PHY215-03: The Experimental Basis of Quantum Physics

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1 Standing Waves

Consider two traveling waves with the same amplitude A, angular frequency $\omega(= 2\pi f)$, and wave number $k(= \frac{2\pi}{\lambda})$, moving in opposite directions along the x-axis, where $\omega = kv$ with v being the speed of the wave.

The wave traveling to the right is given by

$$y_1(x,t) = A\sin(kx - \omega t),$$

and the wave traveling to the left is

$$y_2(x,t) = A\sin(kx + \omega t).$$

The resultant wave $y(x,t)$ is the sum of these two waves:

$$\begin{aligned} y(x,t) &= y_1(x,t) + y_2(x,t) \\ &= A\sin(kx - \omega t) + A\sin(kx + \omega t) \end{aligned}$$

Using the trigonometric identity

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta,$$

we can expand the above equation:

$$\begin{aligned} y(x,t) &= A[\sin(kx)\cos(\omega t) - \cos(kx)\sin(\omega t)] \\ &+ A[\sin(kx)\cos(\omega t) + \cos(kx)\sin(\omega t)] \\ &= 2A\sin(kx)\cos(\omega t) \end{aligned}$$

For a string of length L fixed at both ends, the allowed wavelengths are given by

$$\lambda_n = \frac{2L}{n},$$

where $n = 1, 2, 3, \cdots$. The wave number $k = \frac{2\pi}{\lambda}$, so

$$k_n = \frac{n\pi}{L}.$$

Substituting k_n into the above equation, we get the standing wave equation

$$y(x,t) = 2A\sin\left(\frac{n\pi x}{L}\right)\cos(\omega t).$$

1.1 Quantum physics

In quantum physics, a photon is considered as a "quanta" of the electromagnetic field, meaning it is the smallest discrete unit of electromagnetic energy that can exist within that field; essentially, it's the particle representation of light when considering the electromagnetic field at the quantum level.

For photon with energy E, its frequency is

$$f = E/h,$$

where h is the Planck's constant. Its wavelength

$$\lambda = \frac{c}{f} = \frac{ch}{E} = \frac{h}{p},$$

where c is the speed of light, and p is its momentum with

$$p = \frac{E}{c}.$$

Hence, the above condition for the standing (electromagnetic) wave can be written as

$$\lambda_n = \frac{h}{p_n} = \frac{2L}{n}.$$

Namley, the quantization condition for the standing electromagnetic waves is

$$p_n L = n\frac{h}{2}.$$

2 Fluorescent patterns inside a cathode ray tube

Fluorescent patterns inside a cathode ray tube are produced due to a combination of several physical phenomena and principles. The main physics reasoning behind this is as follows:

2.1 Electron Emission and Acceleration

- **Thermionic Emission:**
The cathode in the cathode ray tube is usually made of a material like tungsten. When heated, the thermal energy causes the electrons in the cathode material to gain enough energy to overcome the work function of the material and escape from the surface. This process is called thermionic emission. The heat is typically provided by passing an electric current through the cathode, which raises its temperature.

- **Electrical Acceleration:**
Once the electrons are emitted from the cathode, a high voltage is applied between the cathode and the anode. This creates an electric field within the tube that accelerates the electrons towards the anode. The electrons gain kinetic energy as they are accelerated by the electric field.

2.2 Interaction with the Gas or Phosphor

- **Gas Excitation:**

If there is a small amount of gas present in the tube, the high-energy electrons collide with the gas atoms or molecules. These collisions transfer energy to the gas particles, exciting their electrons to higher energy levels. When the excited electrons return to their lower energy levels, they emit photons of specific wavelengths, which can cause the gas to glow and produce a fluorescent pattern.

- **Phosphor Interaction:**

In many cathode ray tubes, the inner surface of the tube is coated with a phosphor material. When the high-energy electrons strike the phosphor, they transfer their energy to the phosphor atoms. This excites the electrons in the phosphor to higher energy states. As these excited electrons relax back to their ground states, they emit photons of visible light, creating the fluorescent patterns that are observed. Different phosphor materials emit light of different colors depending on their chemical composition and the energy levels of their electrons.

2.3 Electron Beam Focusing and Deflection

- **Focusing:**

To create a clear and well-defined fluorescent pattern, the electron beam needs to be focused. This is often achieved using electrostatic or magnetic focusing systems. Electrostatic focusing uses electric fields to bend the electron beam and bring it to a focus at a specific point on the screen. Magnetic focusing uses magnetic fields to achieve the same effect.

- **Deflection:**

To create different patterns or images, the electron beam needs to be deflected. This can be done using either electrostatic or magnetic deflection systems. Electrostatic deflection plates create electric fields that exert forces on the electrons, causing them to change their direction. Magnetic deflection coils produce magnetic fields that interact with the moving electrons and deflect the beam. By controlling the voltages or currents applied to these deflection systems, the electron beam can be made to scan across the screen in a specific pattern, creating the desired fluorescent patterns.

3 Positron emission tomography (PET)

Positronium, which is the bound state of an electron and a positron, only exists under extreme conditions (such as in a vacuum and with low-energy beams), and the positron emission tomography (PET) imaging scans rely on the direct annihilation of positrons and electrons. The physical environment inside the human body (high electron density and high collision probability) completely excludes the possibility of positronium formation.

