Outline

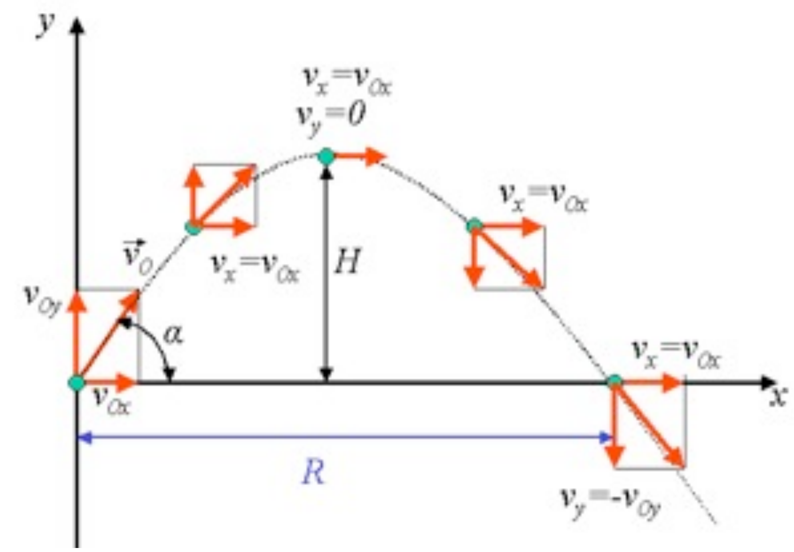
- Waves vs. Particles
    - Review of Wave Diffraction
- deBroglie Waves
- Davison and Germer
- Electron Double Slit

  Electrons Behave Like Waves!
- Complementarity

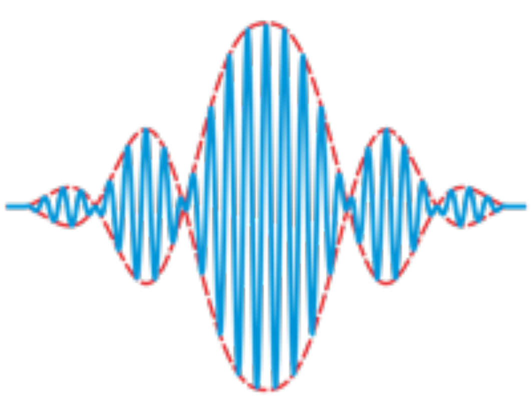
Particles vs. Waves

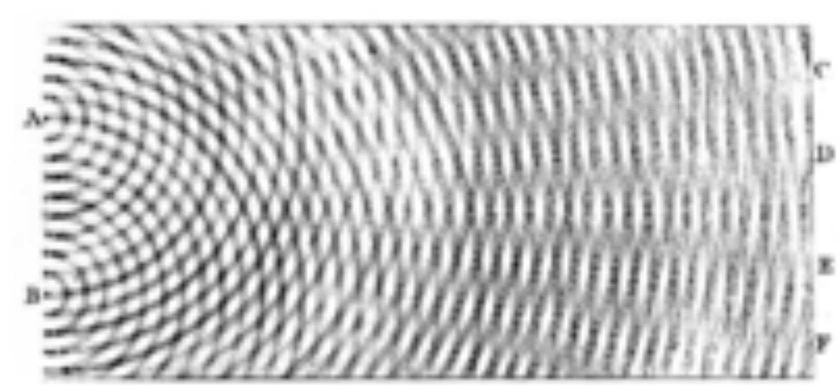
Property	Particle	Wave
Location	Definite	Indefinite
Momentum	Definite	Indefinite
Interference	No	Yes

