# Twin Paradox

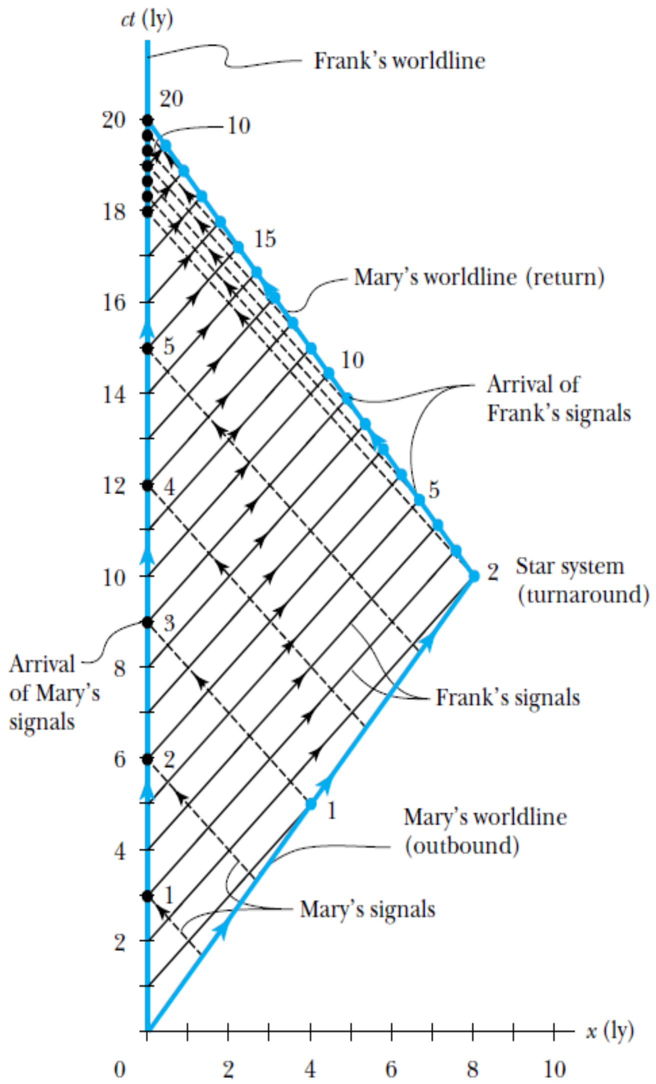



**Figure 2.26** The spacetime diagram for Mary's trip to the star system and back. Notice that Frank's worldline is a vertical line at $x = 0$, and Mary's two worldlines have the correct slope given by the magnitude $c/v$. The black dashed lines represent light signals sent at annual intervals from Mary to Frank. Frank's annual signals to Mary are solid black. The solid dots denote the time when the light signals arrive.

