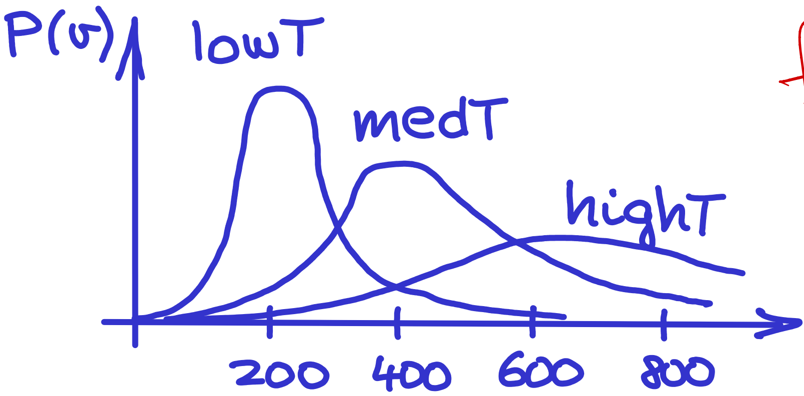
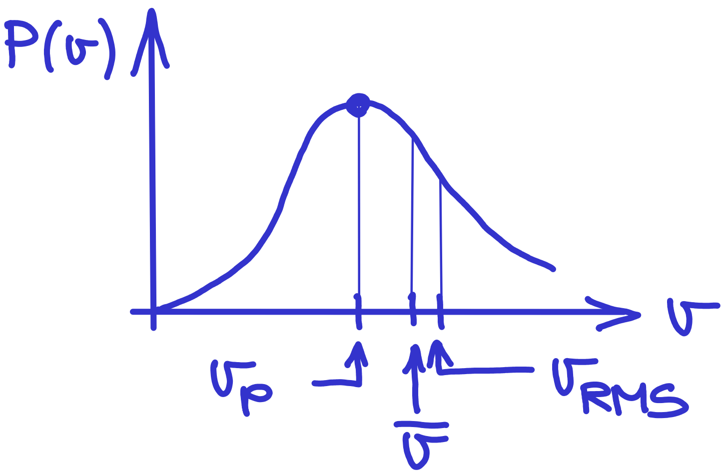
Maxwell-Boltzmann distribution of molecular speeds



$$\underbrace{f(v)\,dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}\,dv}_{\text{distribution function}}$$



v_p : most probable

$\bar{v}$: average

v_{RMS} : Root-Mean-Square

$$v_p = \sqrt{2}\sqrt{\frac{RT}{M}} = \sqrt{2}\sqrt{\frac{k_B T}{m}} \qquad \sqrt{2} = 1.4142$$

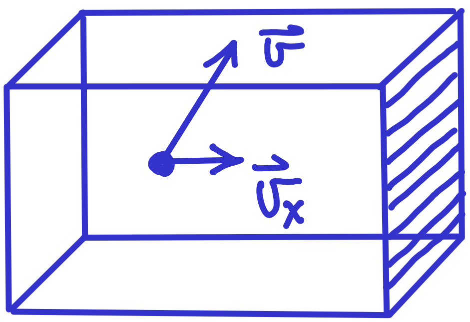
$$\bar{v} = \sqrt{\frac{8}{\pi}}\sqrt{\frac{RT}{M}} = \sqrt{\frac{8}{\pi}}\sqrt{\frac{k_B T}{m}} \qquad \sqrt{\frac{8}{\pi}} = 1.5958$$

$$v_{RMS} = \sqrt{3}\sqrt{\frac{RT}{M}} = \sqrt{3}\sqrt{\frac{k_B T}{m}} \qquad \sqrt{3} = 1.7320$$

M : molar mass in kg/mol

m : mass of a single atom or molecule in kg

Pressure: the micro-macro connection



The gas pressure is the force per unit area resulting from the collisions between the wall and the atoms/molecules.

$$\boxed{p = \frac{2}{3}\underbrace{\left(\frac{N}{V}\right)}_{\text{number density}} \cdot \underbrace{\left(\frac{1}{2} m \overline{v^2}\right)}_{\text{average translational kinetic energy of one molecule}}}$$

Note: elastic collisions do exist in the microscopic world.

$$\left.\begin{aligned} pV &= \frac{2}{3} N \left(\frac{1}{2} m \overline{v^2}\right) \\ pV &= N k_B T \end{aligned}\right\} \quad \frac{2}{3} N \left(\frac{1}{2} m \overline{v^2}\right) = N k_B T$$

$$\boxed{\frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T}$$

The temperature of a gas is a direct measure of the average molecular kinetic energy.

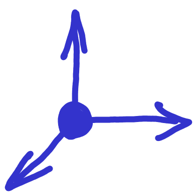
$v_{RMS} = \sqrt{\overline{v^2}}$: Root Mean Square

$$v_{RMS} = \sqrt{\frac{3 k_B T}{m}}$$

Degrees of freedom, internal energy

monoatomic gases:
He, Ne, Ar
Kr, Xe, Rn

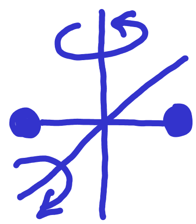
$f = 3$



3 trans-lations

linear molecules
H_2, N_2, O_2, CO_2

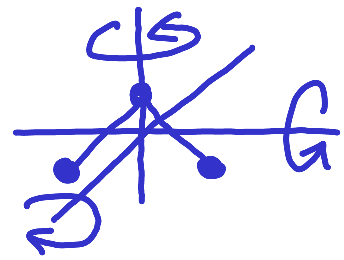
$f = 5$



3 trans. + 2 rotations

polyatomic molecules
NH_3, CH_4
H_2O

$f = 6$



3 trans. + 3 rotations

<u>Equipartition theorem</u>: the internal energy of a system is <u>equally</u> divided among <u>all</u> degrees of freedom. Every degree of freedom stores the same $\frac{1}{2} N k_B T = \frac{1}{2} nRT$ energy. The total internal energy of a system is then:

$$\boxed{\begin{aligned} E_{th} &= U = \frac{f}{2} N k_B T = \frac{f}{2} nRT \\ E_{th} &= U = N \cdot \epsilon \quad \text{where } \epsilon = \frac{f}{2} k_B T \end{aligned}}$$

ϵ : energy of one particle (atom or molecule)