Particle



Wave

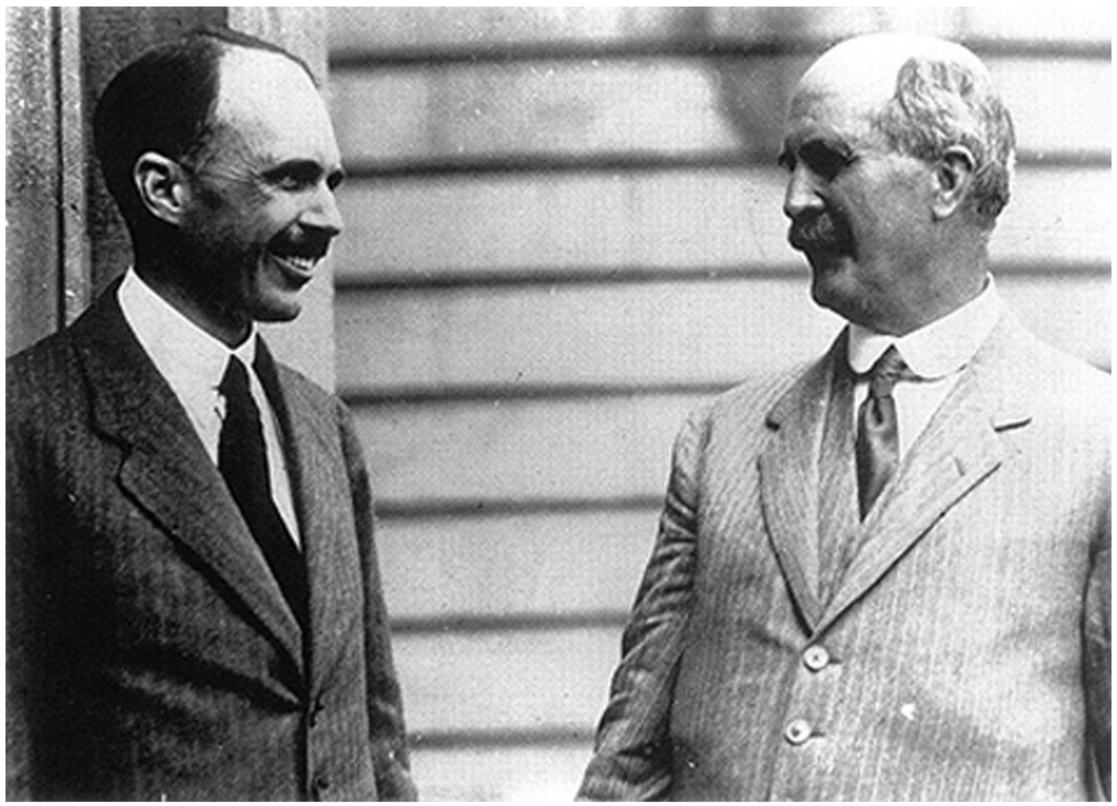




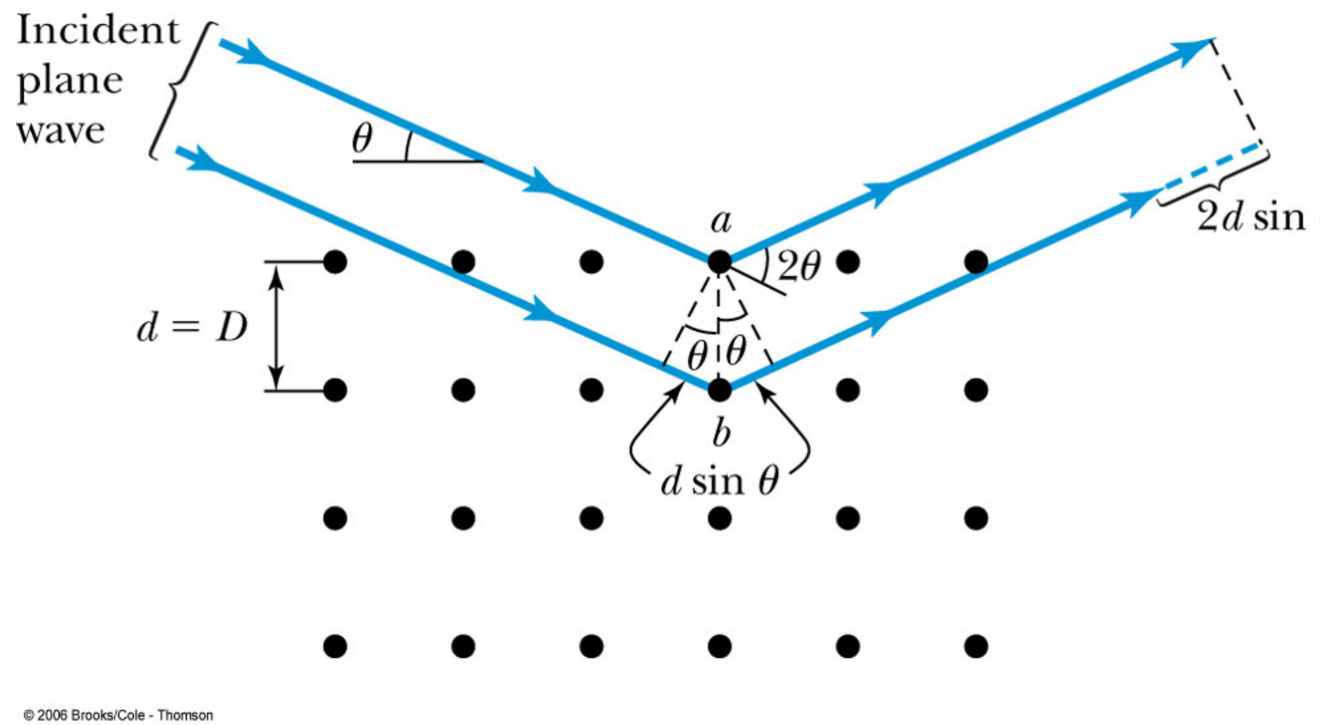
Diffraction, Thomas Young (1803)

Images: http://en.wikipedia.org
http://www.staff.amu.edu.pl
http://micro.magnet.fsu.edu

Bragg Diffraction



William Lawrence Bragg
1890-1972
William Henry Bragg
1862-1942
Nobel Prize 1915



Bragg's Law:

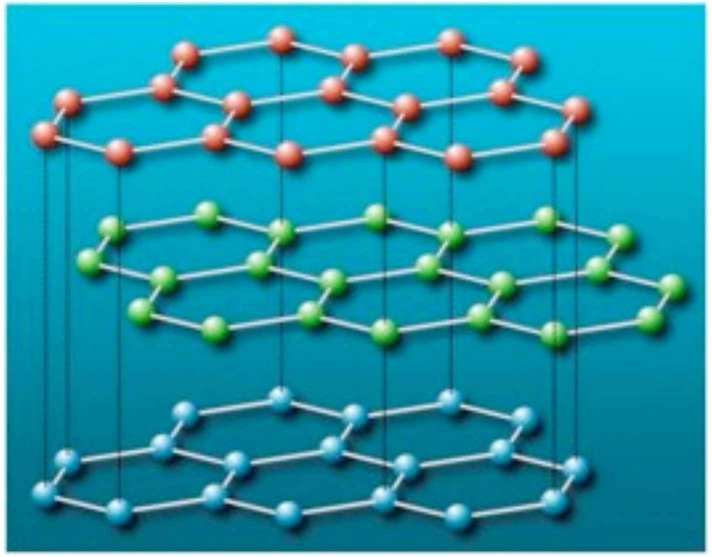
$$n\lambda = 2d\sin\theta$$

Classically, light behaves as a wave!

Images: Thornton and Rex

Electrons

- Are parts of an atom.
- Atoms are particles. →
- Ergo, electrons are particles.
- Or are they...



Graphite

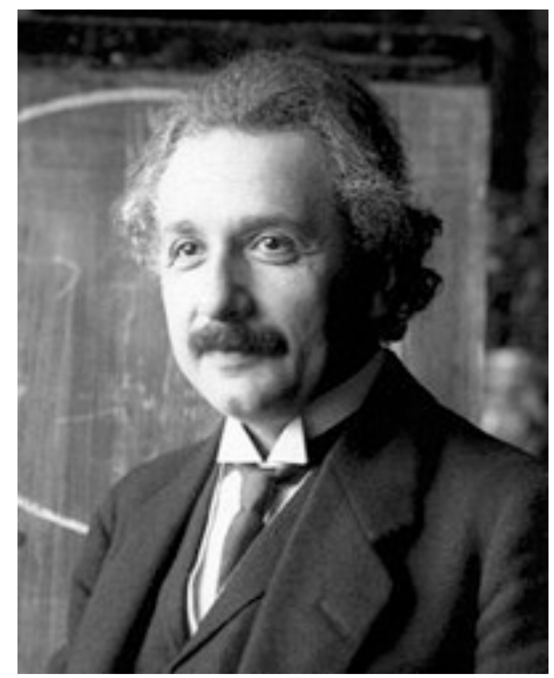
Concept Test

- For a light wave with wavelength λ and frequency ν, the following relation is always true (in a vacuum):
  - λν=c ← $[c] = \frac{\mathrm{m}}{\mathrm{s}} = [\lambda] \cdot [\nu]$
  - λc=ν
  - νc=λ

where c is the speed of light.

For Light

- $\lambda\nu = c$
- $E = pc$ (Einstein)
- $E = h\nu$ (Planck)
- $pc = h\nu = hc/\lambda$
- $\therefore\ \lambda = h/p$



Images: http://www.wikipedia.org

A Crazy Idea...

<u>Matter waves</u>

An electron with momentum p
"has" a wavelength

$\lambda = h/p$!

Louis de Broglie
1892-1987
Nobel Prize: 1929

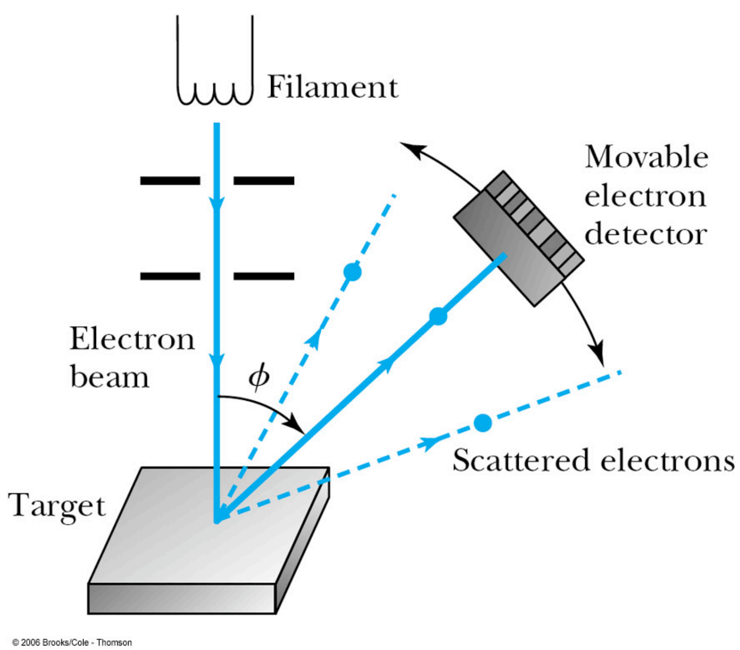
Quantum "fuzziness": a wave is hard to localize within a size of order λ

