Dynamics

Momentum:

Classical: $\vec{p} = m\vec{u}$

Relativistic: $\vec{p} = \dfrac{m\vec{u}}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma m \vec{u}$

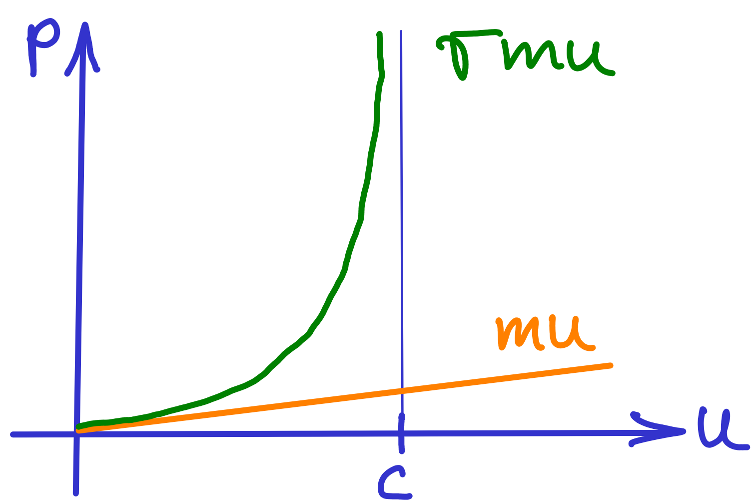
Notation:

- v: speed between the two reference frames K and K'
- Lorentz gamma factor:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

- u: speed, $\vec{u}$: velocity } of the particle
- factor in the momentum equation

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$



Energy

Classical kinetic: $K = \frac{1}{2} m u^2$

Relativistic kinetic: $K = (\gamma - 1) mc^2$

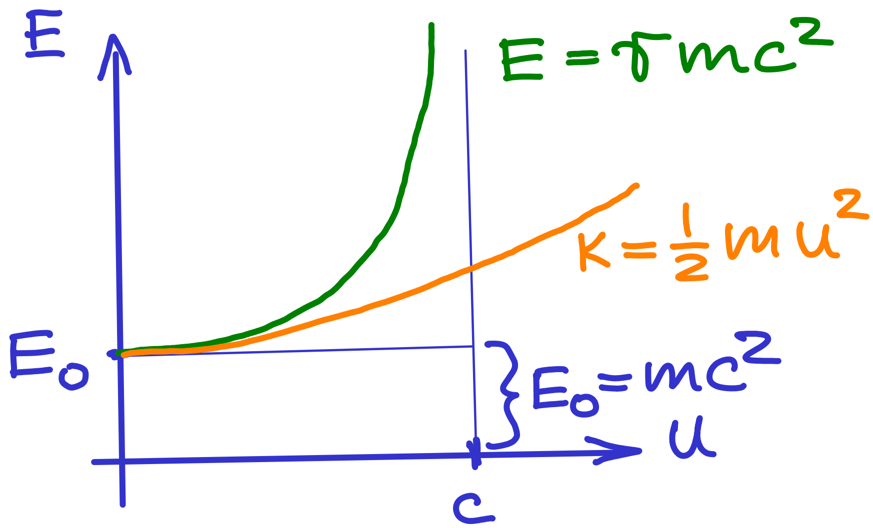
Relativistic total: $E = \gamma mc^2$

$$\underbrace{E}_{\text{total}} = \gamma mc^2 = mc^2 + (\gamma - 1) mc^2 = E_0 + \underbrace{K}_{\text{kinetic}}$$

$\left.\boxed{E_0 = mc^2}\right\}$ rest energy:

$1\,\text{kg} \longleftrightarrow 9 \cdot 10^{16}\,\text{J} = 90\,\text{PJ}$

PJ: petajoule!



Classical kinetic energy:

$$\left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}} \cong 1 + \frac{1}{2}\frac{u^2}{c^2} \quad \text{when } u \ll c$$

$$K = (\gamma - 1) mc^2 = \left(1 + \frac{1}{2}\frac{u^2}{c^2} - 1\right) mc^2 =$$

$= \frac{1}{2} m u^2$: as we learned it before in classical mechanics.

