Rydberg constant

$$E_u - E_\ell = \Delta E = hf \qquad c = \lambda f \Rightarrow$$

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{hf}{hc} = \frac{E_u - E_\ell}{hc}$$

$$\frac{1}{\lambda} = \frac{1}{hc}\left(-\frac{E_0}{n_u^2} - \left(-\frac{E_0}{n_\ell^2}\right)\right)$$

$$\frac{1}{\lambda} = \frac{E_0}{hc}\left(\frac{1}{n_\ell^2} - \frac{1}{n_u^2}\right)$$

$$R = \frac{E_0}{hc} = \frac{me^4}{4\pi c\hbar^3 (4\pi\varepsilon_0)^2}$$

$$R = \frac{13.6\,eV}{1240\,eV\,nm} = 0.01096776\ \frac{1}{nm}$$

Reduced mass:

$$\frac{1}{\mu} = \frac{1}{m} + \frac{1}{M} \Rightarrow \mu = \frac{mM}{m+M}$$

If $M = 1836m$, then $\mu = \frac{1836}{1837}m = 0.999456m$

$$R_H = 0.999456\,R_\infty$$

$$R_\infty = 1.097373 \cdot 10^7\ 1/m$$

$$R_H = 1.096776 \cdot 10^7\ 1/m$$

Velocity

Bohr condition: $v = \frac{n\hbar}{mr}$

$$v_n = \frac{n\hbar}{mn^2 a_0} = \frac{1}{n} \cdot \frac{\hbar}{m a_0} = \frac{1}{n} \cdot \frac{e^2}{4\pi\varepsilon_0 \hbar}$$

$$\beta_n = \frac{v}{c} = \frac{1}{n} \cdot \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{1}{n} \cdot \alpha$$

$$\underbrace{\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137}}_{\text{fine structure constant}}$$

$$\beta_n = \frac{Z}{n} \cdot \alpha$$

Summary

$$r_{n,Z} = \frac{n^2}{Z} a_0 \qquad a_0 = 0.529\,\mathring{A}$$

$$E_{n,Z} = -\frac{Z^2}{n^2} \cdot E_0 \qquad E_0 = 13.6\,eV$$

$$\beta_{n,Z} = \frac{Z}{n} \cdot \alpha \qquad \alpha \cong \frac{1}{137}$$

	n	Z	μ
$r_{n,Z}$	n^2	$\frac{1}{Z}$	$\sim \frac{1}{\mu}$
$E_{n,Z}$	$-\frac{1}{n^2}$	Z^2	$\sim \mu$
$\beta_{n,Z}$	$\frac{1}{n}$	Z	X