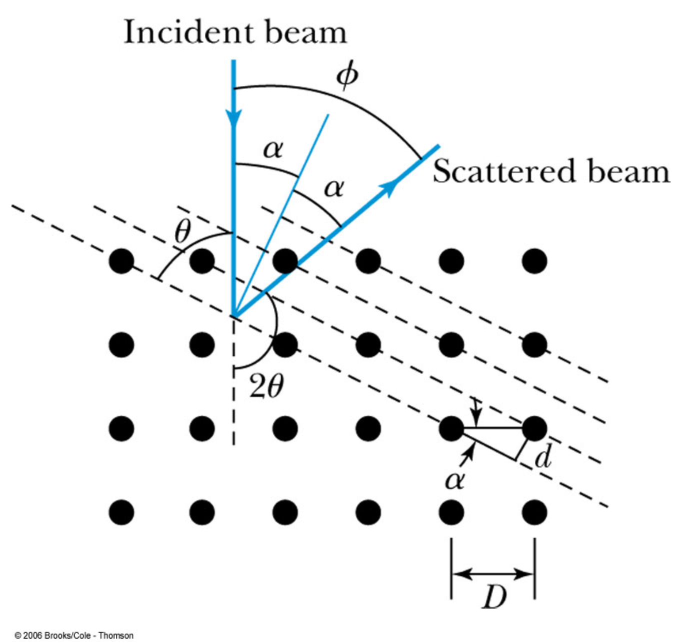
Heisenberg will make this more precise!

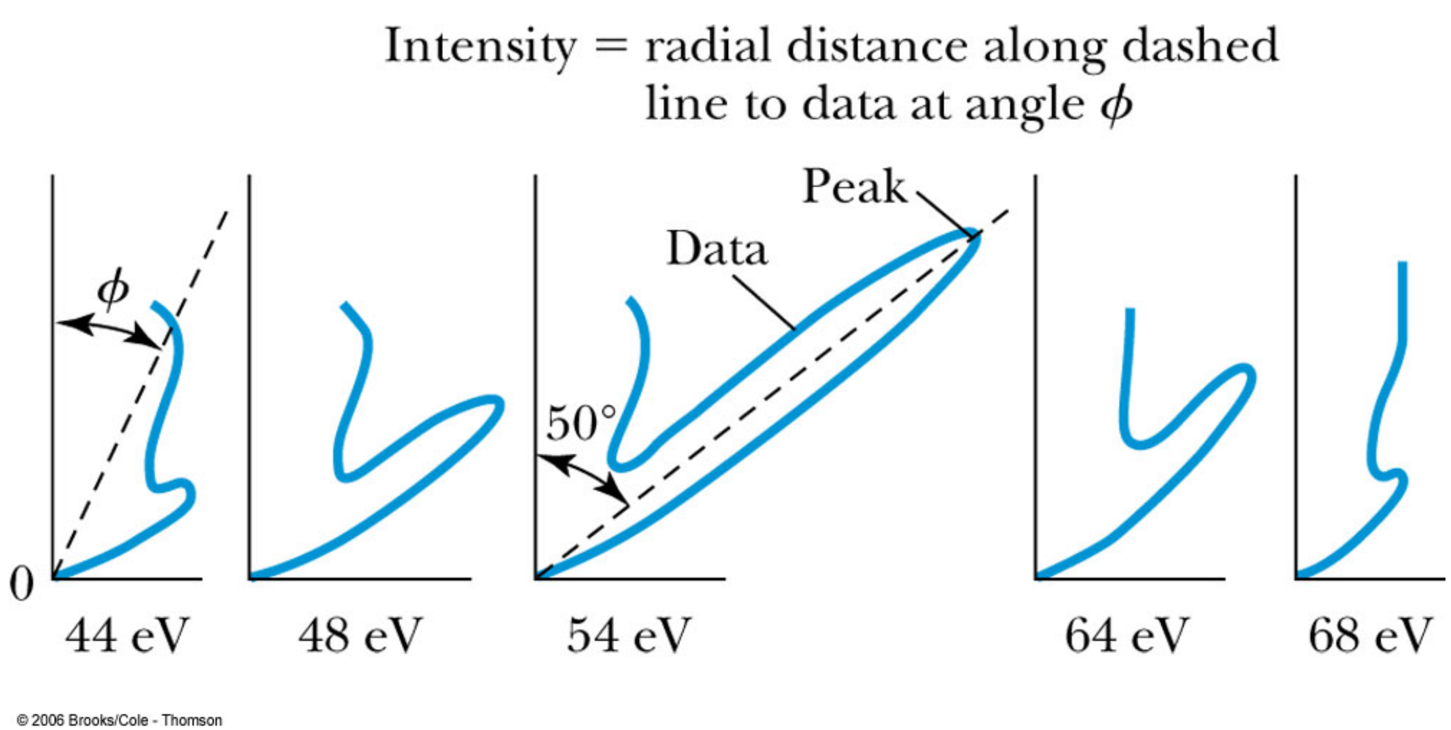
Image: http://www.wikipedia.org

Experimental Proof!

1925: Diffraction of Electrons by a Nickel Crystal







Clinton Davission (R)

(1881-1958)

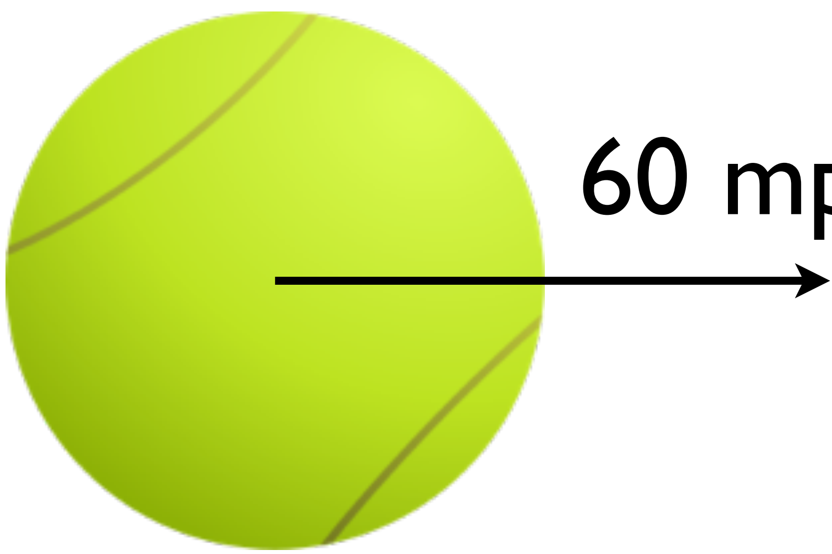
Nobel Prize 1937

Lester Germer

(1896-1971)

Images: Thornton and Rex

What is the wavelength of a tennis ball?



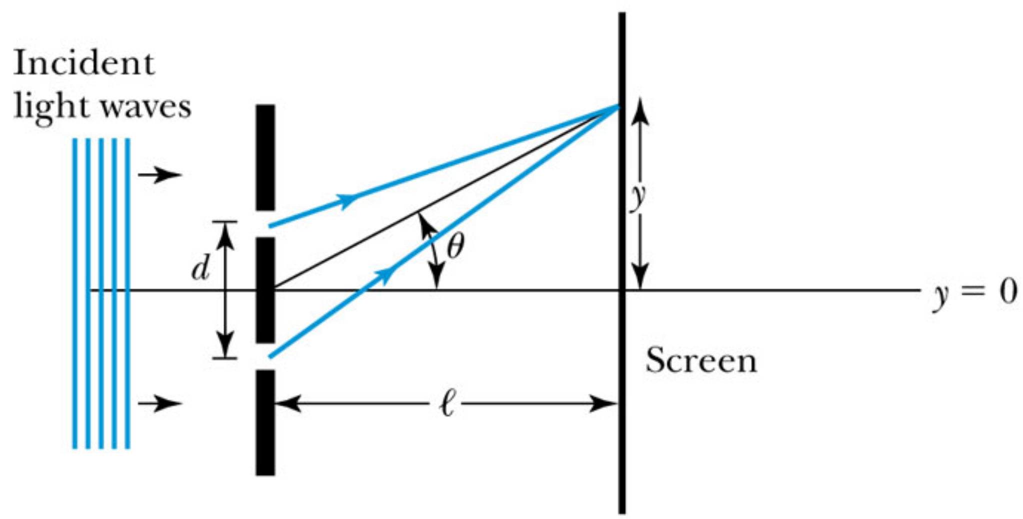
$$\begin{aligned}\lambda &= \frac{6.63 \times 10^{-34}\,\mathrm{J\,s}}{60\,\frac{\mathrm{mi}}{\mathrm{hr}} \cdot 1600\,\frac{\mathrm{m}}{\mathrm{mi}} \cdot \frac{1}{3600}\,\frac{\mathrm{hr}}{\mathrm{s}} \cdot 0.1\mathrm{kg}} \\ &= 2.5 \times 10^{-34}\,\mathrm{m} \\ &= \mathcal{O}\left(10^{-24}\right) \times \text{size of atom}\end{aligned}$$

