Radio transmission from a spaceship

The frequency of the radio on the spaceship: $f_0 = 3.175$ GHz.

Speed of the spaceship: $\beta = 0.727$.

When the ship is moving away, the frequency shifts down:

$$f_\downarrow = f_0 \cdot \sqrt{\frac{1-\beta}{1+\beta}} \quad \Big\} \text{ redshift}$$

When the ship is coming back, the frequency shifts up:

$$f_\uparrow = f_0 \cdot \sqrt{\frac{1+\beta}{1-\beta}} \quad \Big\} \text{ blue shift.}$$

The arithmetic mean of the two frequencies:

$$\frac{f_\downarrow + f_\uparrow}{2} = \frac{f_0}{2}\left(\sqrt{\frac{1-\beta}{1+\beta}} + \sqrt{\frac{1+\beta}{1-\beta}}\right) \quad \Big\} \text{ not very interesting}$$

Geometric mean:

$$\sqrt{f_\downarrow \cdot f_\uparrow} = \sqrt{f_0\sqrt{\frac{1-\beta}{1+\beta}} \cdot f_0\sqrt{\frac{1+\beta}{1-\beta}}} = \sqrt{f_0^2} = f_0$$

$$\boxed{\sqrt{f_\downarrow \cdot f_\uparrow} = f_0 \quad \text{or} \quad f_\downarrow \cdot f_\uparrow = f_0^2}$$

Relativistic speeding ticket

red light: $\lambda_0 = 680\,nm$
green light: $\lambda = 530\,nm$

$$c = \lambda f \Rightarrow f = \frac{c}{\lambda}$$

$$f_{\uparrow} = f_0 \sqrt{\frac{1+\beta}{1-\beta}}$$

$$\frac{c}{\lambda} = \frac{c}{\lambda_0} \cdot \sqrt{\frac{1+\beta}{1-\beta}}$$

$$\lambda = \lambda_0 \cdot \sqrt{\frac{1-\beta}{1+\beta}}$$

Doppler effect with wavelengths.
It is a blue shift: shift toward shorter wavelengths.

$$\left(\frac{\lambda}{\lambda_0}\right)^2 = \frac{1-\beta}{1+\beta}$$

$$\left(\frac{\lambda}{\lambda_0}\right)^2 + \left(\frac{\lambda}{\lambda_0}\right)^2 \cdot \beta = 1 - \beta$$

$$\left(\frac{\lambda}{\lambda_0}\right)^2 \cdot \beta + \beta = 1 - \left(\frac{\lambda}{\lambda_0}\right)^2$$

$$\beta = \frac{1 - \left(\frac{\lambda}{\lambda_0}\right)^2}{1 + \left(\frac{\lambda}{\lambda_0}\right)^2}$$

$\beta = 0.244 \Rightarrow v = 7.32 \cdot 10^7\, m/s$

It's a speeding ticket for sure!
But NASA will buy your car for big bucks!

Relativistic cookie

Mass (i.e. rest mass) of the cookie: $m = 179\,g = 0.179\,kg$

How much energy does this mass correspond to?

Einstein said:

$$E_0 = mc^2 = 0.179 \cdot (3 \cdot 10^8)^2 =$$
$$= 1.611 \cdot 10^{16} = 16.11\ PJ$$

This is a lot of energy, but how can we achieve this conversion?

