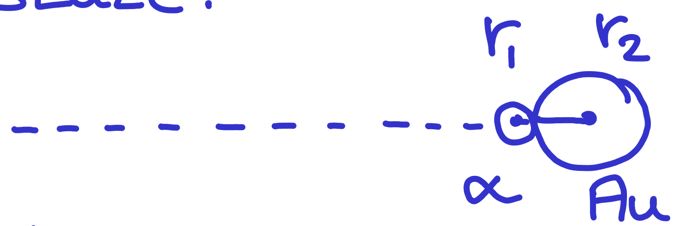
Alpha particle touching gold nucleus

initial state:

final state:

$r_1 = 2.85\,\text{fm}$

$r_2 = 7.53\,\text{fm}$



Energy balance:

$$KE_i + \underbrace{PE_i}_{=0} = \underbrace{KE_f}_{=0} + PE_f$$

$$KE_i = PE_f$$

$$KE_i = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(Z_1 e)\cdot(Z_2 e)}{(r_1 + r_2)}$$

$Z_1 = 2 : \alpha$

$Z_2 = 79 : \text{Au}$

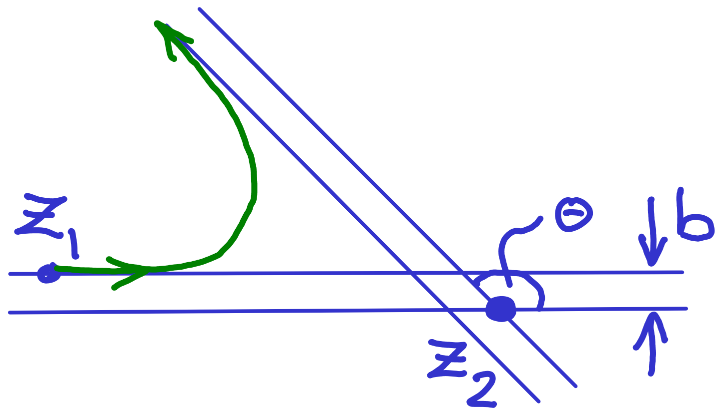
$r_1 = 2.85 \cdot 10^{-6}\,\text{nm}$

$r_2 = 7.53 \cdot 10^{-6}\,\text{nm}$

$$KE_i = \underbrace{\frac{e^2}{4\pi\varepsilon_0}}_{1.44\,\text{eVnm}} \cdot \frac{Z_1 \cdot Z_2}{(r_1 + r_2)}$$

$$KE_i = 2.19 \cdot 10^{7}\,\text{eV} = 21.9\,\text{MeV}$$

Impact parameter of an alpha particle



$Z_1 = 2$: alpha

$Z_2 = 79$: Gold

$\Theta = 134.3°$

$KE = 3.29\ MeV =$
$= 3.29 \cdot 10^6\ eV$

$$b = \frac{1}{2} \cdot \frac{k Z_1 e Z_2 e}{KE} \cdot \cot\left(\frac{\Theta}{2}\right) \qquad k = \frac{1}{4\pi\varepsilon_0}$$

$$b = \frac{1}{2} \cdot \underbrace{\frac{e^2}{4\pi\varepsilon_0}}_{1.44\ eV\,nm} \cdot \frac{Z_1 \cdot Z_2}{KE} \cdot \cot\left(\frac{\Theta}{2}\right)$$

$$b = 14.6 \cdot 10^{-6}\ nm = 14.6\ fm$$

$$b \approx 10 - 30\ fm$$

$KE_{\alpha,f} = KE_{\alpha,i}$: the scattering is elastic

Rutherford scattering

Integral: unit: 1

$$f = \frac{\pi}{4} \cdot nt \cdot (kZ_1eZ_2e)^2 \cdot \frac{1}{KE^2} \cdot \cot^2\left(\frac{\theta}{2}\right)$$

$$f = \frac{\pi}{4} \cdot nt \; (ke^2)^2 \cdot \frac{(Z_1Z_2)^2}{KE^2} \cdot \cot^2\left(\frac{\theta}{2}\right)$$

Differential: unit: $1/m^2$

$$\frac{N(\theta)}{N_i} = \frac{1}{16} \cdot nt \cdot (kZ_1eZ_2e)^2 \cdot \frac{1}{KE^2} \cdot \frac{1}{r^2} \cdot \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

$$\frac{N(\theta)}{N_i} = \frac{1}{16} \cdot nt \; (ke^2)^2 \cdot \frac{(Z_1Z_2)^2}{KE^2} \cdot \frac{1}{r^2} \cdot \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

$$ke^2 = \frac{e^2}{4\pi\varepsilon_0} = 1.44\,eVnm = 1.44 \cdot 10^{-9}\,eVm$$

$$n = 5.90 \cdot 10^{28}\,\frac{1}{m^3}\,; \quad t = 16.7\,nm = 16.7 \cdot 10^{-9}\,m$$

