Time dilatation or dilation

Moving clocks run slower.

$$T = \gamma T_0 = \frac{T_0}{\sqrt{1-\beta^2}} = \frac{T_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

T_0 : proper time : time measured in the resting frame

T : time measured in the moving frame

Lorentz contraction

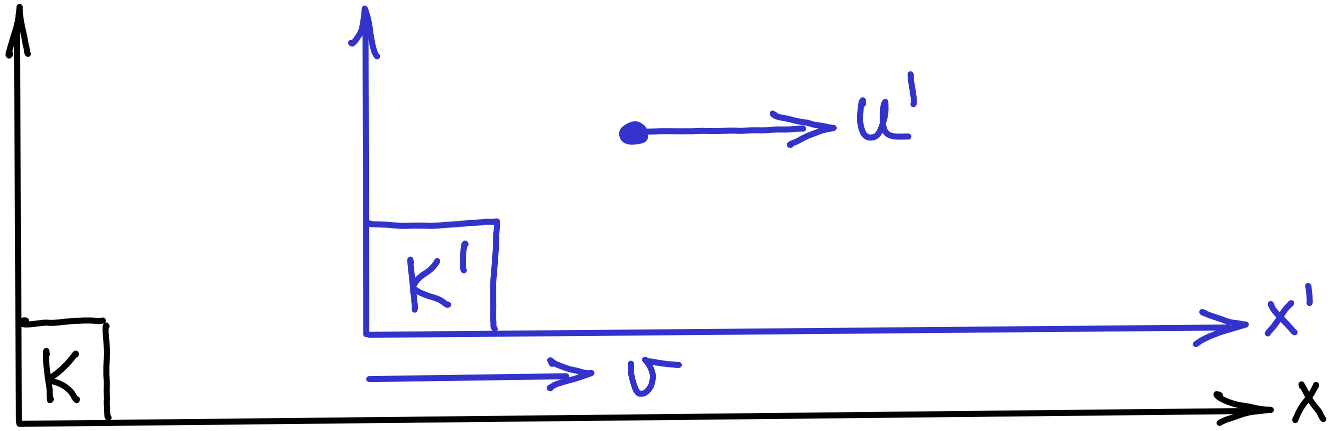
or length contraction

$$L = \frac{L_0}{\gamma} = L_0 \cdot \sqrt{1-\beta^2} = L_0 \cdot \sqrt{1-\frac{v^2}{c^2}}$$

L_0 : proper length

L : length measured in the moving frame

Velocity addition



$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}} \Rightarrow \beta = \frac{B + \beta'}{1 + B\beta'}$$

Special cases:

- If $u' \ll c$ and $v \ll c$, then $u = v + u'$ (Galilean velocity addition)

- If $u' = c$ and $v \ll c$, then:

$$u = \frac{v + c}{1 + \frac{vc}{c^2}} = \frac{v + c}{1 + \frac{v}{c}} = \frac{v + c}{\frac{c + v}{c}} = c$$

- If $u' = c$ and $v = c$, then:

$$u = \frac{c + c}{1 + \frac{c^2}{c^2}} = \frac{2c}{1 + 1} = \frac{2c}{2} = c$$

Distances, geometry

Eucledian distance:

$$(\Delta r)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$

Spacetime interval:

$$(\Delta s)^2 = \underbrace{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2} - (c\Delta t)^2$$

$$(\Delta s)^2 = (\Delta r)^2 - (c\Delta t)^2$$

The spacetime interval is invariant under the Lorentz transformation:

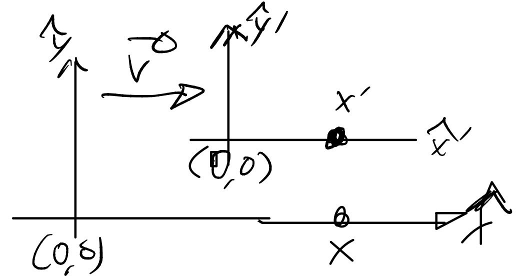
$$(\Delta s)^2 = (\Delta s')^2$$

Three vector: $\vec{r} = (x, y, z)$

Four vector: (x, y, z, ict)

(i: imaginary unit : $i = \sqrt{-1}$)

$$\Delta x' = \gamma(\Delta x - v\Delta t)$$
$$\Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)$$
$$\left(\begin{array}{l}\Delta y' = \Delta y \\ \Delta z' = \Delta z\end{array}\right.$$



$$(\Delta x')^2 - (c\Delta t')^2 = (\Delta x)^2 - (c\Delta t)^2$$

$$(\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - (c\Delta t')^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2 \equiv (\Delta s)^2$$

$$(\Delta x')^2 = \gamma^2\left[(\Delta x)^2 + v^2(\Delta t)^2 - 2v\,\Delta x\,\Delta t\right]$$
$$(c\Delta t')^2 = \gamma^2\left[(c\Delta t)^2 + \frac{v^2}{c^2}(\Delta x)^2 - 2v(\Delta t)(\Delta x)\right]$$
$$\Rightarrow (\Delta x')^2 - (c\Delta t')^2 = \underbrace{\gamma^2\left(1 - \frac{v^2}{c^2}\right)}_{=1}\left[(\Delta x)^2 - (c\Delta t)^2\right]$$

