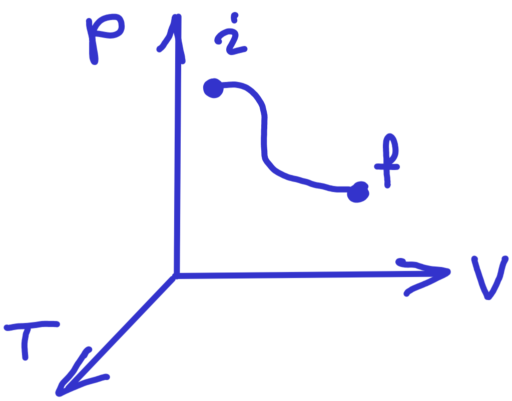
Ideal gas processes

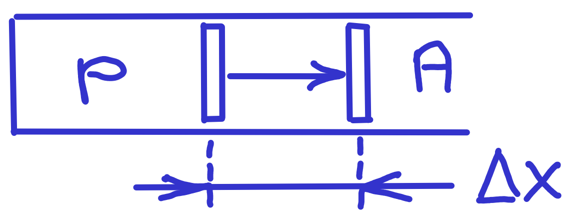


$pV = nRT$

●—● : reversible

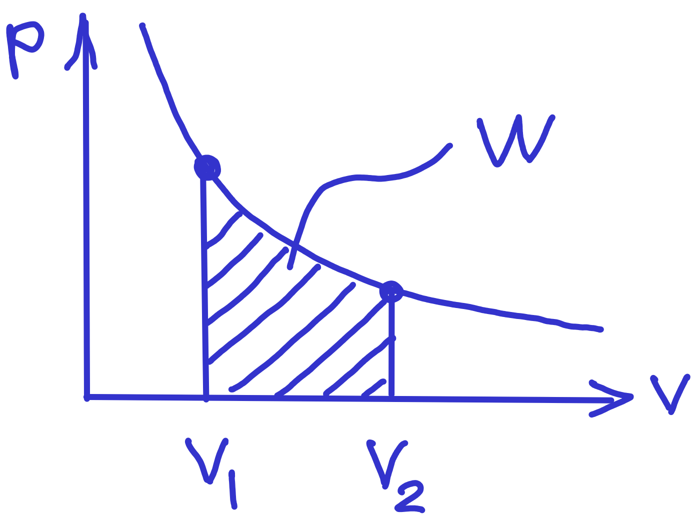
●- - -● : irreversible

Why is the p-V plane special?



Work done by the gas:

$$W = F \cdot \Delta x = pA\Delta x = p \cdot \Delta V$$



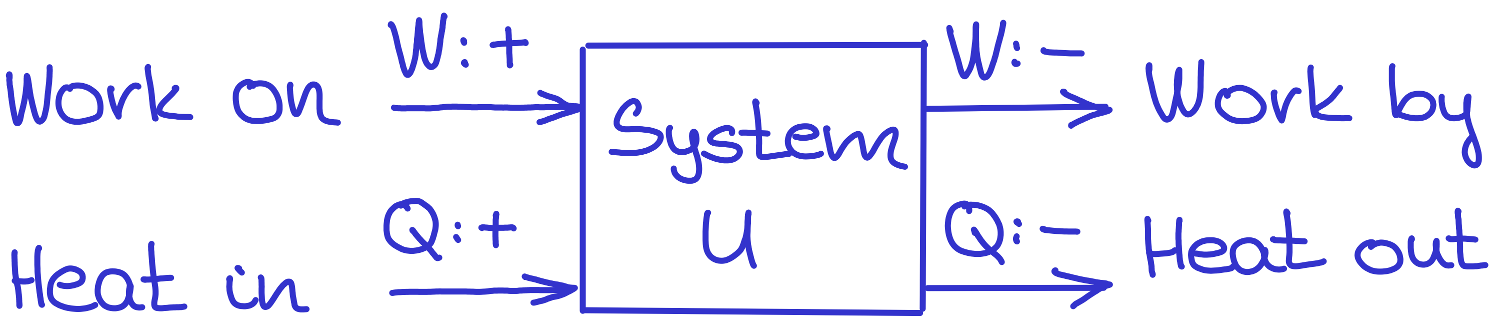
The work done by the gas is the area under the p-V curve.

