Carbon-14 dating

Willard Libby, 1960 Chemistry Nobel

$^{14}_{6}C \longrightarrow {}^{14}_{7}N + \beta^- + \bar{\nu}_e$ with $T_{1/2} = 5730\,yrs$

$$\frac{^{14}C}{^{12}C} = 1.2 \cdot 10^{-12} \quad (\text{currently})$$

$$R_0 = 14 \frac{\text{decays}}{\text{min} \cdot g} \quad \text{of Carbon}$$

$$R = R_0 \cdot 2^{-t/T_{1/2}} \Rightarrow q = \frac{R}{R_0} = 2^{-t/T_{1/2}} = 0.710$$

$$\ln q = -\frac{t}{T_{1/2}} \cdot \ln 2$$

$$-\frac{T_{1/2} \cdot \ln q}{\ln 2} = t$$

$$t = -\frac{5730\,yrs \cdot \ln 0.710}{\ln 2} = 2831\,yrs$$

The sample is ≈ 2830 years old.

Atomic number

Nucleus : $^{106}Pd \Rightarrow A = 106$

From periodic table: Pd : $Z = 46$

$A = Z + N \Rightarrow N = A - Z = 106 - 46 = 60$

This nucleus has 60 neutrons.

Un-ionized = neutral $\Rightarrow n_{elec} = Z$

This atom has 46 electrons.

Nuclear radius

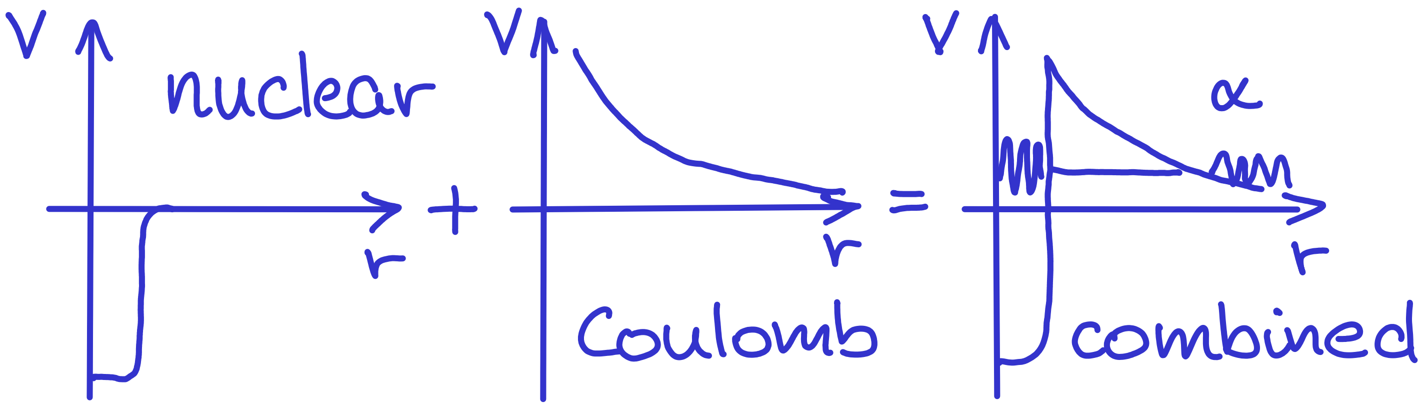
$R = r_0 \cdot A^{1/3}$ $\qquad r_0 = 1.2 - 1.25\ fm$

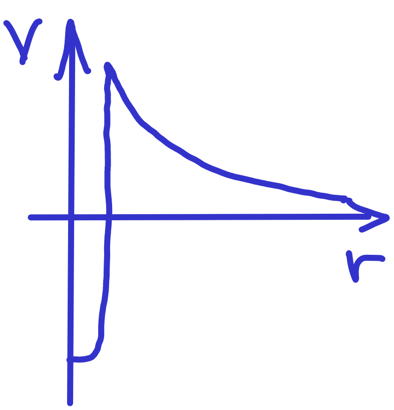
This question uses $r_0 = 1.25\ fm$.

