General Relativity (1915)

(The Physics of Gravity)

Inertial mass: the mass in Newton's second law: $\vec{a} = \vec{F}/m$.

Gravitational mass: the mass in Newton's law of gravitation:

$$F = G\frac{m \cdot M}{d^2} = mg$$

Gravitational Equivalence Principle:

the inertial mass and the gravitational mass are proportional to each-other. (With a proper choice of units, they can be made equal. In the metric system they are equal.) In other words: acceleration in one direction is equivalent to gravitation in the opposite direction.

This principle states that all objects, regardless of their mass or composition, fall at the same rate in a gravitational field when no other forces (like air resistance) act on them. In other words, the trajectory of a freely falling object is independent of its mass and composition.

The large scale implications of the General Theory of Relativity

-- Space and time are really part of a singular structure, named "Spacetime", providing a 4-dimensional framework in which matter and energy can move.

-- General relativity is a theory of spacetime. The "force of gravity" is really due to the fact that mass and energy cause space and time to curve ("bend" or "wrap"). Other bits of matter, or light, that travel in spacetime follow the curvature, and from our perspective in 3-dimensions appear to accelerate as a result.

-- Space and time tell energy and matter how to move; energy and matter tell space and time how to bend.

GPS and Einstein's General Relativity

(1) The Global Positioning System (GPS) and general relativity are closely linked.

How GPS Works:

GPS relies on a network of satellites that orbit the Earth and transmit signals to receivers on the ground. These signals contain the time the message was sent and the satellite’s position. The receiver calculates its position by determining how long it took for the signals to arrive from multiple satellites.

Role of Special relativity

Einstein's theory of special relativity states that moving clocks run slower than stationary clocks.

Role of General Relativity:

Einstein's theory of general relativity explains how gravity can affect time. According to the theory, time passes more slowly in stronger gravitational fields. Since the Earth’s gravitational field is stronger closer to its surface than in the higher altitudes where GPS satellites orbit, the clocks on GPS satellites tick slightly faster than those on the ground.

Corrections

-- GPS engineers incorporate corrections for these effects into the GPS system's algorithms.

-- The satellite clocks are slowed down by about 38 microseconds per day to match the speed of clocks on Earth.

-- Without accounting for general relativity, GPS calculations would become inaccurate, leading to errors of several kilometers in just a day.

-- These corrections allow GPS to deliver position data with a 1-meter level of accuracy and time data with a 100-nanosecond level of accuracy.

Practical confirmation

The GPS system's use of Einstein's theory of relativity is one of the first practical confirmations that the theory applies to our daily lives.

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(2) Applying the Equivalence Principle of General Relativity:

Since there is no difference between gravitational acceleration and the act of changing a whole reference frame into a non-inertial reference frame, we can analyze phenomena in a situation where an inertial frame is considered instantaneously inertial -- although it experiences overall accelerations, at any moment in time all elements of the frame have the same velocity.

Using this imaginary of instantaneously inertial frames of reference, the relativistic Doppler shift of light by a gravitational field yield the relation between the two different time durations

$$\Delta t_{higher} \sim (1- g h/C^2) \Delta t_{lower}$$

$$\Delta t_{higher} \simeq \left(1 - \frac{gh}{c^2}\right) \Delta t_{lower}$$

where a "lower" observer is at located at the surface of the Earth and a "higher" observer is at a height of h above the surface. g is the gravitational acceleration, which is 9.8 m/sec^2 on Earth.

Hence, the clock at satellite (the "higher" observer) clicks faster than the clock on Earth by (g h/C^2).