First Law of thermodynamics

$$\boxed{\Delta U = Q + W}$$

Work (ordered) and heat (disordered) are two ways of transfering energy between the environment and the system. U: internal energy due to moving atoms and molecules.

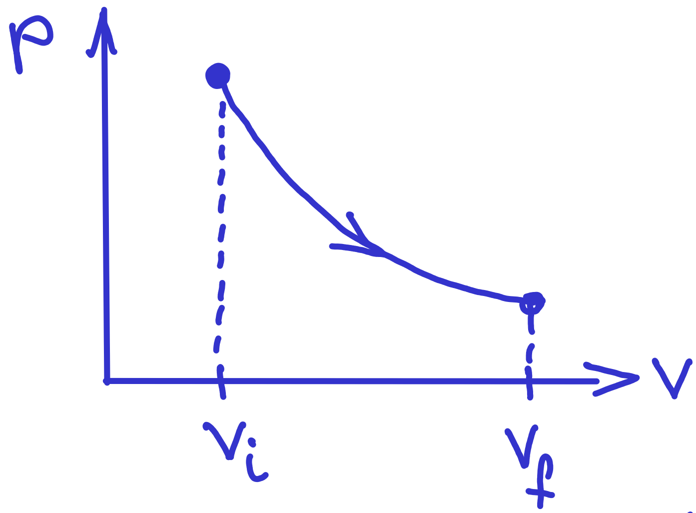
Sign convention:



Important: the First Law doesn't tell anything about the value of U, only about ΔU i.e. the change of U.

Isothermal process

Isothermal: constant temperature



$$pV = nRT$$

$$p = nRT \cdot \frac{1}{V}$$

$\Delta U = 0$ because the temperature is constant:

$$U = \frac{f}{2} nRT$$

$$W = -nRT \ln\left(\frac{V_f}{V_i}\right)$$

(calculus : $\int \frac{1}{x} dx = \ln x$)

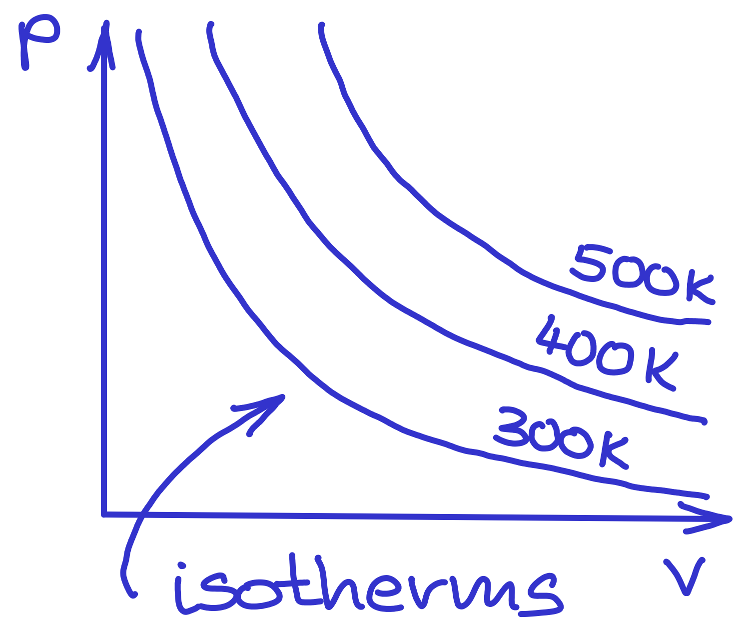
$$\Delta U = Q + W \Rightarrow \boxed{0 = Q + W}$$

