\frac{L}{L_0}\right)^2} = \sqrt{1 - \left(\frac{1050}{1600}\right)^2} = 0.755$$

$$\left(v = \beta c = 0.755 \cdot 3 \cdot 10^8 \,\frac{m}{s} = 2.265 \cdot 10^8 \,\frac{m}{s}\right)$$

If the kitchen timer is set to $T_0 = 2.15\,h$, then how much time does the roast spend in the oven observed by us?

Time dilation: $T = \frac{T_0}{\gamma} = \frac{T_0}{\sqrt{1-\beta^2}}$

$$T = \frac{2.15\,h}{\sqrt{1-0.755^2}} = 3.28\,h$$

(just don't burn the roast!)

Two events

Spacetime intervals are invariant:

$$(\Delta s)^2 = (\Delta s')^2$$

$$(\Delta x)^2 - \underbrace{(c\Delta t)^2}_{\substack{=0 \text{ because } \Delta t = 0 \text{ in one} \\ \text{of the inertial frames}}} = (\Delta x')^2 - (c\Delta t')^2$$

$$(\Delta x)^2 = (\Delta x')^2 - (c\Delta t')^2$$

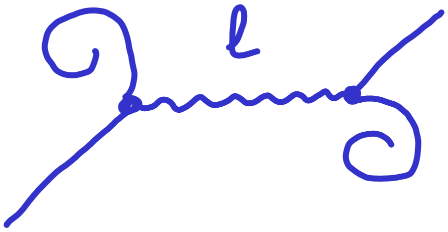
$$(c\Delta t')^2 = (\Delta x')^2 - (\Delta x)^2$$

$$(\Delta t')^2 = \frac{(\Delta x')^2 - (\Delta x)^2}{c^2}$$

$$\Delta t' = \frac{\sqrt{(\Delta x')^2 - (\Delta x)^2}}{c}$$

The two events are simultaneous in one reference frame, but not in the other.

Particle - X



track length: $\ell = 22.7\,\text{cm}$
lifetime : $\tau = 256.2\,\text{ps}$

Wrong answer: $v = \frac{\ell}{\tau} = \frac{22.7\,\text{cm}}{256.2\,\text{ps}} =$

$= \frac{0.227\,\text{m}}{2.562 \cdot 10^{-10}\,\text{s}} = 8.86 \cdot 10^{8}\,\frac{\text{m}}{\text{s}} = 2.95\,c$

Important: the lifetime of a particle is always measured in the frame of the particle, i.e. the lifetime is always proper time. We observed the decay of the particle in the lab frame, therefore we have to transform the proper time to lab time: $t = \gamma \cdot \tau$

With this the speed is:

