The Bohr model of the Hydrogen atom

→ Certain stationary states exist, they are stable.

→ The energies of the states are well defined. When a transition happens, electromagnetic radiation is emitted / absorbed. The frequency of the radiation:

$$E_2 - E_1 = \Delta E = hf$$

→ The angular momentum of a state is quantized:

$$L = mvr = n \cdot \hbar \quad \text{where } \hbar = \frac{h}{2\pi}$$

$n = 1, 2, 3, \ldots$

n: principal quantum number

(Originally: $K = \frac{1}{2}mv^2 = \frac{1}{2}nhf_{orb}$

$$mv^2 = nh\frac{v}{2\pi r}$$

$$mvr = n\frac{h}{2\pi} = n\hbar\ )$$

Successes of the Bohr model

Size of the H-atom:

$$r_n = n^2 \cdot a_0 = n^2 \cdot \frac{4\pi\varepsilon_0 \hbar^2}{me^2}$$

$a_0 = 0.529 \text{Å}$: Bohr radius.

Energies of the states:

$$E_n = -\frac{E_0}{n^2} = -\frac{me^4}{2\hbar^2 (4\pi\varepsilon_0)^2} \cdot \frac{1}{n^2}$$

$E_0 = 13.6\,\text{eV}$: ionization energy of the H-atom.

Rydberg constant:

$$R_\infty = \frac{E_0}{hc} = \frac{me^4}{4\pi c \hbar^3 (4\pi\varepsilon_0)^2}$$

$$R_\infty = 1.097373 \cdot 10^7 \ 1/m$$

$$R_H = 1.096776 \cdot 10^7 \ 1/m$$

Electron velocity:

$$\beta_n = \frac{v_n}{c} = \frac{\alpha}{n} = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \cdot \frac{1}{n}$$

$\alpha \approx \frac{1}{137}$: fine structure constant

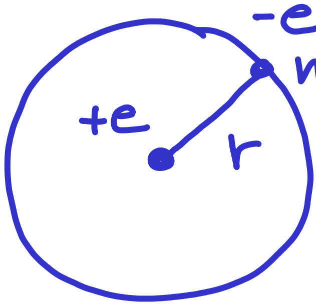
Bohr's Correspondance Principle

In the limit of large quantum numbers, quantum mechanics reduces to classical mechanics.

$$n = \underbrace{1, 2, 3, \ldots}_{\text{quantum}} \qquad \underbrace{\infty}_{\text{classical}}$$

mechanics

Radii of orbits



Newton's second law with electric force:

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2} = m\frac{v^2}{r}$$

$$\frac{e^2}{4\pi\varepsilon_0 m r} = v^2$$

Bohr condition:

$$L = mvr = n\hbar$$

$$v = \frac{n\hbar}{mr}$$

$$v^2 = \frac{n^2\hbar^2}{m^2 r^2}$$

Let's combine them:

$$\frac{e^2}{4\pi\varepsilon_0 m r} = \frac{n^2\hbar^2}{m^2 \cdot r^2}$$

$$r = \frac{4\pi\varepsilon_0 \hbar^2}{m e^2} \cdot n^2 = n^2 \cdot \underbrace{a_0}_{0.529\,\mathring{A}}$$

$$r = \frac{4\pi\varepsilon_0 \hbar^2}{m e^2 Z} \cdot n^2 = n^2 \cdot \frac{a_0}{Z}$$

Energies of orbits

$$E = KE + PE = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\varepsilon_0 r} =$$

$$= \frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = -\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r}$$

$$E_n = -\frac{1}{2}\cdot\frac{e^2}{4\pi\varepsilon_0 n^2 a_0} = -\frac{1}{n^2}\cdot\frac{e^2}{8\pi\varepsilon_0 a_0} =$$

$$= -\frac{1}{n^2}\cdot\frac{e^2}{8\pi\varepsilon_0 \frac{4\pi\varepsilon_0\hbar^2}{me^2}} = -\frac{1}{n^2}\cdot\underbrace{\frac{me^4}{2\hbar^2(4\pi\varepsilon_0)^2}}_{E_0 = 13.6\,eV}$$

$$E_n = -\frac{E_0}{n^2}$$

$$E_n = -\frac{1}{n^2}\cdot\frac{me^4Z^2}{2\hbar^2(4\pi\varepsilon_0)^2} = -\frac{Z^2}{n^2}\cdot E_0$$