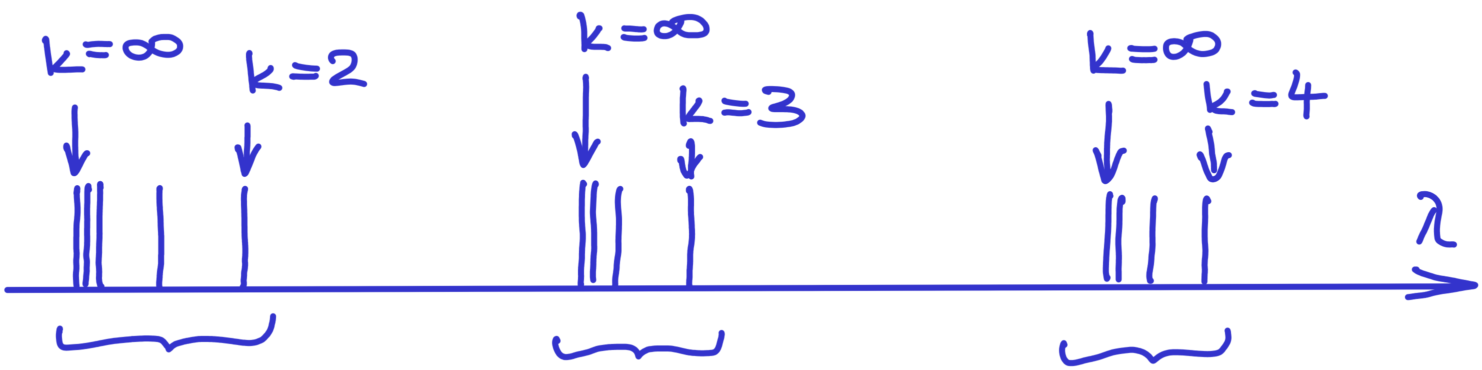
Series limits

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n^2} - \frac{1}{k^2} \right) ; \quad R_H = 1.096776 \cdot 10^7 \, \frac{1}{m}$$



Lyman	Balmer	Paschen
$n=1$	$n=2$	$n=3$
UV	UV, visible	IR

$$\frac{1}{\lambda_{lim}} = R_H \left(\frac{1}{n^2} - \frac{1}{\infty^2} \right) = \frac{R_H}{n^2}$$

$$\lambda_{lim} = \frac{n^2}{R_H}$$ } shortest wavelength in the series

Longest wavelength in a series:

$$\frac{1}{\lambda_{long}} = R_H \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right) = R_H \frac{(n+1)^2 - n^2}{n^2 \cdot (n+1)^2}$$

$$= R_H \frac{2n+1}{n^2 \cdot (n+1)^2}$$

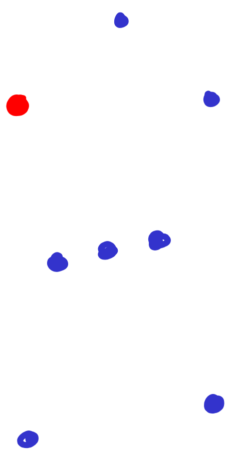
Series limits 2.

$$\lambda_{longest} = \frac{n^2 \cdot (n+1)^2}{R_H \cdot (2n+1)}$$

n	n^2	$\lambda_{lim} = \frac{n^2}{R_H}$	$\frac{n^2(n+1)^2}{2n+1}$	$\lambda_{longest}$
1	1	91.2 nm	$\frac{4}{3} = 1.333$	121.6 nm
2	4	364.7 nm	$\frac{36}{5} = 7.2$	656.5 nm
3	9	820.6 nm	$\frac{144}{7} = 20.57$	1876 nm
4	16	1459 nm	$\frac{400}{9} = 44.44$	4052 nm
5	25	2279 nm	$\frac{900}{11} = 81.81$	7460 nm

visible spectrum: 400-700 nm

Betelgeuse



$\lambda_{max} = 800\,nm$
(infrared)

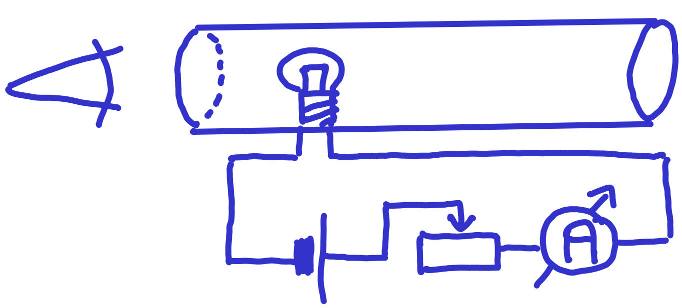
Wien's displacement law:

$$\lambda_{max} \cdot T = b \; ; \quad b = 2.898 \cdot 10^{-3}\, m \cdot K$$

$$T = \frac{b}{\lambda_{max}} = \frac{2.898 \cdot 10^{-3}}{800 \cdot 10^{-9}} = 3622\,K$$