Particle - X

$$\left.\begin{array}{l} u = 0.760c \text{ or } \beta = 0.760 \\ p = 5.260 \cdot 10^{-19}\ \text{kgm/s} \end{array}\right\} \quad \begin{array}{ll} m_0 = ? & E_0 = ? \\ K = ? & E = ? \end{array}$$

$$p = \gamma m_0 u \Rightarrow m_0 = \frac{p}{\gamma u} = \frac{\sqrt{1-\beta^2} \cdot p}{\beta c}$$

$$m_0 = \frac{\sqrt{1-0.76^2} \cdot 5.26 \cdot 10^{-19}}{0.76 \cdot 3 \cdot 10^8} = 1.499 \cdot 10^{-27}\ \text{kg}$$

$$E_0 = m_0 c^2 = \frac{\sqrt{1-\beta^2} \cdot p}{\beta c} \cdot c^2 = \frac{\sqrt{1-\beta^2}}{\beta} \cdot pc$$

$$E_0 = \frac{\sqrt{1-0.76^2}}{0.76} \cdot 5.26 \cdot 10^{-19} \cdot 3 \cdot 10^8 = 1.35 \cdot 10^{-10}\ \text{J}$$

$$E = \gamma E_0 = \gamma m_0 c^2 = \frac{1}{\sqrt{1-\beta^2}} \cdot \frac{\sqrt{1-\beta^2}}{\beta} \cdot pc = \frac{pc}{\beta}$$

$$E = \frac{pc}{\beta} = \frac{5.26 \cdot 10^{-19} \cdot 3 \cdot 10^8}{0.76} = 2.08 \cdot 10^{-10}\ \text{J}$$

$$K = E - E_0 = (\gamma - 1) E_0 = \left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) \cdot \frac{\sqrt{1-\beta^2}}{\beta} \cdot pc =$$

$$= \frac{1-\sqrt{1-\beta^2}}{\sqrt{1-\beta^2}} \cdot \frac{\sqrt{1-\beta^2}}{\beta} \cdot pc = \frac{1-\sqrt{1-\beta^2}}{\beta} \cdot pc$$

$$K = \frac{1-\sqrt{1-0.76^2}}{0.76} \cdot 5.26 \cdot 10^{-19} \cdot 3 \cdot 10^8 = 0.73 \cdot 10^{-10}\ \text{J}$$

Protons from the tevatron

The rest energy of the proton:

$E_0 = 938.27$ MeV (MeV = mega electronvolt, $1\,\text{eV} = 1.6 \cdot 10^{-19}\,\text{J}$)

The total relativistic energy of the proton is $E = 1$ TeV, where $1\,\text{TeV} = 10^3\,\text{GeV} = 10^6\,\text{MeV} = 10^9\,\text{keV} = 10^{12}\,\text{eV}$.

What is the gamma factor and what is the speed of these protons?

$$E = \gamma m c^2 = \gamma E_0 \Rightarrow \gamma = \frac{E}{E_0} = \frac{10^6\,\text{MeV}}{938.27\,\text{MeV}}$$

$\gamma = 1065.8$ — the proton is not a sphere at this energy, but a disk! (or a pancake)

$\bigcirc \Rightarrow |$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow 1-\beta^2 = \frac{1}{\gamma^2} \Rightarrow$$

$$\sqrt{1 - \frac{1}{\gamma^2}} = \beta$$

$$\beta = \frac{\sqrt{\gamma^2 - 1}}{\gamma} = 0.\underbrace{999999}_{\text{six 9-s}}56$$

Relativistic electron

$$\left.\begin{array}{l} E = 135\,E_0 \\ E_0 = 0.511\,\text{MeV} = 511\,\text{keV} \end{array}\right\} \quad \begin{array}{ll} \frac{K}{E_0} = ? & K = ? \\ \beta = ? & p = ? \end{array}$$

$$\left.\begin{array}{l} E = 135 E_0 \\ E = \gamma E_0 \end{array}\right\} \Rightarrow \gamma = 135$$

$$K = (\gamma - 1)E_0 \Rightarrow \frac{K}{E_0} = (\gamma - 1) = 134$$

$$K = (\gamma - 1)E_0 = 134 \cdot 0.511\,\text{MeV} = 68.5\,\text{MeV}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \beta = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} = 0.99997256$$