Energy-momentum relations

$$\left.\begin{aligned} \vec{p} &= \gamma m \vec{u} \\ E &= \gamma m c^2 \end{aligned}\right\} \Rightarrow \boxed{E^2 = p^2c^2 + m^2c^4}$$

$$\frac{p}{E} = \frac{\gamma m u}{\gamma m c^2} = \frac{u}{c^2} = \frac{\beta}{c} \Rightarrow \boxed{\frac{p}{E} = \frac{\beta}{c}}$$

Photon: $m = 0$: no rest mass

$$\Rightarrow E = pc \Rightarrow \frac{p}{E} = \frac{1}{c} \Rightarrow \beta = 1$$

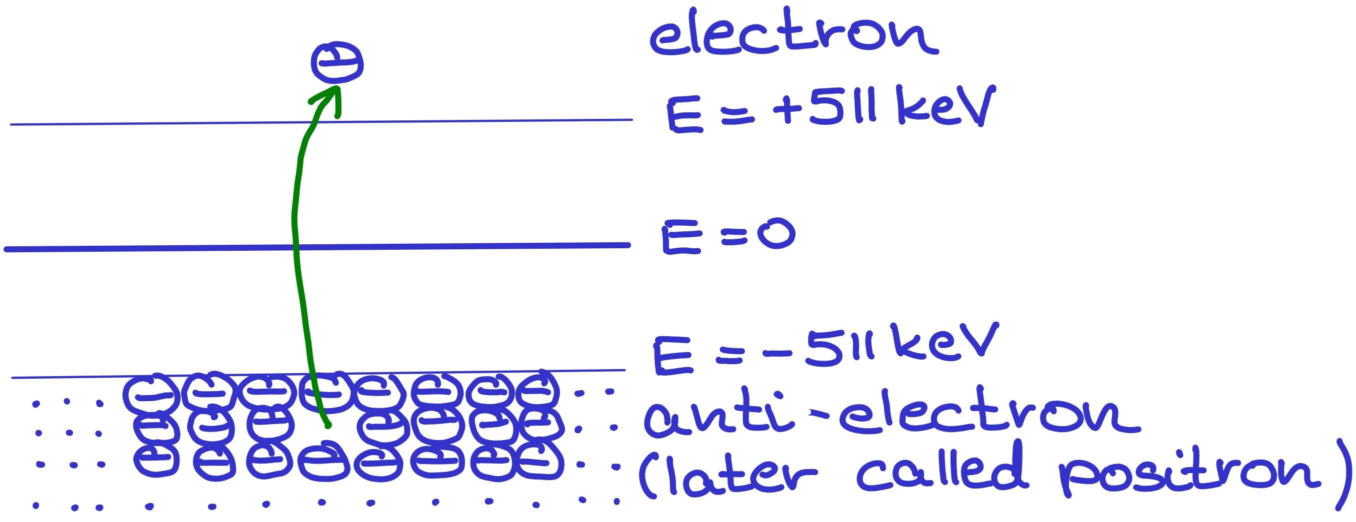
The idea of anti-matter by Dirac:

$$E = \pm\sqrt{p^2c^2 + m^2c^4}$$

$+$: matter

$-$: anti-matter

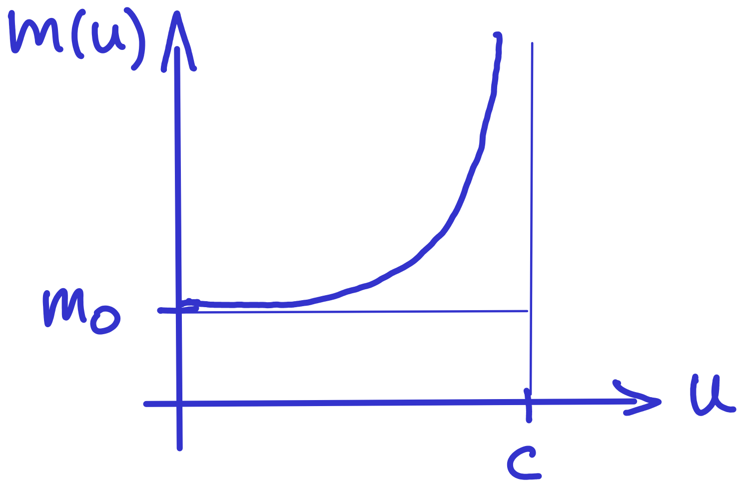
Vacuum by Dirac:



Relativistic mass

m_0 : rest mass

$$m(u) = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \cdot m_0 = \gamma m_0 : \text{ moving mass}$$



Einstein's formula properly:

$$E_0 = m_0 c^2$$

rest mass

rest energy