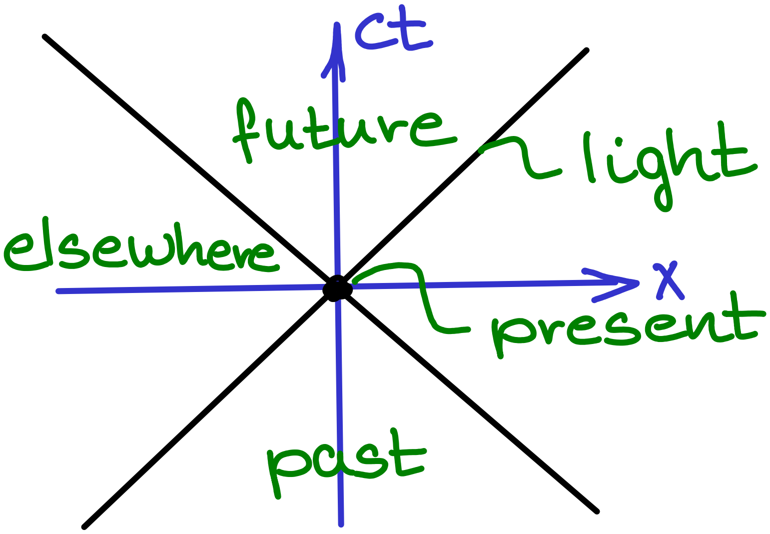
for $\boxed{\gamma^2 = \dfrac{1}{1 - \frac{v^2}{c^2}}}$

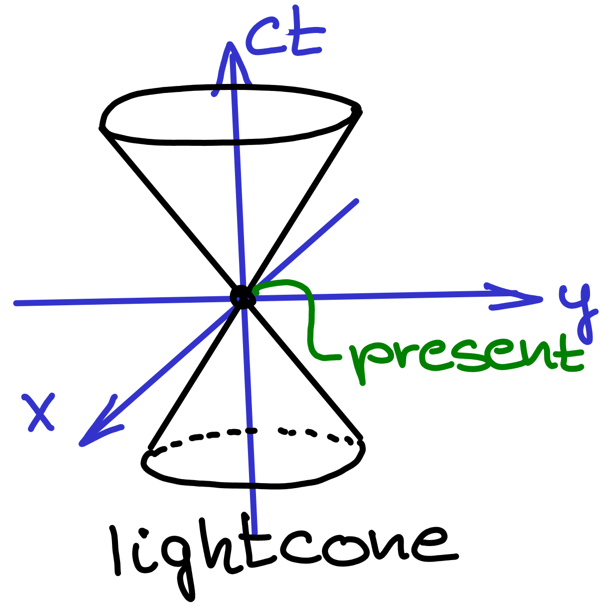
When two events are connected by light signal,

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2$$
$$= 0,$$

$c = 3\times10^8$ m/sec = the speed of light

Spacetime, Minkowski diagram





Spacetime interval:

$$(\Delta s)^2 = (\Delta r)^2 - (c\Delta t)^2$$

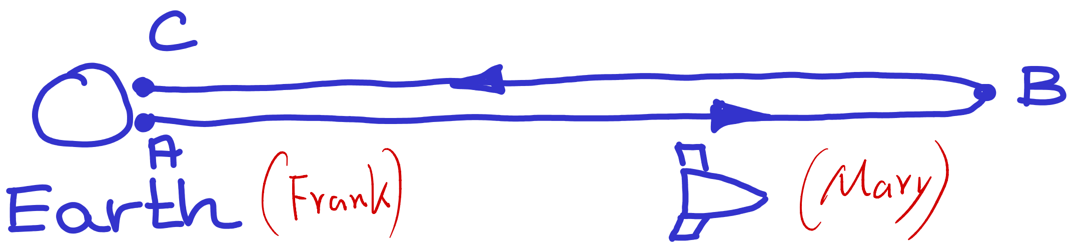
or $$(\Delta s)^2 = (\Delta x)^2 - (c\Delta t)^2$$

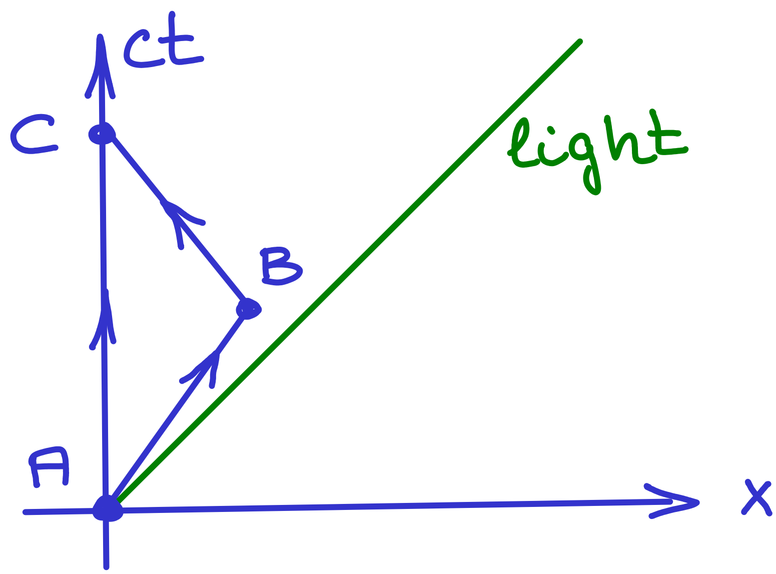
$(\Delta s)^2 = 0$: lightlike

$(\Delta s)^2 > 0$: spacelike

$(\Delta s)^2 < 0$: timelike

Twin paradox





Special relativity: there is no absolute motion, only relative motion. But then which clock will slow down?