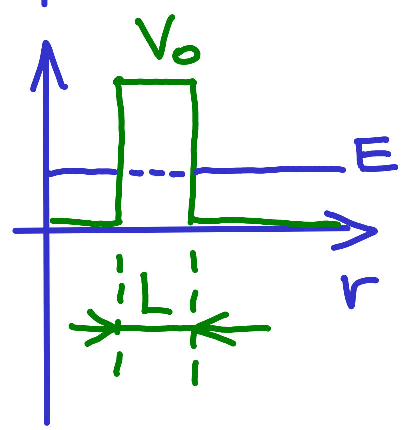
${}^{7}B$: $A = 7$: $R = 1.25 \cdot 7^{1/3} = 2.39\ fm$

${}^{234}U$: $A = 234$: $R = 1.25 \cdot 234^{1/3} = 7.70\ fm$

Alpha particle tunneling







$V_0 = 13.7\,\text{MeV}$

$E = 4.10\,\text{MeV}$

$L = 12.2\,\text{fm}$

$$m_\alpha = 3727.38\ \text{MeV}/c^2$$

$$\hbar c = 197.33\ \text{eV nm}$$

$$T = 16 \cdot \frac{E}{V_0}\left(1 - \frac{E}{V_0}\right) \cdot e^{-2\kappa L}$$

$$\kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar} = \frac{\sqrt{2mc^2(V_0 - E)}}{\hbar c} =$$

$$= \frac{\sqrt{2 \cdot 3727.38 \cdot (13.7 - 4.10)(\text{MeV})^2}}{197.33\,\text{eV nm}} =$$

$$= \frac{267.5\,\text{MeV}}{197.33\,\text{eV nm}} = \frac{267.5\,\text{eV}}{197.33\,\text{eV fm}} =$$

Alpha particle tunneling (2)

$$= 1.356 \frac{1}{fm}$$

$$2kL = 33.08$$

$$e^{-2kL} = 4.306 \cdot 10^{-15}$$

$$16 \cdot \frac{E}{V_o}\left(1 - \frac{E}{V_o}\right) = 16 \cdot \frac{4.10}{13.7}\left(1 - \frac{4.10}{13.7}\right) =$$

$$= 3.355$$

$$T = 1.445 \cdot 10^{-14}$$

Binding energies

$M(^1H) = 1.007825$ AMU

$m_n = 1.008665$ AMU

$1\,\text{AMU} = 931.494061\ \text{MeV}/c^2$

$$B(^A_Z X_N) = \underbrace{\left[Z \cdot M(^1H) + N \cdot m_n - M(^A_Z X_N)\right]}_{\Delta M\,:\ \text{mass defect}} c^2$$

	^{23}Na	^{56}Fe	^{244}Pu
A/Z/N	23/11/12	56/26/30	244/94/150
M (AMU)	22.989770	55.934942	244.064198
ΔM (AMU)	0.200285	0.528458	1.971102
B (MeV)	186.564	492.255	1836.070
$\frac{B}{A}$ (MeV)	8.111	8.790	7.525

Q-value of alpha decay

$$^{235}_{92}U_{143} \longrightarrow {}^{231}_{90}Th_{141} + \alpha$$

$$\alpha = {}^{4}_{2}He_{2}$$

$$M(^{4}He) = 4.002603\ AMU$$

$$M(^{235}U) = 235.043923\ AMU$$

$$M(^{231}Th) = 231.036297\ AMU$$

$$Q = [M(^{235}U) - M(^{231}Th) - M(^{4}He)]c^2$$

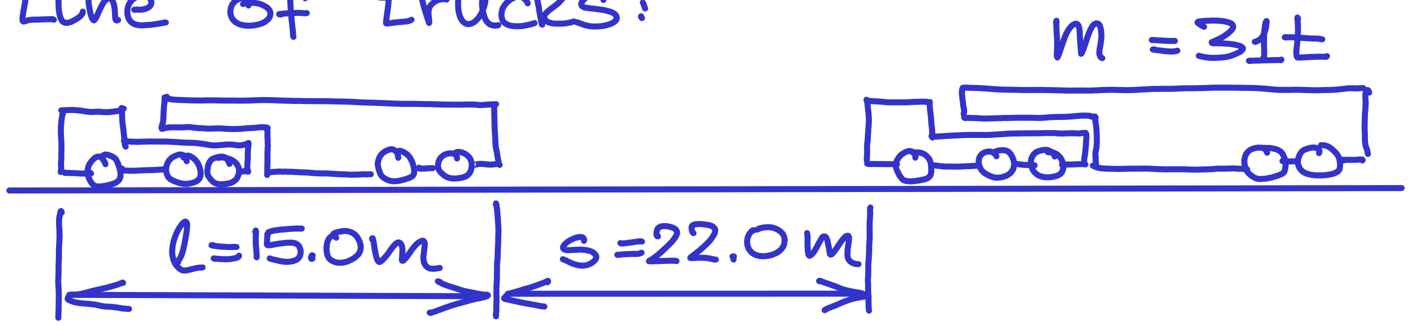
$$Q = 0.005023\ AMU \cdot c^2$$

$$Q = 4.68\ MeV$$

Yield of nuclear bombs

1 kt = 1000 t = 1,000,000 kg of TNT yields $4.2 \cdot 10^{12}$ J of energy.

