

## Constant volume gas thermometer

$$p_1 = 318 \text{ kPa} \text{ at } T_1 = 100^\circ\text{C} = 373 \text{ K}$$

$$p_2 = 450 \text{ kPa} \text{ at } T_2 = ?$$

Gay-Lussac / Charles :

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow T_2 = T_1 \cdot \frac{p_2}{p_1}$$

$$T_2 = 373 \text{ K} \cdot \frac{450 \text{ kPa}}{318 \text{ kPa}} = 527.8 \text{ K}$$

Unified gas law :

$$\frac{p_1 V_0}{T_1} = \frac{p_2 V_0}{T_2} \Rightarrow \frac{p_1}{T_1} = \frac{p_2}{T_2}$$

Volume  $V_0$  drops out of the equation, because it is constant.

## Tire pressure

$$P_0 = 445 \text{ kPa}$$

$$T_i = 10.5^\circ\text{C} = 283.5 \text{ K}$$

$$T_f = 57.5^\circ\text{C} = 330.5 \text{ K}$$

First case: constant volume

$$\frac{P_0}{T_i} = \frac{P}{T_f} \Rightarrow P = P_0 \cdot \frac{T_f}{T_i}$$

$$P = 445 \text{ kPa} \cdot \frac{330.5 \text{ K}}{283.5 \text{ K}} = 519 \text{ kPa}$$

Second case:  $V_i \rightarrow V_f$  expansion

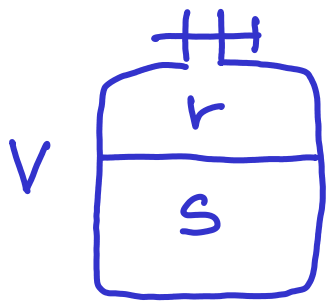
Unified Gas Law:

$$\frac{P_0 V_i}{T_i} = \frac{P \cdot V_f}{T_f} \Rightarrow P = P_0 \cdot \frac{T_f}{T_i} \cdot \frac{V_i}{V_f}$$

$$P = 445 \text{ kPa} \cdot \frac{330.5 \text{ K}}{283.5 \text{ K}} \cdot \frac{24.9 \text{ l}}{25.6 \text{ l}} = 505 \text{ kPa}$$

Important: the final pressure is lower, if the gas is allowed to expand from its initial volume.

## Releasing gas from a bottle



released:  $r = 43\% = 0.43$

stays :  $s = 57\% = 0.57$

$$r + s = 1$$

$$P_i = 3.84 \text{ MPa } (\approx 38 \text{ atm})$$

$$T_i = 23.3^\circ\text{C} = 296.3 \text{ K} \quad : \text{ initial}$$

$$T_f = 5.8^\circ\text{C} = 278.8 \text{ K} \quad : \text{ final}$$

Ideal Gas Law for the initial state:

$$P_i V = n_i R T_i \quad / \cdot s \text{ both sides}$$

$P_i \cdot (sV) = (s n_i) R T_i$  :  $(s n_i)$  amount of gas occupies  $(sV)$  of the volume.

Ideal Gas Law for the final state:

$P_f V = (s n_i) R T_f$  :  $(s n_i)$  amount of gas occupies the full  $V$  volume, b/c fraction  $r$  of the gas was released.

The ratio of the two equations:

$$\frac{P_i V}{P_f V} = \frac{n_i R T_i}{s n_i R T_f}$$

$$\frac{P_i}{P_f} = \frac{T_i}{sT_f}$$

$$P_f = P_i \cdot \frac{sT_f}{T_i} = 3.84 \cdot \frac{0.57 \cdot 278.8}{296.3}$$

$$P_f = 2.06 \text{ MPa}$$

## Gas densities at STP

$$\text{STP: } \left. \begin{array}{l} T = 273\text{K} \\ p = 101.3\text{kPa} \end{array} \right\} \Rightarrow V = 22.4\text{ l for 1 mol} \\ \text{or } V_{\text{molar}} = 22.4\text{ l/mol}$$

Xenon: atomic mass: 131.3 AMU.

$$1\text{ AMU} = 1.66 \times 10^{-27}\text{ kg}$$

$m = 131.3\text{ AMU}$  means that 1 mol of this substance has a mass of 131.3 g.

$$\rho = \frac{m}{V} = \frac{M}{V_{\text{molar}}} = \frac{131.3\text{ g/mol}}{22.4\text{ l/mol}}$$

$$\rho = 5.86\text{ g/l} = 5.86\text{ kg/m}^3$$

Carbon-dioxide:  $\text{CO}_2$  at STP

$$\text{Molar mass: } (\underbrace{12}_{\text{C}} + \underbrace{2 \cdot 16}_{2 \cdot \text{O}})\text{ g/mol} = 44\text{ g/mol}$$

$$\rho = \frac{44\text{ g/mol}}{22.4\text{ l/mol}} = 1.96\text{ g/l} = 1.96\text{ kg/m}^3$$


LON-CAPA uses L for liter instead of l.