$$Q = -W = nRT \ln(V_f / V_i)$$

$$W_{gas} = -W$$

$\Delta U = 0$

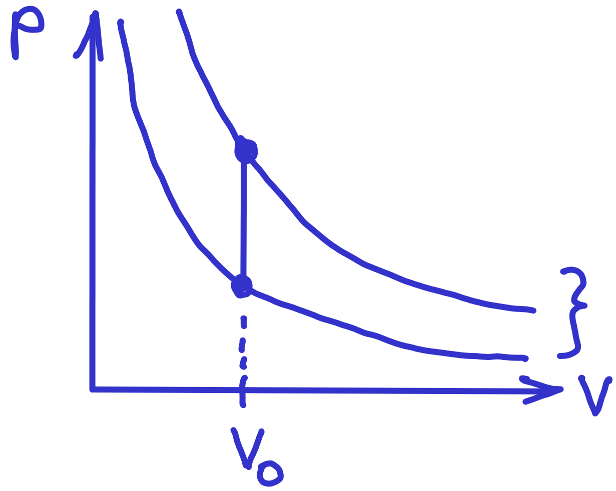
$$Q \xrightarrow{100\%} W_{gas}$$



Isochoric process

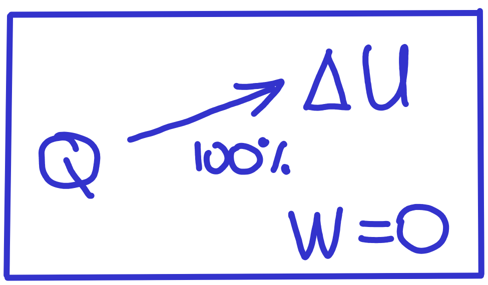
Isochoric : constant volume

Newer names: isovolumetric, isometric



$W = 0$ because $\Delta V = 0$
(V_0 is constant)

$\Delta U = Q + W \implies \boxed{\Delta U = Q}$



$Q = n C_v \cdot \Delta T$

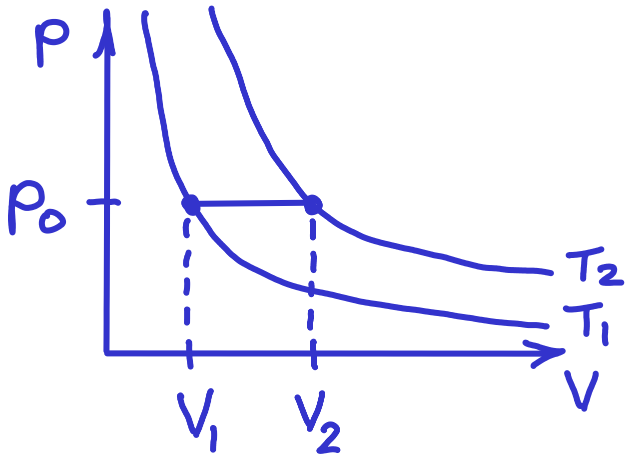
$\Delta U = n C_v \cdot \Delta T \qquad \Delta U = \frac{f}{2} n R \cdot \Delta T$

$\boxed{C_v = \frac{f}{2} R} \qquad R = 8.31 \frac{J}{mol K} \; ; \; f = 3, 5, 6$

C_v : constant volume molar heat

Isobaric process

Isobaric : constant pressure



$$W_{gas} = p_0 \cdot \Delta V$$

$$W = -p_0 \cdot \Delta V$$

$$Q = n C_p \cdot \Delta T$$

(definition of C_p)