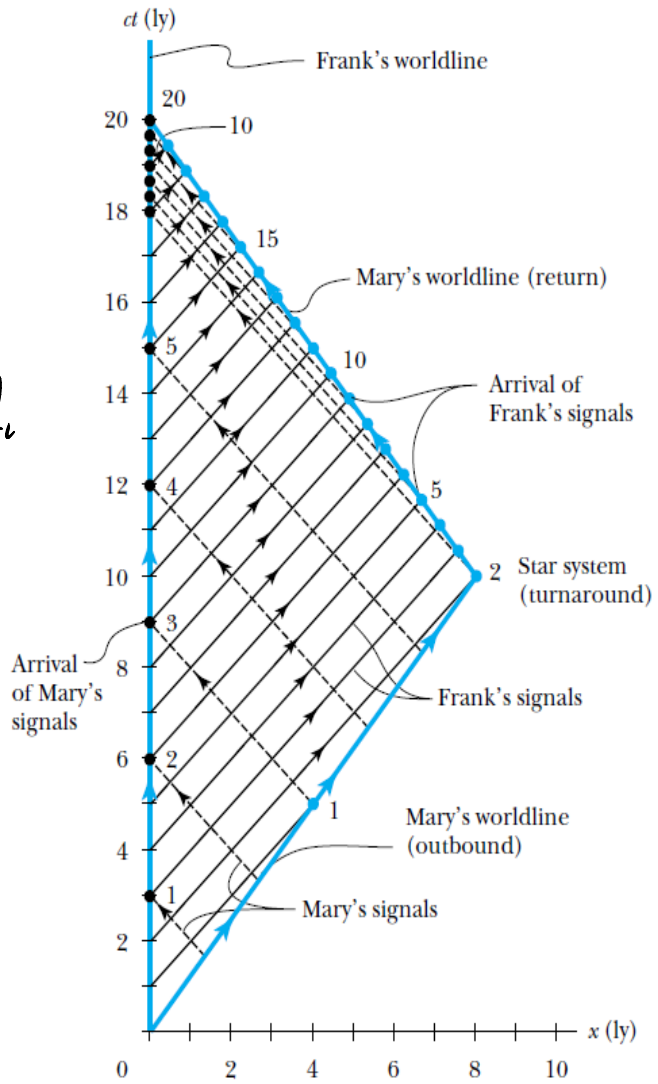
# Twin Paradox

Mary's spaceship travels with the speed of 0.8 C, for the distance of 8 ly to the star, and return to the earth at the same speed.

$\beta = \frac{v}{c} = 0.8\,c = \frac{4}{5}$

$\sqrt{1-\beta^2} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \frac{3}{5}$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{5}{3}$

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$\frac{8\ ly}{0.8\ c} = 10$ years (for Frank)

Time dilation :

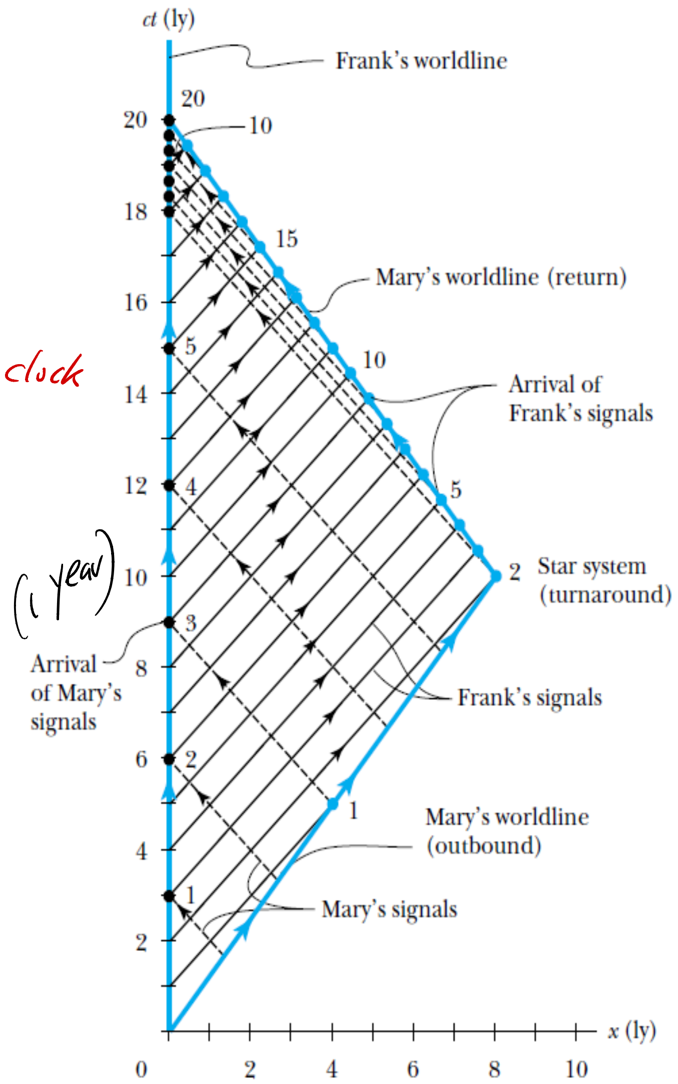
$\frac{10 \text{ years}}{\gamma} = (10)\left(\frac{3}{5}\right) = 6$ years (for Mary)

# Twin Paradox

Mary's spaceship travels with the speed of $0.8c$, for the distance of 8 ly to the star, and return to the earth at the same speed.