Quantum "fuzziness" of tennis ball is *very* small!

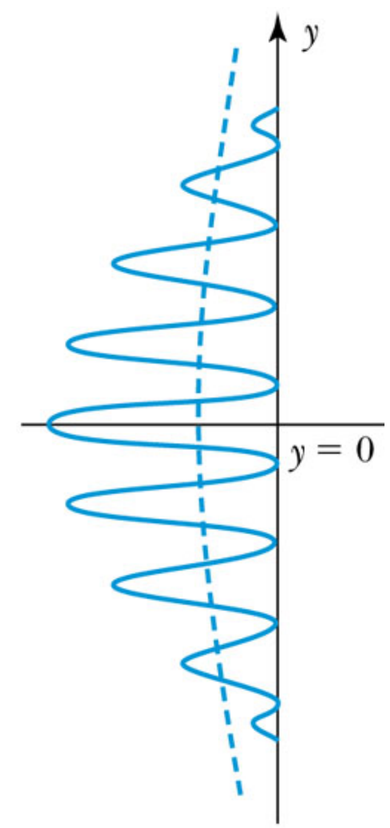
(I can't blame my inability to hit a tennis ball to quantum effects.)

Image: http:/commonswikimedia.org

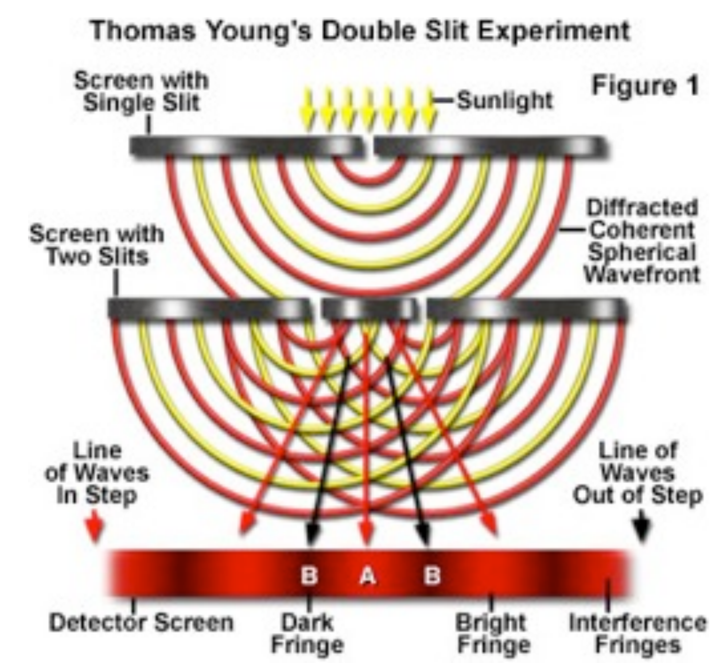
Double Slit Experiment Light



(a)



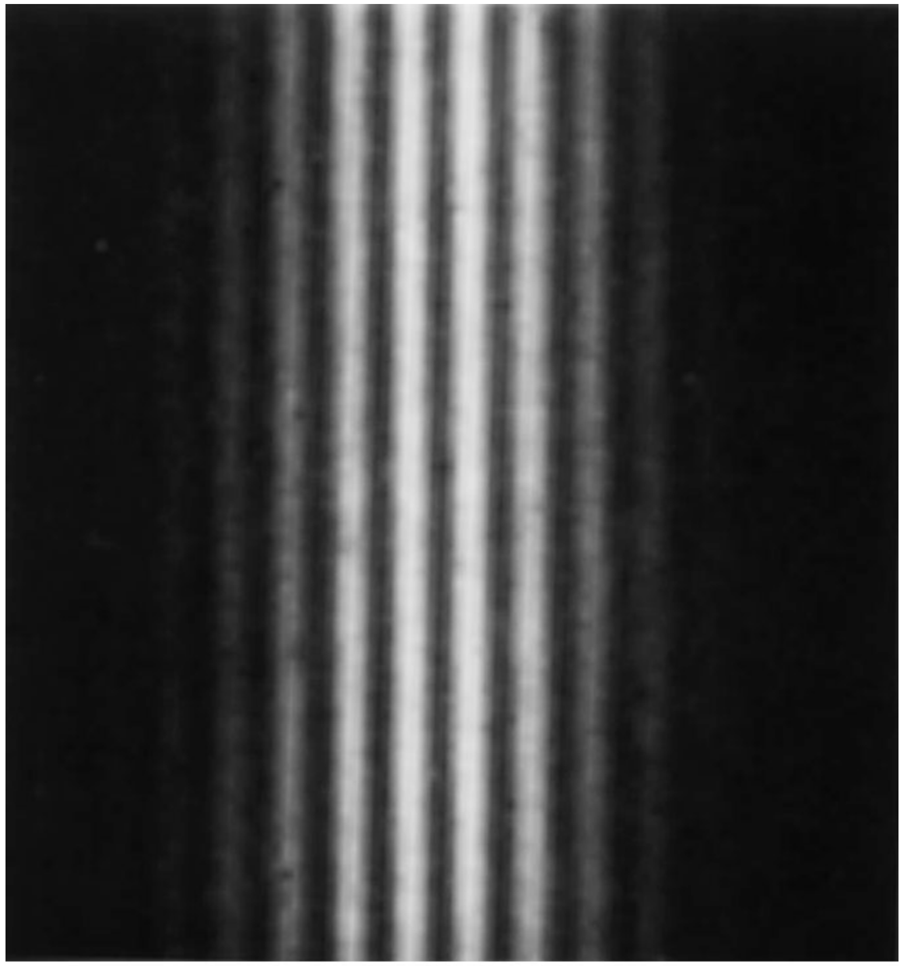
(b)



Diffraction, Thomas Young (1803)

Images: Thornon and Rex
http://micro.magnet.fsu.edu

Double Slit Experiment: Electrons!