$$\Delta U = Q + W$$

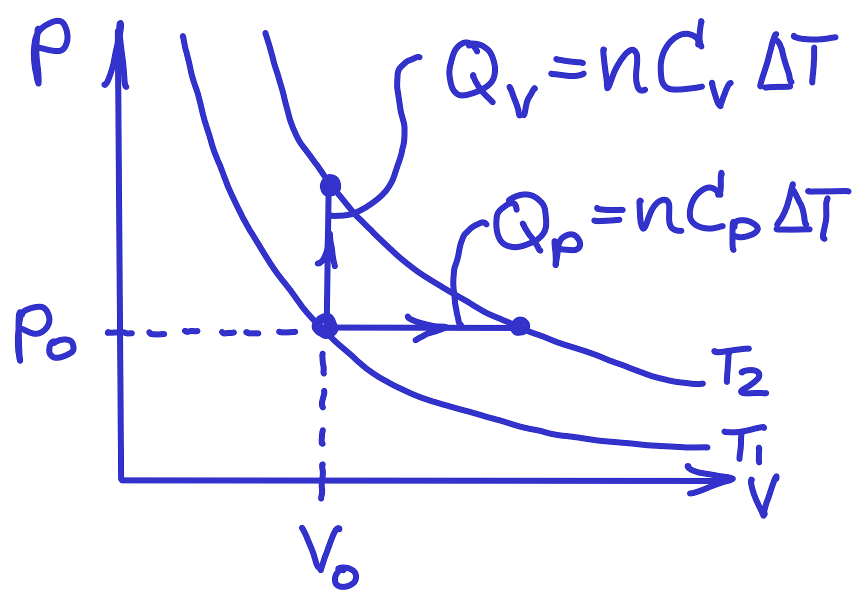
$$\Delta U = n C_p \Delta T - \underbrace{p \Delta V}_{= nR \cdot \Delta T \text{ ideal gas law}} = n C_v \cdot \Delta T$$

$C_p = C_v + R$	$C_p = \frac{f+2}{2} R$

C_p : constant pressure molar heat

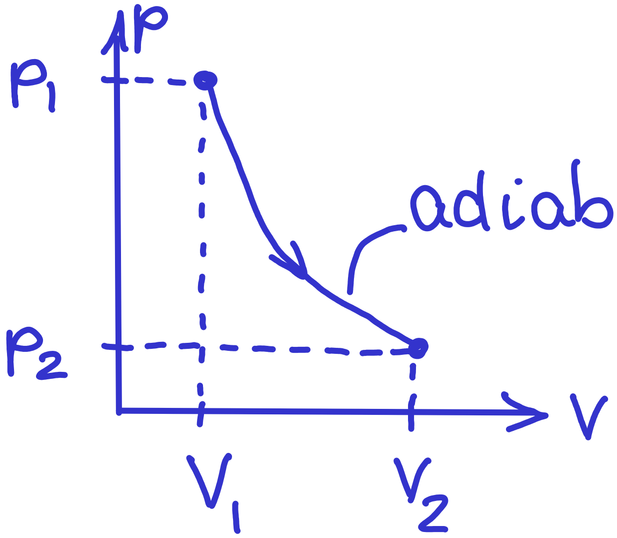
$$Q \xrightarrow{\frac{f}{f+2}} \Delta U$$