Note: ① Frank's clock is in an inertial system during the entire trip while Mary's clock is NOT.

② $1\,\text{ly} \equiv 1$ light-year $= (\text{speed of light}) \times (\text{one year})$
$= (3 \times 10^8\ \text{m/sec}) \cdot (3.16 \times 10^7\ \text{sec/year}) \cdot (1\ \text{year})$
$=$ distance (m)

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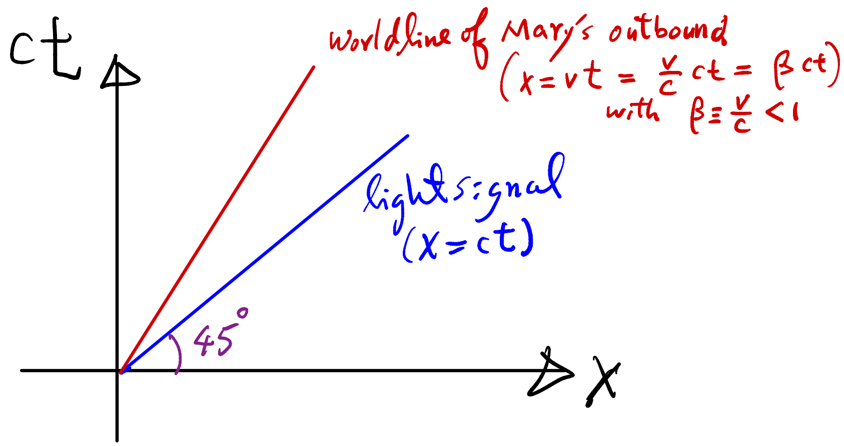
(1) According to Frank:
Mary's travel time to the star is
$(8\,\text{ly})/0.8c = 10\,y$
and the return is also $10\,y$
$\Rightarrow$ Total travel time is $20\,y$.

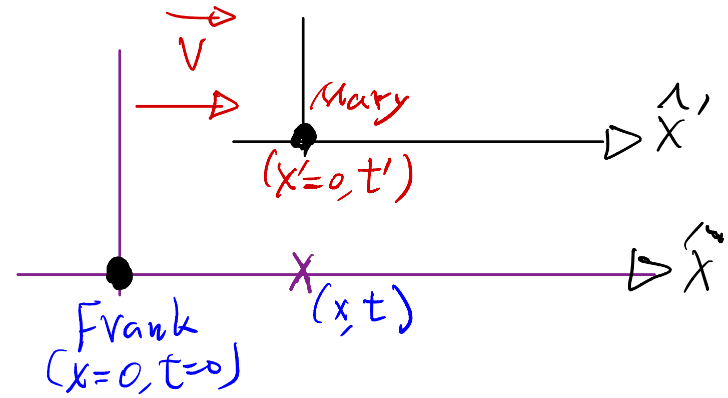
(2) According to Frank, Mary's clock is ticking more slowly.
Her travel time to the star is only
$10\sqrt{1-(0.8)^2}\ y = 6\,y$.
Hence, Frank calculate that Mary will only getting $6+6=12\,y$ older, while Frank is already $20\,y$ older.