C. Jönsson, 1961

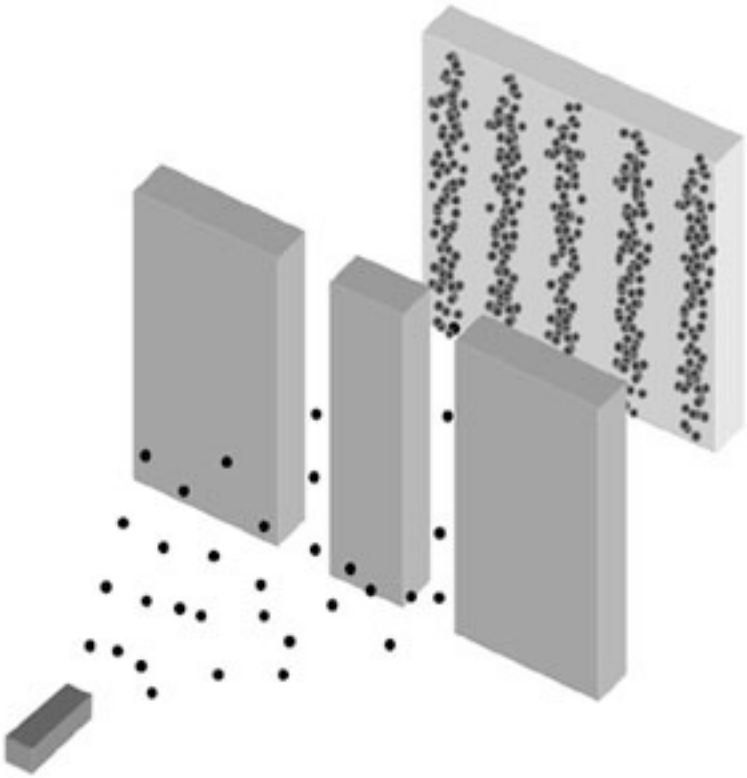
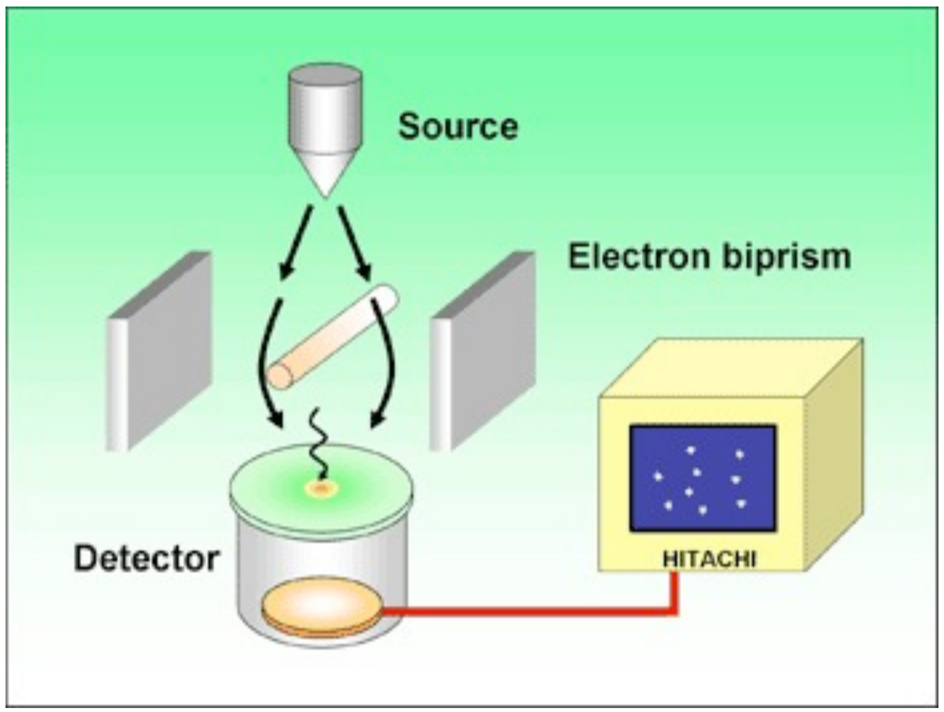


Image: Thornon and Rex
http://stephenwhitt.files.wordpress.com

One electron at a time!



Electrons, even one at a time, behave like waves!

Hitachi Laboratories
Dr. Tonomura Akiro

Movie: www.hitachi.com

Duality and Complementarity

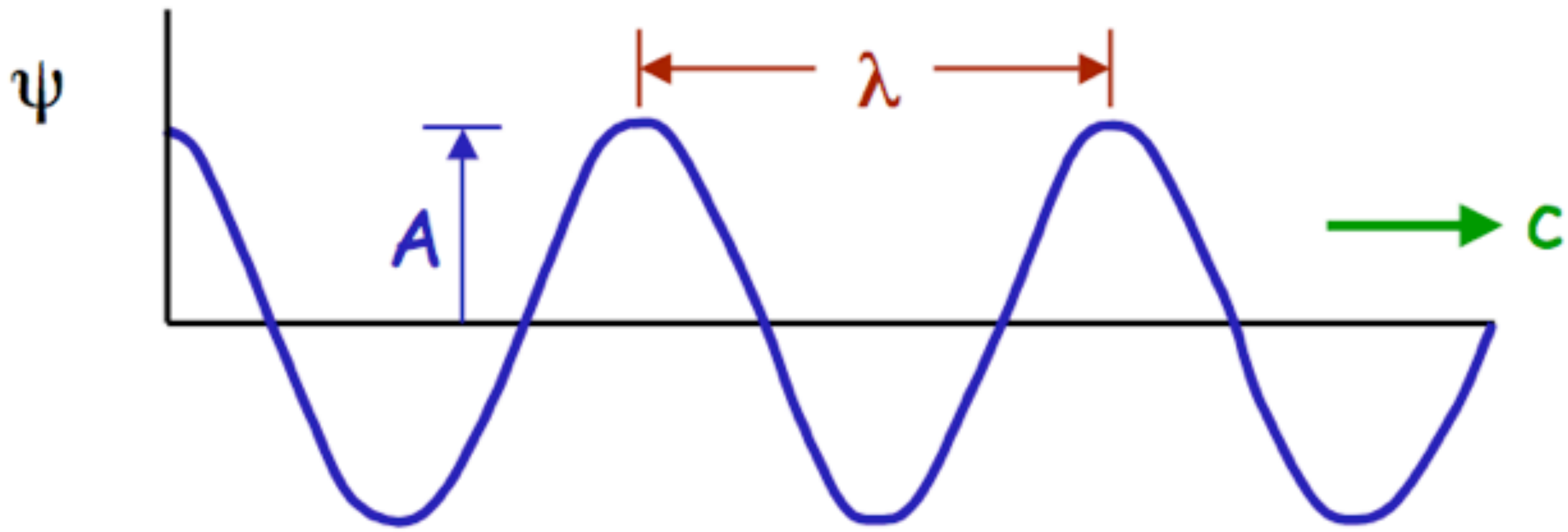
- Wave-Particle Duality: all matter and energy exhibits *both* wave and particle like properties!
- Complementarity: a single quantum-mechanical system can behave like a wave or a particle, *but not both simultaneously!* (Bohr)

Summary, so far

- Light is classically a wave, but can behave like a particle (e.g. the photoelectric effect).
- Electrons are classically described as particles, but can behave like waves (e.g. the Davisson-Germer experimet).
- We need a description that unifies the particle and wave aspectsof natural systems!

Waves

A plane wave: (light)



Amplitude: A
Wavelength: λ
Speed: c
Frequency: $\nu = c/\lambda$

$$\psi(x,t) = A\cos[2\pi(x - ct)/\lambda]$$ (right-moving wave)

It is convenient to rewrite:
$$\psi(x,t) = A\cos(kx - \omega t)$$

Wave number: $k = 2\pi/\lambda$
Angular frequency: $\omega = 2\pi\nu$

Wave relation: $c = \lambda\nu = \omega/k$

All light waves have same speed c in vacuum, independent of wave number k.