Line of trucks:



Fission bomb: $M_1 = 50.0\,kt$

$$n_1 = \frac{M_1}{m} = \frac{50{,}000\,t}{31.0\,t} \cong \underbrace{1613}_{\text{trucks needed}}$$

$$L_1 = n_1 \cdot (l+s) = 1613 \cdot 37\,m = 59.7\,km$$

Fusion bomb: $M_2 = 18.0\,Mt$

$$n_2 = \frac{M_2}{m} = \frac{18.0 \cdot 10^6\,t}{31.0\,t} = \underbrace{5.81 \cdot 10^5}_{\text{trucks needed}}$$

$$L_2 = n_2 (l+s) \cong 21{,}500\,km$$

The Equator is 40,000 km long.

Mass of fuel rods

$E_{1fission} = 185\,MeV = 2.96 \cdot 10^{-11} J$: energy generated by one fission.

$P_e = 410\,MW$: electric power

$\eta = 39\% = 0.39$: thermodynamic efficiency of the steam turbine

$P_t = \frac{P_e}{\eta} = 1051\,MW$: total power with heat included

$E_{1year} = P_t \cdot t = 1051 \cdot 10^6 \cdot 365 \cdot 24 \cdot 3600 =$
$= 3.32 \cdot 10^{16} J$: total energy produced by fission in one year.

$N \text{ of } {}^{235}U = \frac{E_{1year}}{E_{1fission}} = \frac{3.32 \cdot 10^{16} J}{2.96 \cdot 10^{-11} J} = 1.12 \cdot 10^{27}$: number of ${}^{235}U$ fissions in one year.

Enrichment : $5.5\% = 0.055$

$N_{total} = \frac{N \text{ of } {}^{235}U}{0.055} = 2.04 \cdot 10^{28}$: number of total Uranium (235+238 mixture) needed in one year.

Mass of fuel rods 2.

$$n_{total} = \frac{N_{total}}{N_A} = \frac{2.04 \cdot 10^{28}}{6 \cdot 10^{23} \frac{1}{mol}} = 3.39 \cdot 10^{4} mol$$

Total Uranium in mol units.

Molar mass of Uranium fuel with enrichment ignored:

$$M = 238 \frac{g}{mol} = 0.238 \frac{kg}{mol}$$

$$m = n \cdot M = 3.39 \cdot 10^{4} mol \cdot 0.238 \frac{kg}{mol} =$$

$\cong 8080\ kg = 8.08\ t$: total mass of enriched Uranium fuel need for one year.

The density of Uranium is $\rho_u = 19.1 \frac{t}{m^3}$, therefore the volume of this fuel is $V = \frac{m}{\rho_u} \cong 0.42\ m^3$ i.e. less than half of a cubic meter.

Weizsäcker formula

Nucleus: ^{58}Fe: $A=58, Z=26, N=32$

Volume energy:

$a_V \cdot A = 14\,MeV \cdot 58 = 812\,MeV$

Surface energy: (negative)

$a_A \cdot A^{2/3} = 13\,MeV \cdot 58^{2/3} = 194.8\,MeV$

Coulomb energy: (negative)

$a_c \cdot \frac{Z(Z-1)}{A^{1/3}} = 0.72\,MeV \cdot \frac{26 \cdot 25}{58^{1/3}} = 120.9\,MeV$

Symmetry: (negative)

$a_s \cdot \frac{(N-Z)^2}{A} = 19\,MeV \cdot \frac{(32-26)^2}{58} = 11.79\,MeV$

Pairing: (positive b/c even-even)

$\delta = +\Delta = 33\,MeV \cdot A^{-3/4} = 1.57\,MeV$

All combined together:

$812 - 194.8 - 120.9 - 11.79 + 1.57 = 486.1\,MeV$

From table: $M(^{58}Fe) = 57.933280\,AMU$

$\Delta M = 0.547450\,AMU = 509.95\,MeV$