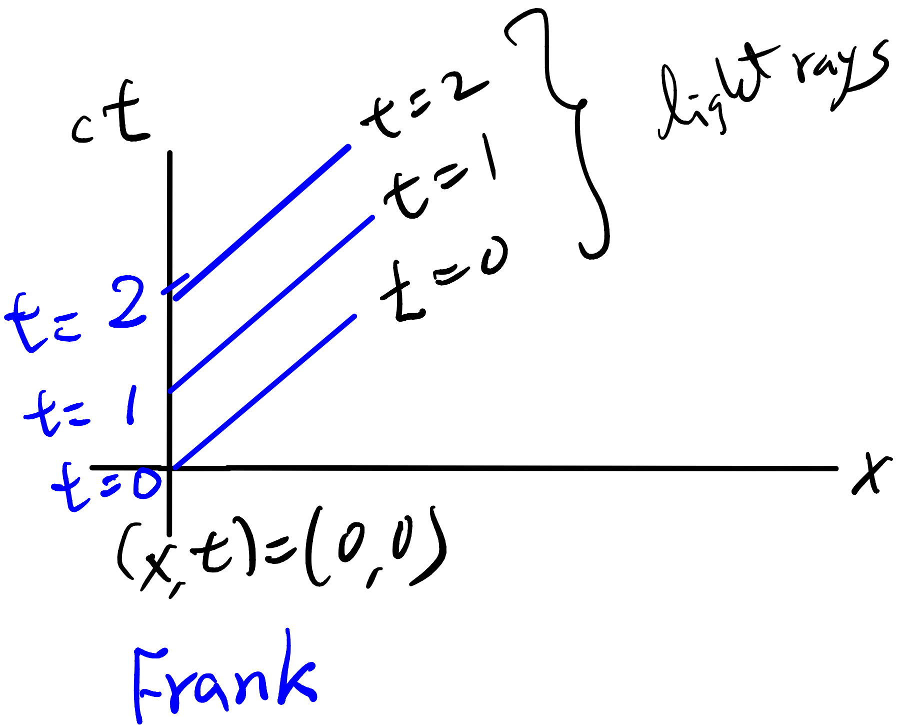
$\Rightarrow$ Mary is younger by
$20-12 = 8\,y$