3.1 Basic principle of PET

The positron emission tomography (PET) imaging is based on detecting the annihilation reaction between positrons (the anti-particle of electron) emitted by radioactive tracers (such as fluorine-18 labeled FDG) and electrons in the body. When a positron meets an electron, annihilation occurs, generating a pair of gamma photons with an energy of 511 keV. These photons are captured by the detector for image reconstruction.

3.2 Annihilation process in the human body

- Extremely short positron lifetime: In soft tissues, a positron has an average survival distance of about $1-3$ millimeters (corresponding to approximately 10^{-10} seconds), and then immediately annihilates with an electron.

- Environmental interference: The electron density in human tissues is extremely high. A positron cannot "pair up" with a single electron for a long time, but instead randomly collides and annihilates with the surrounding electrons.

3.3 Typical energies of photons detected in PET

In positron emission tomography (PET), when a positron emitted from radioactive decay annihilates with an electron, the rest mass of both particles is converted into energy according to Einstein's equation $E = mc^2$. The key details are:

1. Rest Mass Energy:

- The rest mass of an electron (and positron) is $m_e \approx 0.511\ \mathrm{MeV}/c^2$.

- Total energy from annihilation:

$$E_{\text{total}} = 2 \times m_e c^2 = 2 \times 0.511\ \text{MeV} = 1.022\ \text{MeV}.$$

2. Energy Distribution:

- The annihilation produces two gamma photons emitted in opposite directions (to conserve momentum).

- Each photon carries approximately 511 keV (or 0.511 MeV) of energy.

3. Kinetic Energy Consideration:

- Positrons emitted in PET (e.g., from ^{18}F) have kinetic energy (e.g., ~ 0.635 MeV for ^{18}F), but this energy is typically lost through interactions with matter before annihilation.

- At annihilation, positrons are nearly stationary, so their kinetic energy contributes little to the photon energy.

Conclusion: The typical energy of each photon in the annihilation process is 511 keV (or 0.511 MeV), which is detected in PET scanners to form images.

3.4 The β^+ decay of fluorine-18 (^{18}F)

Considering the β^+ decay of fluorine-18 (^{18}F), we must account for two electron masses due to the interplay between nuclear and atomic mass conservation. Here's why:

1. β^+ Decay Process

^{18}F undergoes β^+ decay according to:

$$^{18}F \to {}^{18}O + e^+ + \nu_e$$

A proton in the nucleus converts to a neutron, releasing a positron (e^+) and a neutrino (ν_e).

2. Mass Conservation in β^+ Decay

The energy released in β^+ decay comes from the mass difference between the parent (^{18}F) and daughter (^{18}O) nuclei. However, atomic mass calculations (not nuclear mass) are used for simplicity:

- ^{18}F atom: Contains 9 electrons.

- ^{18}O atom: Contains 8 electrons.

The mass difference is:

$$\Delta m = m(^{18}F \text{ atom}) - \left[m(^{18}O \text{ atom}) + 2m_e\right]$$

Here, $2m_e$ accounts for:

1. Positron mass (m_e): The positron is a fundamental particle with the same mass as an electron.

2. Electron deficit (m_e): The daughter nucleus (^{18}O) has one fewer electron than the parent (^{18}F), effectively requiring an additional electron mass to balance the system.

3. Energetic Requirement

For β^+ decay to occur, the mass difference must be positive:

$$\Delta m > 0 \quad \Rightarrow \quad m(^{18}F \text{ atom}) > m(^{18}O \text{ atom}) + 2m_e$$

This ensures there is enough energy to:

- Overcome the proton-to-neutron mass difference (neutrons are slightly heavier than protons).
- Create a positron (equivalent to $m_e c^2$).
- Account for the electron deficit in the daughter atom.

4. Example with ^{18}F

- $m(^{18}F \text{ atom}) \approx 18.000938$ u
- $m(^{18}O \text{ atom}) \approx 17.999159$ u
- $2m_e \approx 0.001097$ u

Mass difference:

$$\Delta m \approx 18.000938 - (17.999159 + 0.001097) \approx 0.000682 \text{ u}$$

Energy released:

$$E = \Delta m \cdot c^2 \approx 0.000682 \text{ u} \times 931.5 \text{ MeV/u} \approx 0.635 \text{ MeV},$$

which matches the measured maximum positron energy in ^{18}F decay.

Key Takeaway

The two electron masses in β^+ d decay account for:

1. The positron's mass.
2. The electron deficit in the daughter atom.

This adjustment ensures energy conservation and explains why β^+ decay is only feasible if the parent atom's mass exceeds the daughter's mass by at least $2m_e$.

3.4.1 Isotopes of oxygen

Oxygen-18 (^{18}O) is an isotope of oxygen.

Isotopes are atoms of the same element that have the same number of protons but different numbers of neutrons. Oxygen has an atomic number of 8, which means all oxygen atoms have 8 protons.

The most common isotope of oxygen is oxygen-16, which has 8 protons and 8 neutrons. Oxygen-18 has 8 protons and 10 neutrons.

So, oxygen-18 is one of the several isotopes of oxygen, along with oxygen-17 and others.

3.5 Formation conditions of positronium

Positronium is a hydrogen-like atom formed by the combination of an electron and a positron in a vacuum, and the following conditions need to be met:

- Low-energy environment: The positron needs to be in a state of extremely low kinetic energy (close to the thermal motion energy).
- No interfering substances: There should be no collisions with other electrons or atomic nuclei around.

However, inside the human body, once a positron is produced, it will directly annihilate with the surrounding electrons in high density, and a stable bound state cannot be formed.