The best and safest unit for density is  $\text{kg/m}^3$ .

## Thermal energy of a gas

$k_B = 1.38 \cdot 10^{-23}$  J/K : Boltzmann constant

$T = 311$  K : temperature

Hydrogen:  $H_2$ :   $\Rightarrow f = 5$  : degrees of freedom: 3 translations + 2 rotations

$E_1 = \frac{1}{2} k_B T$  : average energy of one degree of freedom of one molecule.

$E = \frac{f}{2} k_B T$  : average energy of one molecule.

$E_{th,1} = \frac{1}{2} N k_B T$  : energy carried by one degree of freedom of the whole system.

$E_{th} = \frac{f}{2} N k_B T$  : internal or thermal energy of the whole system:  $N$  molecules,  $f$  degrees of freedom for each molecule.

# RMS speed of Helium atoms

$T = 446\text{K}$  : temperature

$k_B = 1.38 \cdot 10^{-23} \text{ J/K}$  : Boltzmann constant

Helium :  $m_{\text{He}} = 4.00 \text{ AMU} \Rightarrow$

$\Rightarrow M_{\text{He}} = 4.00 \text{ g/mol}$

( $1 \text{ AMU} = 1.66 \cdot 10^{-27} \text{ kg}$ )

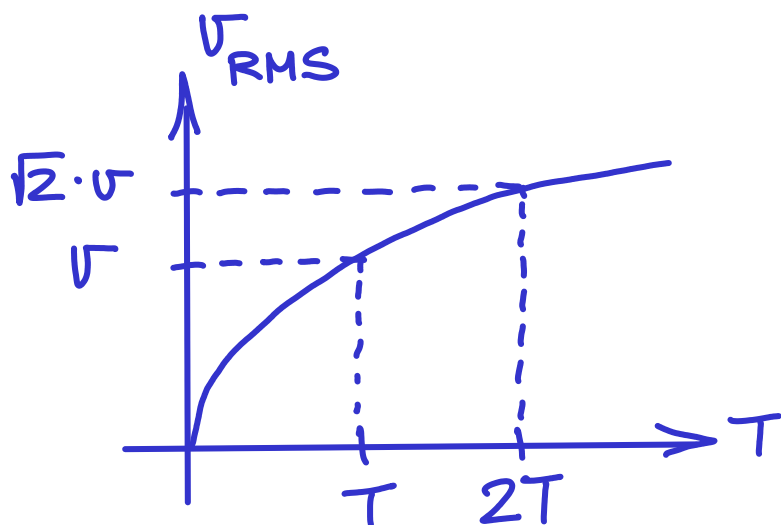
$$v_{\text{RMS}} = \sqrt{\frac{3k_B T}{m}}$$

$$v_{\text{RMS}} = \sqrt{\frac{3 \cdot 1.38 \cdot 10^{-23} \cdot 446}{4 \cdot 1.66 \cdot 10^{-27}}} = 1667 \frac{\text{m}}{\text{s}}$$

Alternatively :

$$v_{\text{RMS}} = \sqrt{\frac{3RT}{M}}$$

$$v_{\text{RMS}} = \sqrt{\frac{3 \cdot 8.31 \frac{\text{J}}{\text{molK}} \cdot 446\text{K}}{0.004 \text{ kg/mol}}} = 1667 \frac{\text{m}}{\text{s}}$$



$$v_{\text{RMS}} = \sqrt{\frac{3k_B T}{m}}$$

$$v_{\text{RMS}} \propto \sqrt{T}$$

If  $T \rightarrow 2T$ ,  
then  $v \rightarrow \sqrt{2} v$ .

$$\sqrt{2} = 1.4142$$

## RMS speed at nanokelvin temperatures

$$T = 157 \text{ nK} ; 1 \text{ nK} = 10^{-9} \text{ K} ; \text{n: nano}$$

$$\text{Rubidium: } M = 86.91 \frac{\text{g}}{\text{mol}} = 0.08691 \frac{\text{kg}}{\text{mol}}$$

$$v_{\text{RMS}} = \sqrt{\frac{3k_{\text{B}}T}{m}} = \sqrt{\frac{3RT}{M}}$$

$$v_{\text{RMS}} = \sqrt{\frac{3 \cdot 8.31 \cdot 157 \cdot 10^{-9}}{0.08691}}$$

$$v_{\text{RMS}} = 0.00671 \frac{\text{m}}{\text{s}} = \underline{6.71 \frac{\text{mm}}{\text{s}}} \quad \downarrow$$

Diameter of atoms:  $1-10 \text{ \AA} = 10^{-10} - 10^{-9} \text{ m}$ .

Even at this very low temperature an atom of an ideal gas would travel a distance of 10-100 million times of its diameter per second.