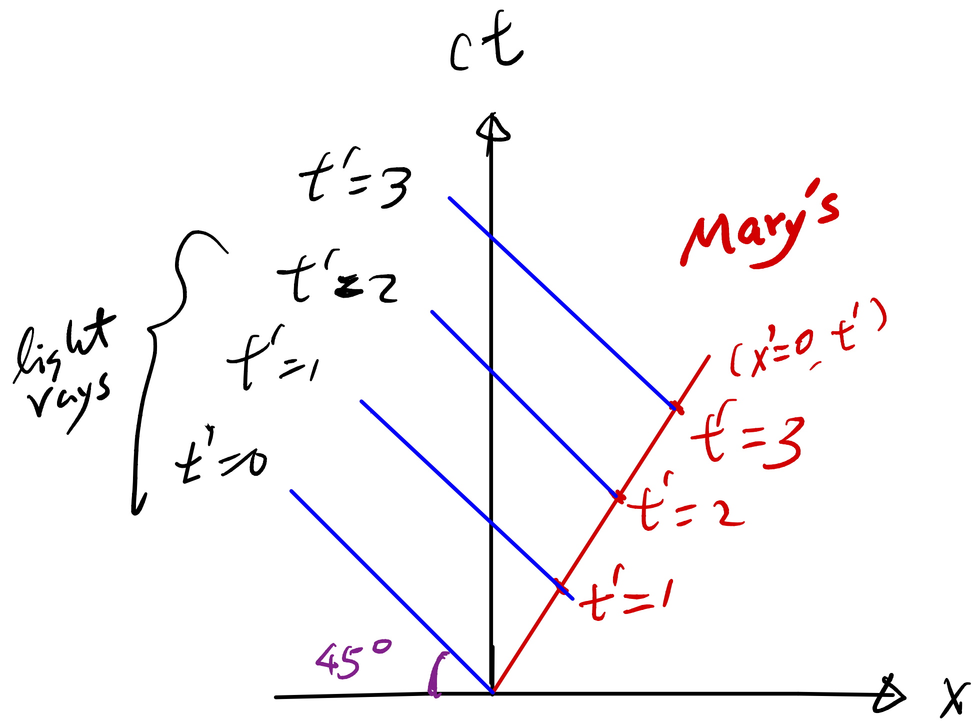
# Spacetime diagrams

According to Frank's observation



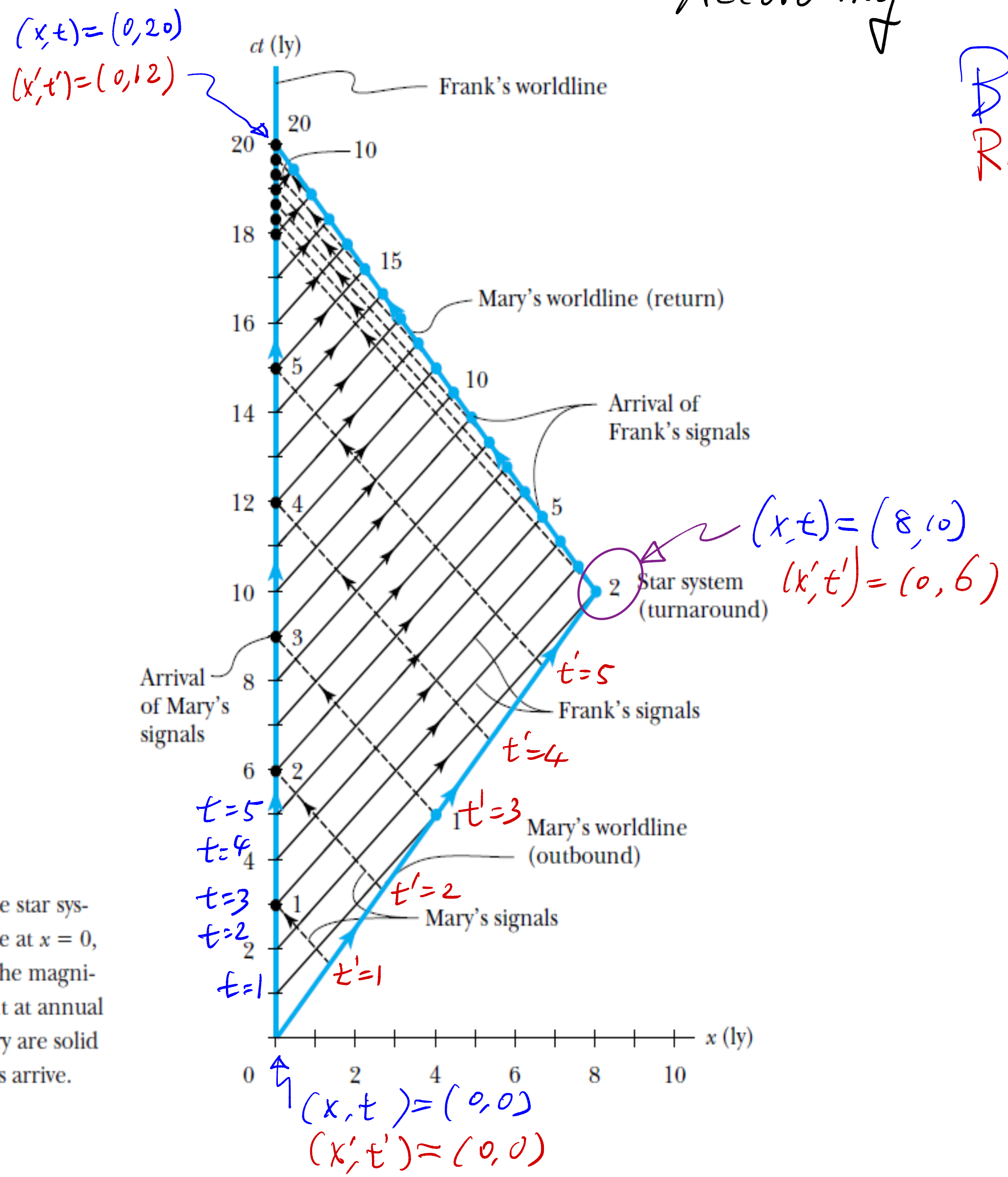



# Twin Paradox

According to Frank's observation

Blue color for Frank

Red Color for Mary

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$$\frac{8\ \text{lys}}{0.8\,c} = 10 \text{ Years}$$ (for Frank)

Time dilation:

$$\frac{10\ \text{Yrs}}{\gamma} = 10 \times \frac{3}{5} = 6 \text{ Years}$$ (for Mary)