$$v = \frac{\ell}{t}$$

$$v = \frac{\ell}{\gamma \tau}$$

$$v = \sqrt{1 - \frac{v^2}{c^2}} \cdot \frac{l}{\tau}$$

$$v^2 = \left(1 - \frac{v^2}{c^2}\right) \cdot \frac{l^2}{\tau^2}$$

$$v^2 = \frac{l^2}{\tau^2} - \frac{v^2}{c^2} \cdot \frac{l^2}{\tau^2}$$

$$v^2\left(1 + \frac{l^2}{c^2\tau^2}\right) = \frac{l^2}{\tau^2}$$

$$v^2 = \frac{1}{1 + \frac{l^2}{c^2\tau^2}} \cdot \frac{l^2}{\tau^2}$$

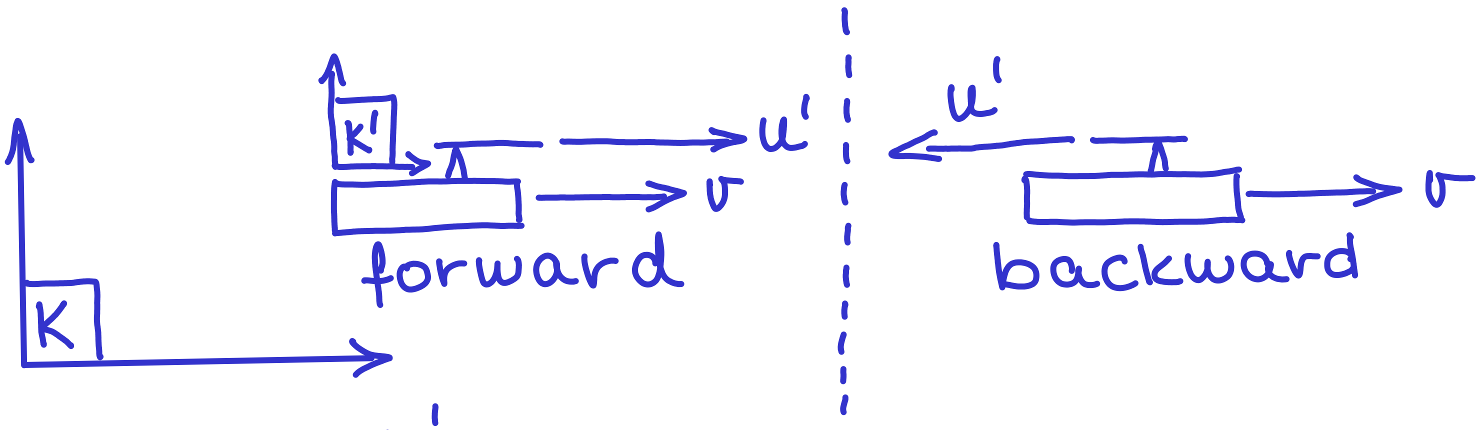
$$v = \frac{1}{\sqrt{1 + \frac{l^2}{c^2\tau^2}}} \cdot \frac{l}{\tau}$$

$$v = \frac{1}{\sqrt{1 + 2.95^2}} \cdot 2.95c$$

$$v = \frac{1}{3.11} \cdot 2.95c = 0.947c$$

$$\beta = 0.947$$

Millennium Falcon



$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$

$$B = \frac{v}{c} = 0.511$$

$$\frac{u}{c} = \frac{\frac{v}{c} + \frac{u'}{c}}{1 + \frac{v}{c} \cdot \frac{u'}{c}}$$

$$\beta' = \frac{u'}{c} = 0.706$$

$$\beta = \frac{u}{c} = ?$$

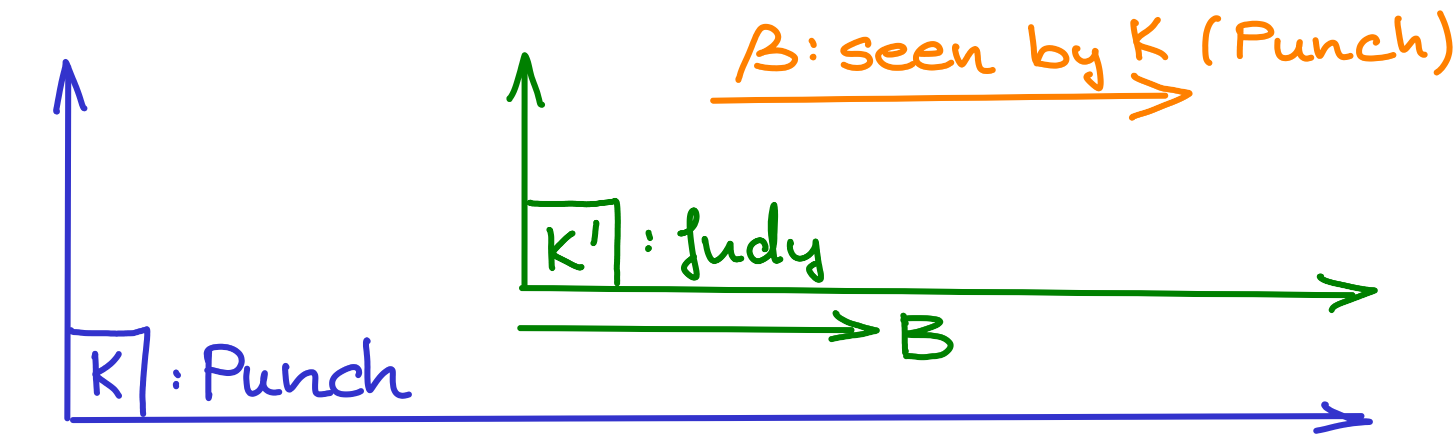
$$\beta = \frac{B + \beta'}{1 + B\beta'}$$

$$\beta_{forward} = \frac{0.511 + 0.706}{1 + 0.511 \cdot 0.706} = \frac{1.217}{1.361} = 0.894$$

$$\beta_{backward} = \frac{0.511 - 0.706}{1 + 0.511 \cdot (-0.706)} = \frac{-0.195}{0.639} = -0.305$$

B, β and β' can have minus signs, the relativistic velocity addition formula will work fine!