Complex Exponentials

- $i=\sqrt{(-1)}$
- $e^{i\theta}=\cos\theta + i \sin\theta$
- All complex numbers have a polar form: $z=x+iy=re^{i\theta}$, $x=r\cos\theta$, $y=r\sin\theta$
- $z^*=x-iy$, $zz^*=x^2+y^2=r^2$
- Plane wave: $\psi=Ae^{i(kx-\omega t)}$
- Classical Physics: use only real or imaginary part

Matter Waves

- Quantum Mechanics: Complex waves
- $\psi = Ae^{i(kx-\omega t)}$

Planck: $E = h\nu = \hbar\omega$

deBroglie: $p = \frac{h}{\lambda} = \hbar k$

→

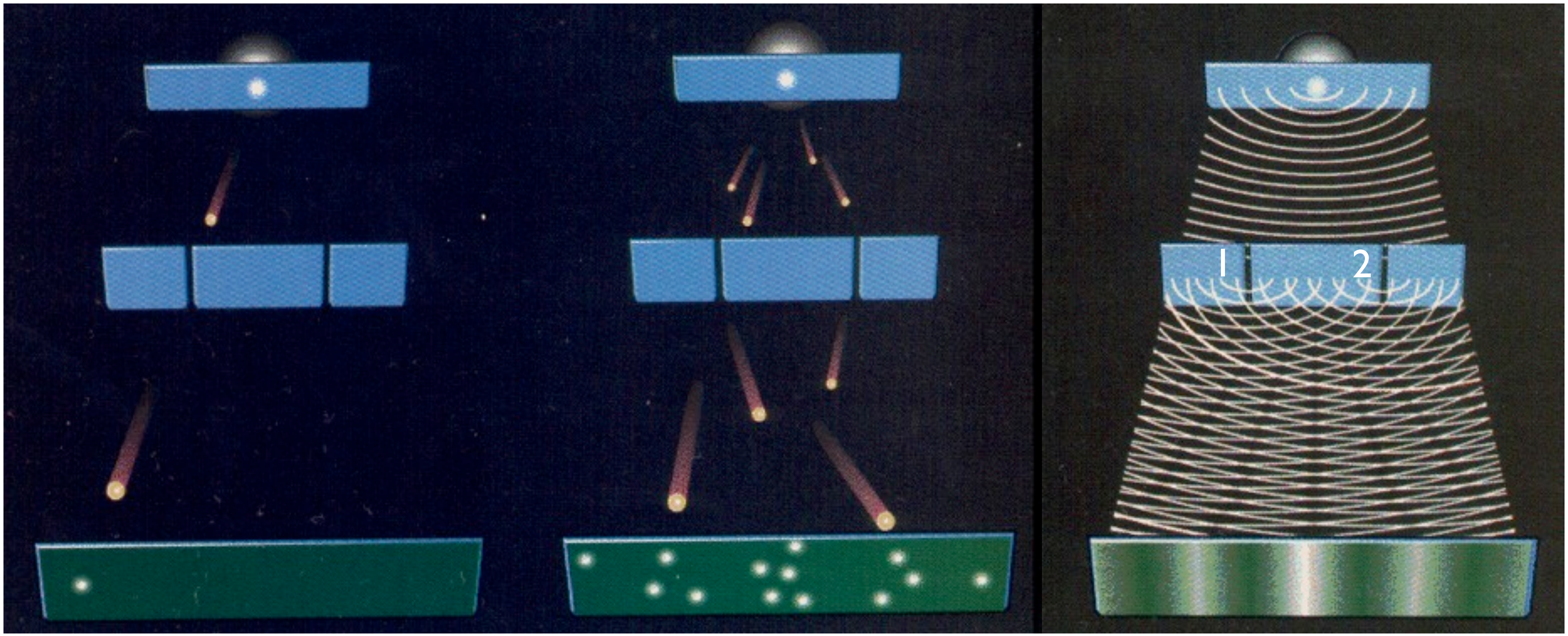
$$E = \frac{p^2}{2m}$$ free particle!

$$\omega = \frac{\hbar k^2}{2m}$$ dispersion relation

$$v = \frac{p}{m} = \frac{d\omega}{dk} \neq \frac{\omega}{k}$$

How do we interpret ψ(x,t)?
How do we calculate it in general?

Interference & Superposition

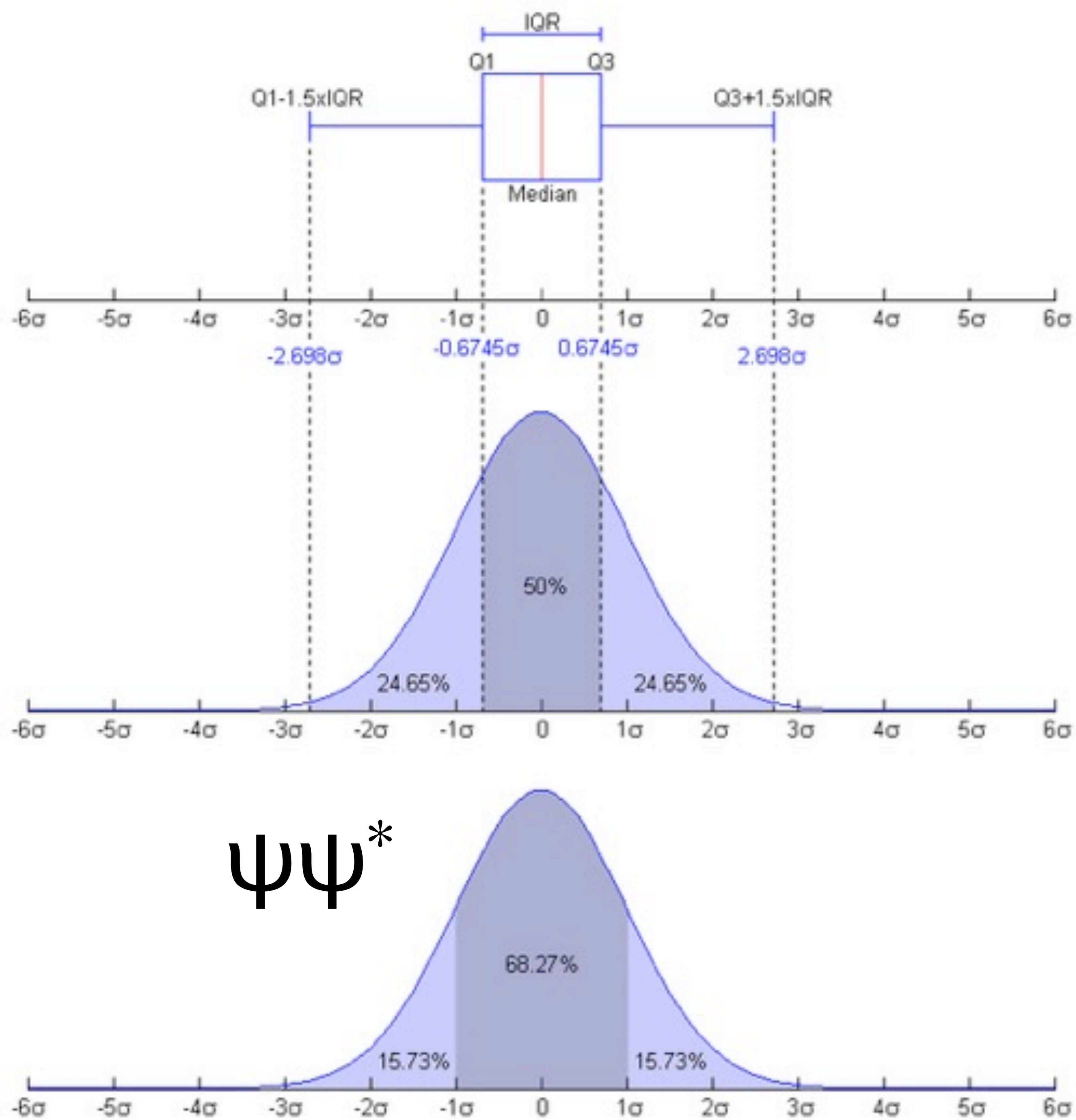


Electron diffraction due to interference of matter waves coming from the two slits:

$$\psi_{screen}=\psi_1+\psi_2$$

Images: http://abyss.uoregon.edu

Probability Density



Gaussian PDF

Image: wikipedia.org

Summary

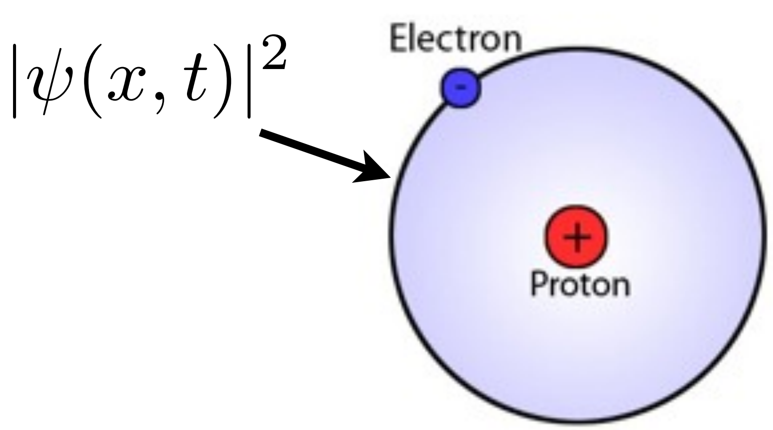
- The Bohr model reproduces the Rydberg formula for Hydrogen spectra.
- The "shell hypothesis" explains peaks in X-ray spectra, and implies periodic table should be ordered by Z.
- Matter waves correspond to complex wavefunction $\psi(x,t)$.
- *Probability* of finding particle at position x is proportional to $|\psi(x,t)|^2$.

Outline

- Born's Interpretation of ψ (cont'd)
- Heisenberg Uncertainty Principle
- "Copenhagen Interpretation"
- Complementarity and the single photon
- Where are we now? What remains to be done?

Born's Interpretation of ψ

- Born suggested that the *probability* of finding a particle at position x and time t is proportional to $|\psi(x,t)|^2 = \psi\psi^*$.
- Positions of destructive interference have $\psi=0$, and no *probability* to find the particle at that location.



Max Born
1882-1970
Nobel Prize 1954

Concept Test

- Born's interpretation of the wavefunction implies that $|\psi(x,t)|^2 = \psi\psi^*$ is the *probability density* of finding a particle in an infinitesmal volume d^3x. All of the following are true except

- $\psi\psi^*$ is a real number

- $\psi\psi^*$ is positive

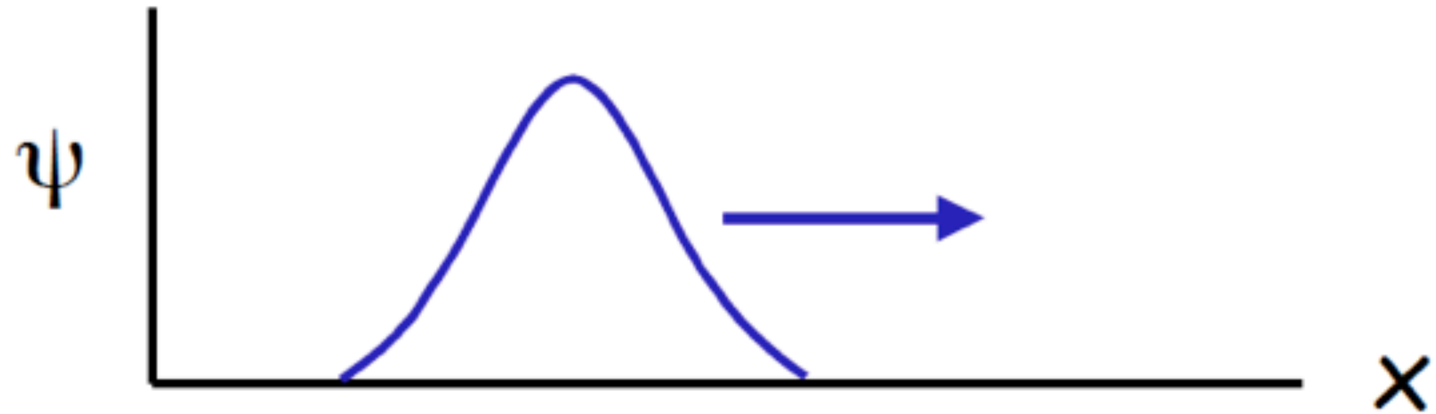
- $\psi\psi^*$ must be between 0 and 1 ←

- $\int \psi\psi^* \, d^3x = 1$

"Localized Particle"

A wave packet can be constructed as a continuous sum (integral) of plane waves.