$$\theta = 19.6^\circ\,; \quad \alpha : Z_1 = 2\,; \quad Au : Z_2 = 79$$

$$KE = 5.70\,MeV = 5.70 \cdot 10^6\,eV$$

$$r = 21.2\,cm = 0.212\,m$$

Bohr-model: radius, energy, speed

$Z=1$

	$n=1$	$n=2$	$n=3$	$n=4$
r_n:	$1a_0$	$4a_0$	$9a_0$	$16a_0$
E_n:	$-E_0$	$-\frac{E_0}{4}$	$-\frac{E_0}{9}$	$-\frac{E_0}{16}$
β_n:	α	$\frac{\alpha}{2}$	$\frac{\alpha}{3}$	$\frac{\alpha}{4}$

$$r_n = n^2 a_0 \,; \qquad a_0 = \frac{\hbar^2}{m e^2 k} = 0.529\,\text{Å}$$

$$E_n = -\frac{E_0}{n^2} \,; \qquad E_0 = \frac{m e^4 k^2}{2\hbar^2} = 13.6\,\text{eV}$$

$$\beta_n = \frac{\alpha}{n} \,; \qquad \alpha = \frac{e^2 k}{\hbar c} \cong \frac{1}{137}$$

Coulomb constant: $\quad k = \frac{1}{4\pi\varepsilon_0}$

Hydrogen-like ions

$E=0$

H ($Z=1$)

$n=2$: $-\frac{1}{4}E_0$ = $-3.4\,eV$

$n=1$: $-E_0$ = $-13.6\,eV$

He⁺ ($Z=2$)

$n=4$: $-\frac{4}{16}E_0$ = $-3.4\,eV$

$n=3$: $-\frac{4}{9}E_0$ = $-6.04\,eV$

$n=2$: $-\frac{4}{4}E_0$ = $-13.6\,eV$

$n=1$: $-4E_0$ = $-54.4\,eV$

Li^{++} ($Z=3$)

$n=4$: $-\frac{9}{16}E_0$ = $-7.65\,eV$

$n=3$: $-\frac{9}{9}E$ = $-13.6\,eV$

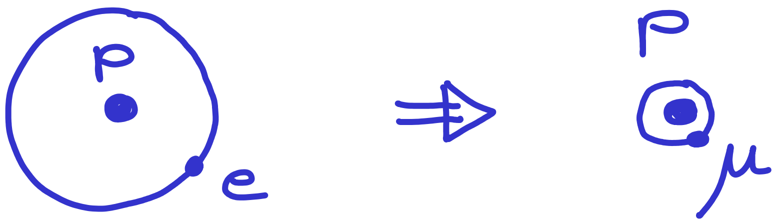
$n=2$: $-\frac{9}{4}E$ = $-30.6\,eV$

$n=1$: $-9E_0$ = $-122.4\,eV$

$$\boxed{E_{n,Z} = -\frac{Z^2}{n^2}E_0}$$

$E_0 = 13.6\,eV$

Muonic atom



electron: $m_e = 0.511\ MeV/c^2 = 1 m_e$

muon : $m_\mu = 105.7\ MeV/c^2 = 206.8\, m_e$

proton : $m_p = 938.3\ MeV/c^2 = 1836\, m_e$

$$E_{n,Z} = -\frac{me^4k^2Z^2}{2\hbar^2n^2} = -\frac{Z^2}{n^2}E_o \ ; \ E_o = 13.6 eV$$

$$r_{n,Z} = \frac{\hbar^2n^2}{me^2kZ} = \frac{n^2}{Z}a_o \ ; \ a_o = 0.529\,\mathring{A}$$

$$\beta_{n,Z} = \frac{e^2kZ}{\hbar cn} = \frac{Z}{n}\alpha \ ; \ \alpha = \frac{1}{137}$$

If we just use the muon-mass ($m_\mu = 206.8\, m_e$) in these formulas, we will not get the correct values. We need to use the reduced mass.

Muonic atom 2.

Reduced mass of the muon when it orbits the proton:

$$\mu_\mu = \frac{m_\mu \cdot m_p}{m_\mu + m_p} = \frac{206.8 m_e \cdot 1836 m_e}{206.8 m_e + 1836 m_e}$$

$$\mu_\mu = 185.9 m_e$$

(This is significantly different from $m_\mu = 206.8 m_e$.)

The energy of the muonic atom is 185.9 times larger than the regular Hydrogen atom, and it is 185.9 times smaller than the Hydrogen atom. However the speed of the muon is the same as the speed of the electron.

$$k\frac{e \cdot e}{r^2} = m\frac{v^2}{r} \Rightarrow ke^2 = mrv^2$$

m : 185.9 times larger

r : 185.9 times smaller

v : same