Punch and Judy



Question: what speed β' does Judy in K' see?

Relativistic velocity addition:

$$\beta = \frac{B + \beta'}{1 + B\beta'} \qquad (\beta = B \oplus \beta')$$

Can we solve this for β'?

$$\beta + \beta B \beta' = B + \beta'$$

$$\beta - B = \beta'(1 - \beta B)$$

$$\frac{\beta - B}{1 - \beta B} = \beta' \qquad (\beta \ominus B = \beta')$$

$$\beta' = \frac{0.85 - 0.60}{1 - 0.85 \cdot 0.60} = \frac{0.25}{0.49} = 0.51$$

Judy will see the rocket moving with $\beta' = 0.51$ instead of the classical 0.25.

Fizeau's experiment

Light travels with a speed of $\frac{c}{n}$ in a material with an index of refraction n. For water $n = 1.33$.

Therefore the speed of light in water is $c_w = \frac{c}{n} = 2.26 \cdot 10^8 \, m/s$ which is still a very high speed, therefore we will need relativistic physics, to deal with it.

The water in Fizeau's interferometer flows with a speed of $v = 1\text{-}3 \, \frac{m}{s}$.

How should we add and subtract c_w and v: classically or relativistically?

Classical method:

$$\left.\begin{aligned} u_+ &= \frac{c}{n} + v \\ u_- &= \frac{c}{n} - v \end{aligned}\right\} \Delta u_{cl} = u_+ - u_- = 2v$$

Relativistic method:

$$\left.\begin{aligned} u_+ &= \frac{c}{n} + \left(1 - \frac{1}{n^2}\right) v \\ u_- &= \frac{c}{n} - \left(1 - \frac{1}{n^2}\right) v \end{aligned}\right\} \begin{aligned} \Delta u_{rel} &= u_+ - u_- = \\ &= 2\left(1 - \frac{1}{n^2}\right) v \end{aligned}$$

Fizeau's experiment 2.

Where did we get the $\left(1-\frac{1}{n^2}\right)$ term?

Relativistic addition:

$$u_+ = \frac{\frac{c}{n}+v}{1+\frac{\frac{c}{n}v}{c^2}} = \frac{\frac{c}{n}+v}{1+\frac{1}{n}\cdot\frac{v}{c}} = \frac{c}{n}\cdot\frac{1+\frac{nv}{c}}{1+\frac{v}{nc}}$$

Since $v \ll c$: $\dfrac{1}{1+\frac{v}{nc}} \cong 1-\dfrac{v}{nc}$

Therefore: $u_+ \cong \frac{c}{n}\cdot\left(1+\frac{nv}{c}\right)\left(1-\frac{v}{nc}\right) =$

$$= \frac{c}{n}\left(1+\frac{nv}{c}-\frac{v}{nc}-\underbrace{\frac{v^2}{c^2}}_{\text{very small}}\right) \cong$$

$$\cong \frac{c}{n}\left(1+\left(n-\frac{1}{n}\right)\frac{v}{c}\right) =$$

$$= \frac{c}{n}+\left(1-\frac{1}{n^2}\right)v$$

Similarly: $u_- = \frac{c}{n}-\left(1-\frac{1}{n^2}\right)v$

Therefore $\Delta u = u_+ - u_- = 2\left(1-\frac{1}{n^2}\right)v$

The $\left(1-\frac{1}{n^2}\right)$ term is called Fresnel's drag coefficient. For water it is $\left(1-\frac{1}{1.33^2}\right) = 0.435$. Fizeau's experiment confirmed this value.