Sun: $\lambda_{max} = 500\,nm$
$\rightarrow T = 5800\,K$

Bolometer:



Cosmic background radiation

$\lambda_{max} = 1.07mm \longleftrightarrow T = 2.7K$

$$f_{max} = \frac{c}{\lambda_{max}} = \frac{3 \cdot 10^{8} m/s}{1.07 \cdot 10^{-3} m} = 280 GHz$$

Penzias, Wilson (1978, Nobel)

COBE: Cosmic Background Explorer: $T = 2.73K$

WMAP: Wilkinson Microwave Anisotropy Probe: $T = 2.725K$

Efficiency of an incandescent bulb

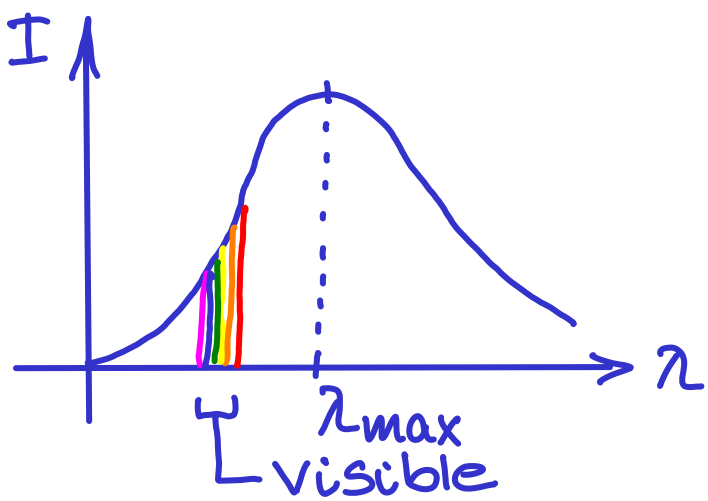
$P = \sigma \varepsilon A T^4$ $\qquad T = 2330\,K$ $\quad$ Tungsten

$\frac{P}{A} = \sigma \varepsilon T^4$ $\qquad \varepsilon = 1$

$\sigma = 5.6704 \cdot 10^{-8} \, \frac{W}{m^2 K^4}$

$$\frac{P}{A} = 5.67 \cdot 10^{-8} \cdot 2330^4 = 1.67 \cdot 10^6 \, \frac{W}{m^2}$$

(100W lightbulb $\Rightarrow A \approx 60\,mm^2$)



$\lambda_{max} = \frac{b}{T} \cong 1240\,nm$

$\lambda_{blue} = 400\,nm$

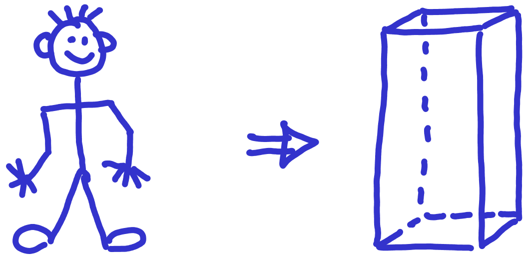
$\lambda_{red} = 700\,nm$

Max Planck:

$$I(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

$\eta \approx 2-6\%$

Power emitted by the human body



$H = 1.88\,m$
$W = 0.315\,m$
$L = 0.26\,m$

$\varepsilon = 97\% = 0.97$
$a = 97\% = 0.97$

$T_{body} = 35.0^\circ C = 308\,K$

$P_{out} = \sigma \varepsilon A T_{body}^4$

$\sigma = 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4}$

$A = 2(HW + WL + LH)$

$P_{out} \approx 1000 - 1500\ W\,!$

$T_{env} = 22.5^\circ C = 295.5\,K$

$P_{in} = a \cdot P_{env} = a \cdot \sigma \cdot 1 \cdot A T_{env}^4$

↖ black body rad.