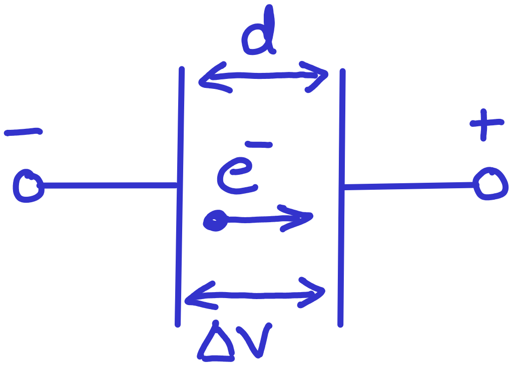
$$\frac{p}{E} = \frac{\beta}{c} \Rightarrow p = \frac{\beta}{c} E = \frac{\beta}{c}\gamma E_0 = \frac{E_0}{c}\beta\gamma =$$

$$= \frac{E_0}{c} \cdot \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} \cdot \gamma = \frac{E_0}{c}\sqrt{\gamma^2 - 1}$$

$$\boxed{p = \frac{E_0}{c}\sqrt{\gamma^2 - 1}}$$

$$p = \frac{0.511\,\text{MeV}}{c} \cdot \sqrt{135^2 - 1} = 68.98\,\frac{\text{MeV}}{c}$$

Classical vs. relativistic electron



$\Delta V = 117\,\text{kV}$

$(d = 16.1\,\text{cm})$

electron:

$m_0 c^2 = 511\,\text{keV}$

$m_0 = 511\,\dfrac{\text{keV}}{c^2}$

$W = e\cdot\Delta V = KE$ → $\beta_{class} = ?$ → $\beta_{rel} = ?$

Classical:

$$e\Delta V = \frac{1}{2} m_0 v^2 \Rightarrow v = \sqrt{\frac{2eV}{m_0}} \Rightarrow$$

$$\Rightarrow \beta_c = \sqrt{\frac{2eV}{m_0 c^2}} = \sqrt{\frac{2\cdot 1e\cdot 117\,\text{kV}}{511\,\text{keV}}} = 0.6767$$

Relativistic:

$$e\Delta V = (\gamma - 1) m_0 c^2$$

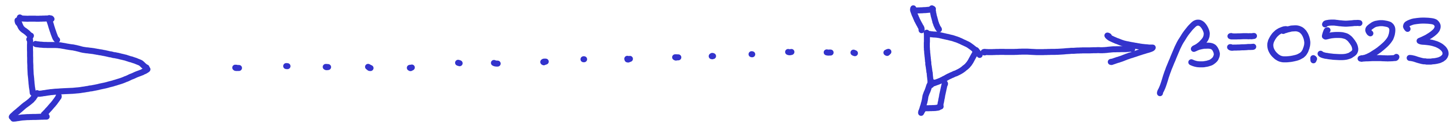
$$\frac{e\Delta V}{m_0 c^2} + 1 = \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\frac{1}{\sqrt{1-\beta^2}} = \frac{e\Delta V + m_0 c^2}{m_0 c^2}$$

$$\beta = \sqrt{1 - \left(\frac{m_0 c^2}{e\Delta V + m_0 c^2}\right)^2}$$

$$\beta_r = \sqrt{1 - \left(\frac{511\,\text{keV}}{117\,\text{keV} + 511\,\text{keV}}\right)^2} = 0.5813$$

Number of missions



$$\text{Work} = K = (\gamma - 1)\, mc^2 = \left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) \cdot mc^2$$

If $m = 118\,t = 1.18 \cdot 10^5$ kg, then

$$W = K = 1.84 \cdot 10^{21}\,J$$

Mission: to send a spaceship away without any extra fuel to stop or turn around. This seems cruel, but it is a robotic mission. There are no humans on board.

Warning: efficiency:

$$\eta = 0.75\% = 0.0075$$

We need to use Sun's energy for other things as well, not just space exploration. We have to grow food, for example.

The number of missions will come out between 10 and 30 per year.