$$Q \xrightarrow{\frac{2}{f+2}} W_{gas}$$



Adiabatic process

Adiabatic: no heat is transferred



$$\Delta U = Q + W \Rightarrow \boxed{\Delta U = W}$$

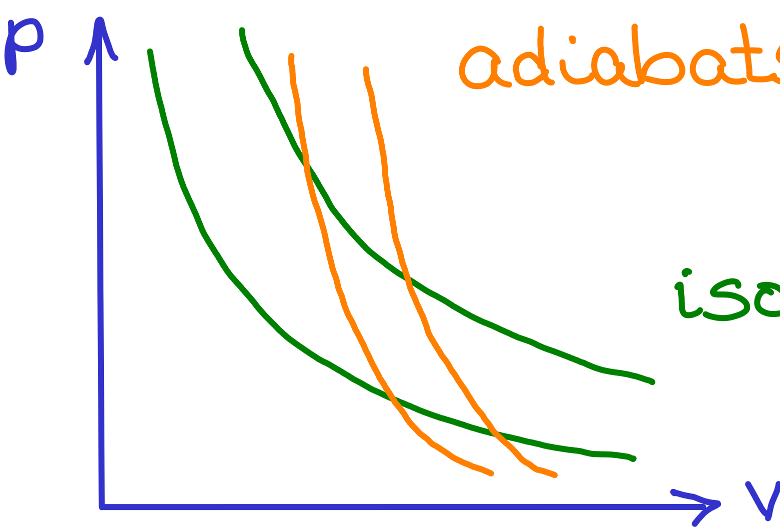
$$Q = 0$$

$$\boxed{Q = 0 \quad \Delta U \quad W}$$

$$p_1 \cdot V_1^{\gamma} = p_2 \cdot V_2^{\gamma} \qquad \gamma = \frac{c_p}{c_v} = \frac{f+2}{f}$$

γ: adiabatic index or specific heat ratio

$$\gamma = \begin{cases} 5/3 = 1.67 & \text{when } f = 3 \\ 7/5 = 1.40 & \text{when } f = 5 \\ 8/6 = 4/3 = 1.33 & \text{when } f = 6 \end{cases}$$

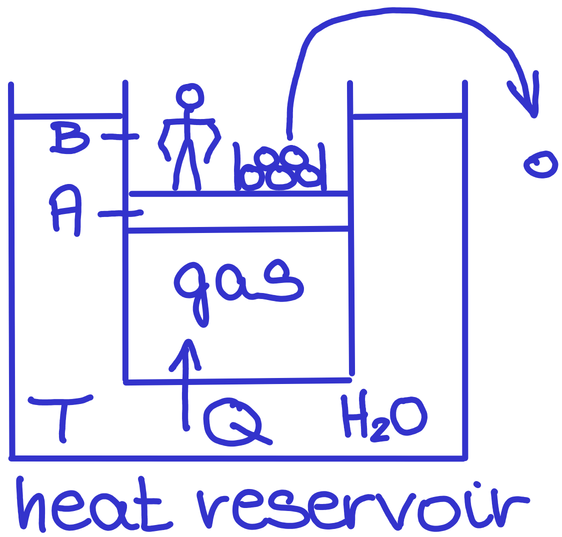


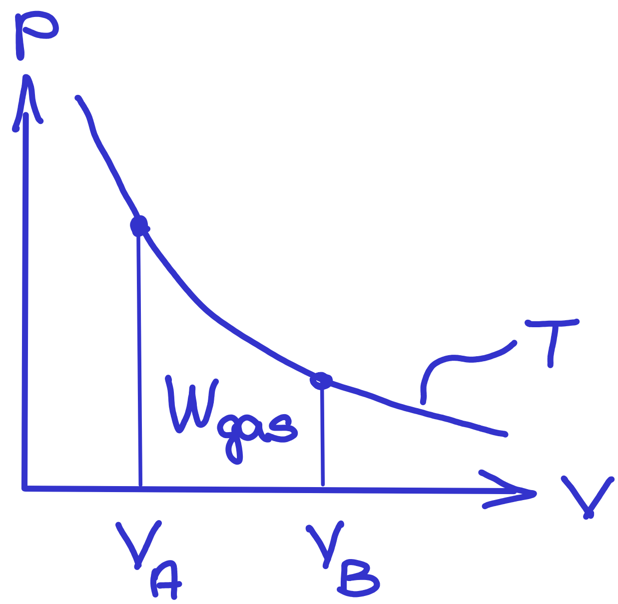
adiabats : $p \sim \frac{1}{V^{\gamma}}$

isotherms : $p \sim \frac{1}{V}$

The adiabats fall faster than the isotherms, because $\gamma > 1$.