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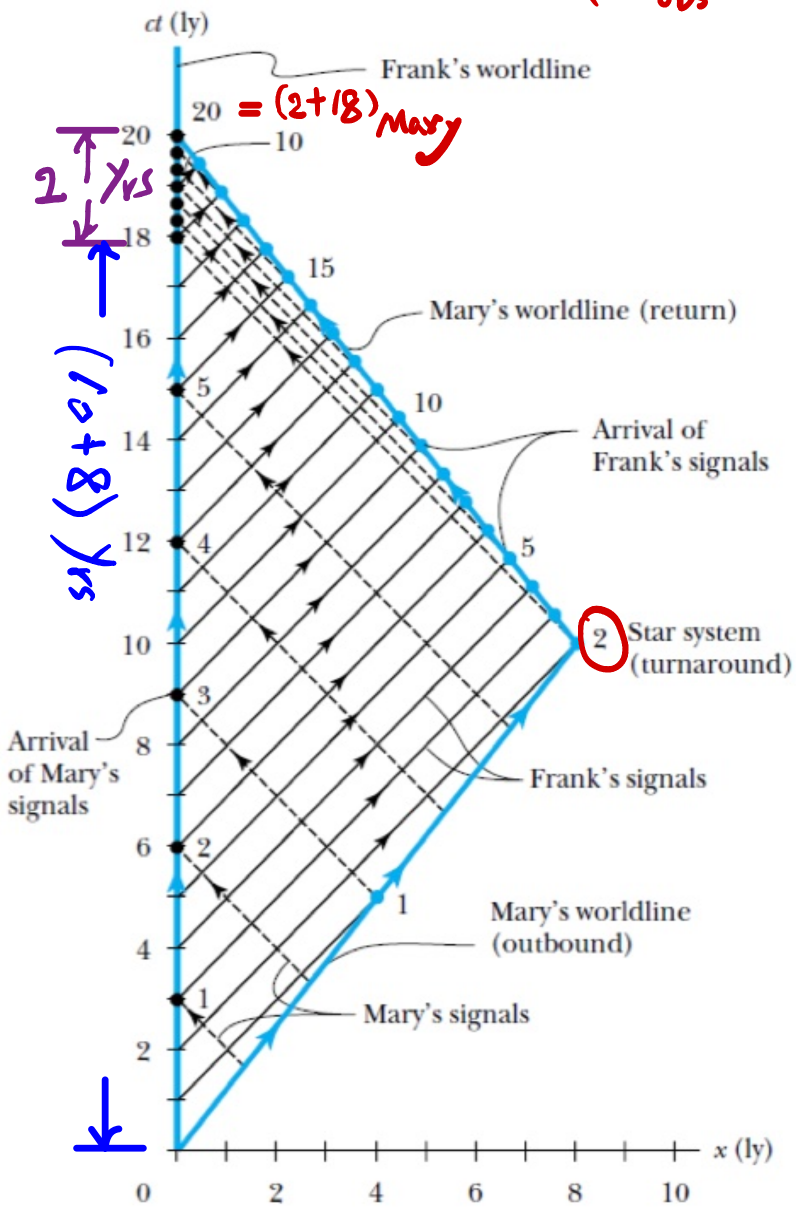


Relativistic Doppler effect

$$\left( f_{obs} = f_{emit}\sqrt{\frac{1\mp\beta}{1\pm\beta}} \right)$$

$V=0.8c=\frac{4}{5}c$, $\beta=\frac{v}{c}=\frac{4}{5}$, $\gamma=\frac{1}{\sqrt{1-\beta^2}}=\frac{5}{3}$

outbound: $\sqrt{\frac{1-\beta}{1+\beta}}=\sqrt{\frac{1-\frac{4}{5}}{1+\frac{4}{5}}}=\sqrt{\frac{1}{9}}=\frac{1}{3}$ (1 flash per 3 years)

Inbound: $\sqrt{\frac{1+\beta}{1-\beta}}=3$ (3 flashes per year)

## According to Frank's viewpoints

(It takes 10 years to reach the turning point, and 8 years for the flash emmited at that point to reach the Earth)

| | Mary's experience | Frank's experience |
|---|---|---|
| Outbound: | $(6\text{ yrs})(\frac{1}{3})=2$ | $(10+8)\text{ yrs}\cdot(\frac{1}{3})=\frac{18}{3}=6$ |
| Inbound: | $(6\text{ yrs})(3)=18$ | $(20-10-8)\text{ yrs}\cdot(3)=6$ |
| | ⇒ Mary sees Frank $2+18=20$ years older | ⇒ Frank sees Mary $6+6=12$ years older |

Hence, Mary is younger than Frank when they meet again after the journey.