$P_{in} = P_{out} \cdot \left(\frac{T_{env}}{T_{body}}\right)^4$

$P_{in} \approx 900 - 1200\,W$

$P_{loss} = P_{out} - P_{in} \approx 100 - 200\,W$

$\lambda_{max} = \frac{b}{T_{body}} = \frac{2.898 \cdot 10^{-3}\, m \cdot K}{308\,K} = \underbrace{9410\,nm}_{\text{far infrared}}$

Radio station

$f = 94.9\ \mathrm{MHz}$ (WMMQ)

$P = 46.5\ \mathrm{kW}$ (?)

Planck constant: $h = 6.63 \cdot 10^{-34}\ \mathrm{Js}$

Energy of one photon at the frequency above:

$$E = hf = 6.63 \cdot 10^{-34}\ \mathrm{Js} \cdot 94.9 \cdot 10^{6}\ \frac{1}{s} =$$

$$= 6.29 \cdot 10^{-26}\ \mathrm{J}$$

Number of photons per second:

$$\frac{P}{E} = \frac{46.5 \cdot 10^{3}\ \mathrm{W}}{6.29 \cdot 10^{-26}\ \mathrm{J}} = 7.39 \cdot 10^{29}\ \frac{1}{s}$$

Roughly a million Avogadro number of photons each second.

Photoelectric effect

Work function: $\phi = 3.25\,eV$

Einstein: $hf = \phi + \underbrace{KE}_{=0} \Rightarrow f_{min} = \dfrac{\phi}{h}$

$h = 6.63 \cdot 10^{-34}\,Js = 4.14 \cdot 10^{-15}\,eVs$

$$f_{min} = \frac{3.25\,eV}{4.14 \cdot 10^{-15}\,eVs} = 7.85 \cdot 10^{14}\,Hz$$

$$\lambda_{max} = \frac{c}{f_{min}} = \frac{3 \cdot 10^{8}\,m/s}{7.85 \cdot 10^{14}\,Hz} \cong \underbrace{382\,nm}_{\text{(ultra) violet}}$$

$hc = 1240\,eVnm$

$$\lambda_{max} = \frac{hc}{hf} = \frac{1240\,eVnm}{3.25\,eV} \cong 382\,nm$$

When the photon frequency is doubled, the energy of the photon doubles: $E_\gamma = 6.50\,eV$. Therefore the kinetic energy of the electrons is going to be $KE = 3.25\,eV$.

$$\underbrace{hf}_{6.5eV} = \underbrace{\phi}_{3.25eV} + \underbrace{KE}_{3.25eV}$$

Planck's constant from the photoelectric effect

V_0 (V)	λ (nm)
3.47	276
5.12	216

$$hf = \phi + eV_0$$

$$\frac{hc}{\lambda} = \phi + eV_0$$

$$\left.\begin{aligned} h\frac{c}{\lambda_1} &= \phi + eV_1 \\ h\frac{c}{\lambda_2} &= \phi + eV_2 \end{aligned}\right\}$$

Two equations, two unknowns: h, ϕ

$$h\frac{c}{\lambda_1} - eV_1 = h\frac{c}{\lambda_2} - eV_2$$

$$e(V_2 - V_1) = hc\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$h = \frac{e(V_2 - V_1)}{c\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)}$$

$$h = \frac{1e \cdot (5.12V - 3.47V)}{3 \cdot 10^8 \frac{m}{s}\left(\frac{1}{2.16 \cdot 10^{-7} m} - \frac{1}{2.76 \cdot 10^{-7} m}\right)}$$

$$h = 5.46 \cdot 10^{-15} \text{ eVs}$$

(Official value: $4.14 \cdot 10^{-15}$ eVs)

Planck's constant 2.

$$\phi = \frac{hc}{\lambda_1} - eV_1 =$$

$$= \frac{5.46 \cdot 10^{-15}\,eVs \cdot 3 \cdot 10^{8}\,\frac{m}{s}}{2.76 \cdot 10^{-7}\,m} - 3.47\,eV =$$

$$= 5.93\,eV - 3.47\,eV = 2.46\,eV$$

$$\phi = \frac{hc}{\lambda_2} - eV_2 = \ldots$$

X-ray production by bremsstrahlung

Voltage applied to the X-ray tube:

$\Delta V = 25.4 \text{ kV}$

Energy balance:

$$e\Delta V = hf_{max}$$

$$e\Delta V = \frac{hc}{\lambda_{min}}$$

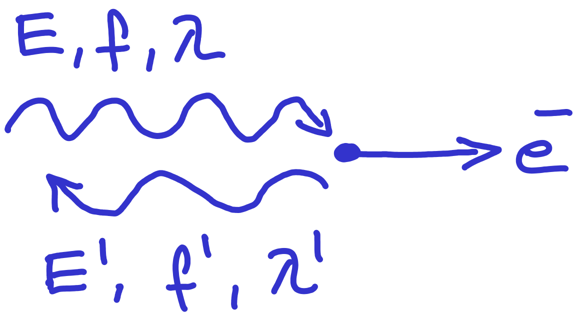
$$\lambda_{min} = \frac{hc}{e\Delta V} = \frac{1240 \text{ eVnm}}{1e \cdot 25{,}400 \text{ V}}$$

$$\lambda_{min} = 0.0488 \text{ nm} = 48.8 \text{ pm}$$

Maximum energy transfer in Compton scattering

$$\Delta\lambda = \lambda' - \lambda = \underbrace{\frac{h}{m_e c}}_{\lambda_c = 2.426\,\text{pm}} (1 - \cos\theta)$$

$\Delta\lambda$ is maximum when $\theta = 180°$:



$E = 42{,}2$ keV

$$\Delta\lambda = 2 \cdot \frac{h}{m_e c} = 2\lambda_c \qquad \text{when } \theta = 180°$$

$$\lambda' = \lambda + 2\lambda_c = \frac{hc}{E} + 2\frac{h}{m_e c} = \frac{hc}{E} + 2\frac{hc}{m_e c^2}$$

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\frac{hc}{E} + 2\frac{hc}{m_e c^2}} = \frac{1}{\frac{1}{E} + \frac{2}{m_e c^2}}$$

$$\Delta E = E - E' = E - \frac{1}{\frac{1}{E} + \frac{2}{m_e c^2}}$$

Maximum energy transfer 2.

$$\Delta E = E - \frac{E \cdot m_e c^2}{m_e c^2 + 2E} = \frac{2E^2}{m_e c^2 + 2E}$$

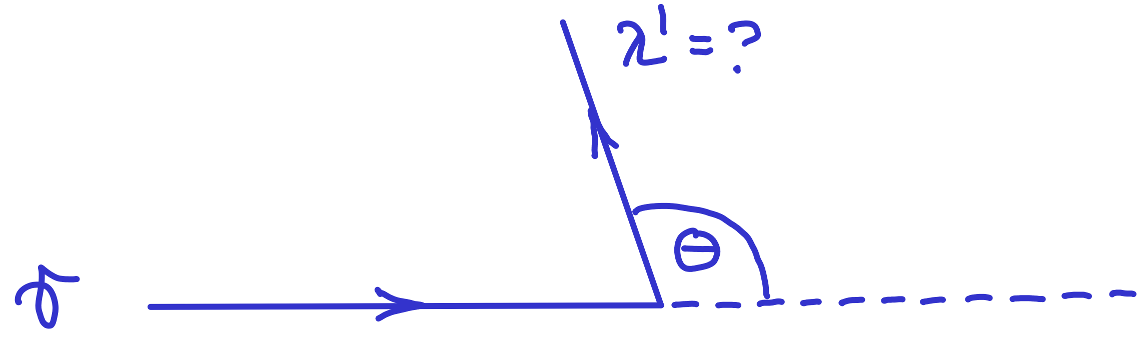
$m_e c^2 = 511\ \text{keV}$ $\quad E = 42.2\ \text{keV}$

$$\Delta E = \frac{2 \cdot 42.2^2}{511 + 2 \cdot 42.2} = 5.98\ \text{keV}$$

$E' = 36.2\ \text{keV}$

This maximum energy transfer happens, when the photon bounces exactly backwards: $\theta = 180°$ and the electron is kicked exactly forward: $\varphi = 0°$.

Compton scattering



$\lambda = 135\,\text{pm}$ $\qquad$ $\theta = 105°$

Compton:

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

$$\lambda' = \lambda + \Delta\lambda = \lambda + \frac{h}{m_e c}(1 - \cos\theta) =$$

$$= \lambda + \frac{hc}{m_e c^2}(1 - \cos\theta) =$$

$$= 0.135\,\text{nm} + \underbrace{\frac{1240\,\text{eVnm}}{511{,}000\,\text{eV}}}_{0.002426\,\text{nm}}(1 - \cos 105°) =$$

$$= 135\,\text{pm} + 2{,}426\,\text{pm} \cdot (1 - \cos 105°) =$$

$$= 138.05\,\text{pm}$$

$$\% = 100 \cdot \frac{\Delta\lambda}{\lambda} = 100 \cdot \frac{3.05}{135} = 2.26\,\%$$