Empirical gas-laws

Boyle-Mariotte: $p \cdot V = \text{constant}$

at a constant temperature for a fixed amount of gas.

Gay-Lussac / Charles:

#1 : $V \sim T$ at constant p
#2 : $p \sim T$ at constant V
} for a fixed amount of gas

Note: T is absolute temperature measured in kelvins.

All three laws together:

$$\frac{p_1 \cdot V_1}{T_1} = \frac{p_2 \cdot V_2}{T_2}$$

for a fixed amount of gas

This is sometimes called "Unified Gas Law".

Ideal gas law

$$pV = nRT$$
or
$$pV = Nk_BT$$

equation of state

p: pressure in $Pa = N/m^2$
V: volume in m^3
T: temperature in K
n: amount of substance in mol
N: number of atoms/molecules

$R = 8.31 \frac{J}{mol\,K}$: Universal gas constant (Regnault constant)

$k_B = 1.38 \cdot 10^{-23} \frac{J}{K}$: Boltzmann's constant

$$nR = Nk_B \Rightarrow R = N_A \cdot k_B$$

$$8.31 \frac{J}{mol \cdot K} = 6.02 \cdot 10^{23} \frac{1}{mol} \cdot 1.38 \cdot 10^{-23} \frac{J}{K}$$

STP: Standard Temperature and Pressure: $T = 0°C = 273K$

$p = 1\,atm = 101.3\,kPa$

at STP the volume of

one mole of any idea gas is 22.4 L

$$V = \frac{nRT}{P} = \frac{1\,mole \times 0.0821\,L \cdot atm/(mol \cdot K) \times 273\,K}{1\,atm} = 22.4\,L$$

Moles, masses, numbers

m : mass of one atom/molecule (kg)

M : molar mass : mass of one mole substance (g/mol → kg/mol)

n : amount of substance (mol)

N : number of particles (1)

$N_A = 6.02 \cdot 10^{23} \frac{1}{mol}$: Avogadro's constant

$$\left.\begin{aligned} pV &= nRT \\ pV &= Nk_BT \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} Nk_B &= nR \\ N_A k_B &= R \end{aligned}}$$

$R = 8.31 \frac{J}{molK}$

$k_B = 1.38 \cdot 10^{-23} \frac{J}{K}$

$$\boxed{M \cdot n = m \cdot N} \text{ } \} \text{ mass of the gas}$$

$$\frac{M}{m} = \frac{N}{n} = \frac{R}{k_B} = N_A$$

$$\frac{M}{m} = \frac{R}{k_B} \Rightarrow \frac{k_B}{m} = \frac{R}{M}$$

$$v_{RMS} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{M}}$$