Converting heat to work





Isothermal expansion: $T = \text{const.}$

$\Rightarrow E_{th} = U = \text{constant} \Rightarrow \Delta U = 0$

1st law : $\Delta U = Q + W$

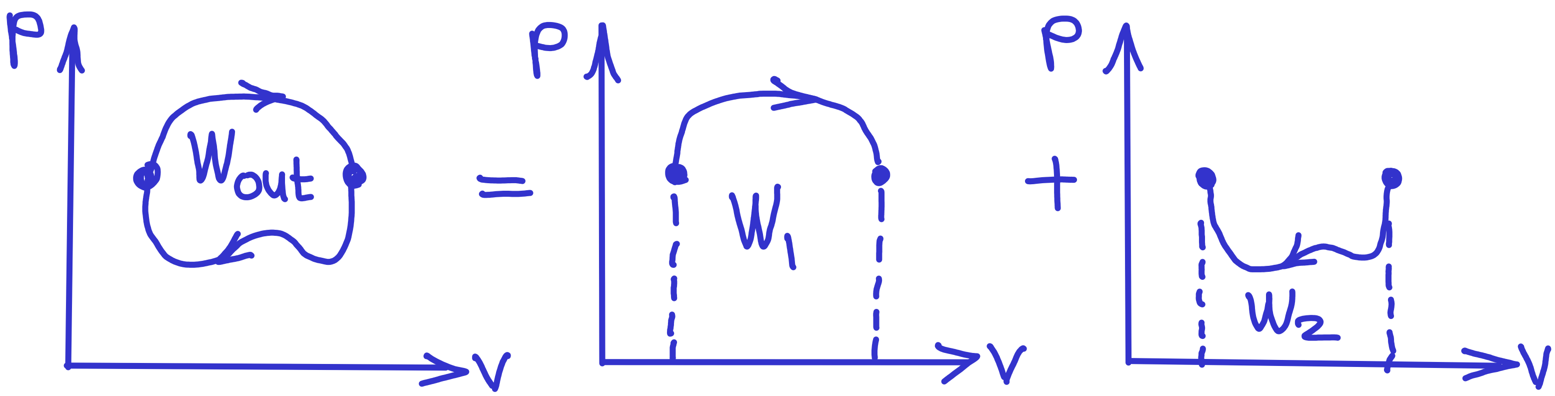
$0 = Q - W_{gas}$

$$\boxed{W_{gas} = Q}$$

We just converted heat Q to mechanical work W_{gas} with 100% efficiency!

Closed-cycle processes

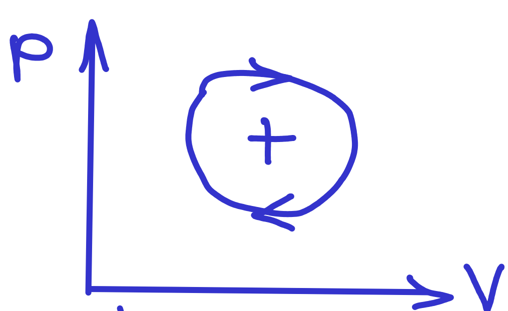
To be practical, a device that transforms heat into work must return to its initial state, and be ready for continued use.



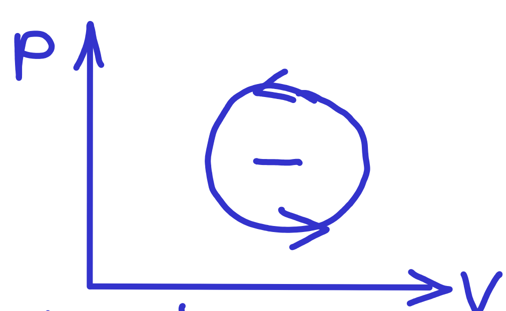
expansion $W_1 > 0$ compression $W_2 < 0$

$W_{out} = W_1 + W_2$ } All these works are defined from the viewpoint of the gas!

W_{out} : the total or net work by the gas.



heat engine



heat pump or refrigerator

<u>Reversible engine</u>: an engine that can operate in both modes.