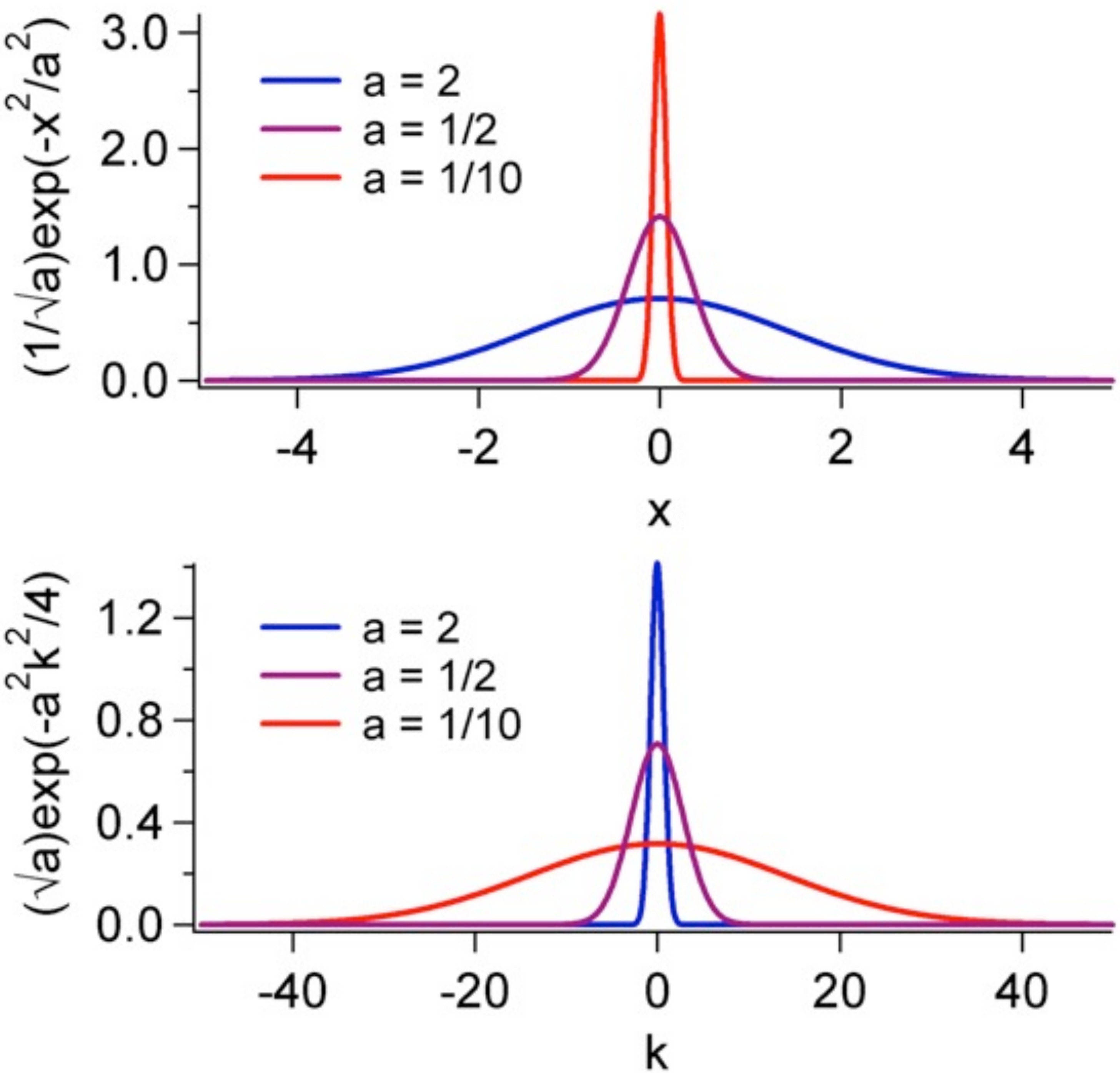
⇒ Fourier Transform



$$\psi(x,t) = \int_{-\infty}^{+\infty} A(k) e^{i(kx-\omega t)} dk$$

Position vs. Momentum

$\sigma_x = \Delta x = a/2$



Position *localization*⇔

Momentum *delocalization*

Momentum *localization* ⇔

Position de*localization*

$\sigma_k = \Delta k = 1/a$

Images: http://chsfpc5.chem.ncsu.edu

Heisenberg Uncertainty Relation

$$\Delta x\, \Delta k \geq 1/2$$

Multiplying by $\hbar$ and using $p = \hbar k$ gives:

$$\Delta x\, \Delta p \geq \hbar/2$$

Werner Heisenberg
1901-1976
Nobel Prize 1932

No measurement can simultaneously measure position and momentum to an accuracy which violates the uncertainty principle!

Image: http://wikipedia.org

Concept Test

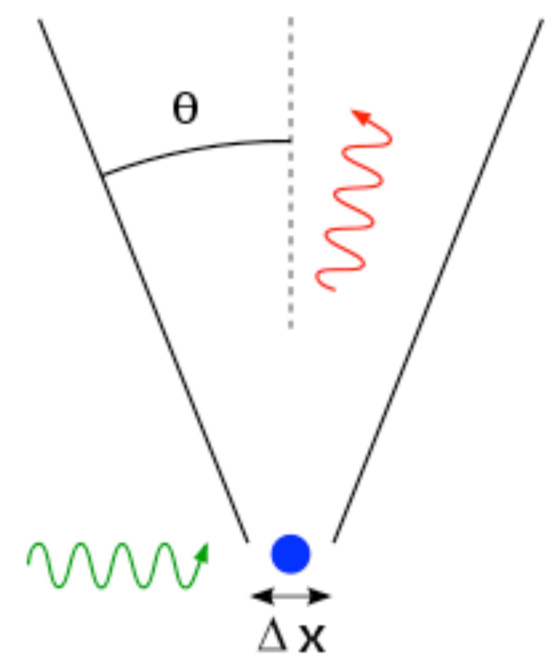
- The de Broglie wavelength of a tennis ball, $\lambda = h/p$, goes to ∞ (the "fuzziness" of the ball) as $p \rightarrow 0$. Why doesn't this prevent us from picking up tennis balls left on the court?
  - de Broglie's formula doesn't apply
  - $h \rightarrow 0$ when the ball isn't moving
  - p is never exactly 0 ←

Concept Test

- Particle A is confined to a (small) region of size L, while particle B is confined to a region of size 2L. From the uncertainty relation, we expect that the kinetic energies are related by
  - $E_A < E_B$
  - $E_A = E_B$
  - $E_A > E_B$ ← $\Delta p_A \approx 2\, \Delta p_B$

Heisenberg's Microscope

- How can we understand the uncertainty relation physically?
  - Suppose we want to locate an electron: we must scatter light off of it!
  - To do so precisely, we must use light with a short wavelength ($\Delta x \approx \lambda$)- but such light also has large energy!
  - The light must scatter off the electron, but this imparts large momentum ($\Delta p \approx \hbar/\lambda$)!



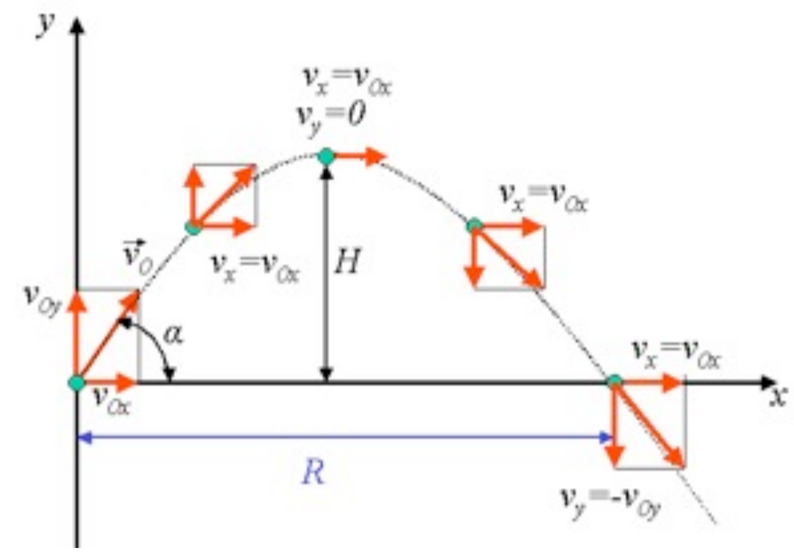
QM crucial to this argument!

Image: http://wikipedia.org

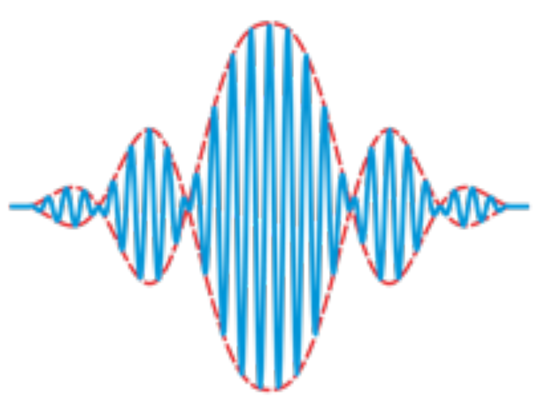
Classical vs. Quantum

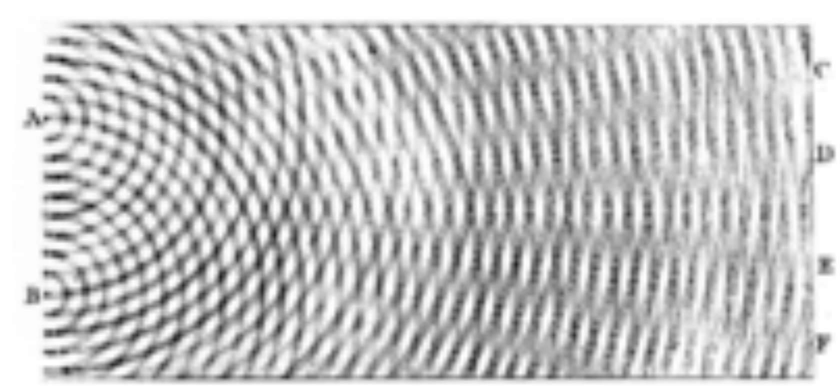
Property	Classical	Quantum
Location	Definite	Indefinite
Momentum	Definite	Indefinite
Interference	No	Yes

Particle



Wave





Diffraction, Thomas Young (1803)

Matter Waves: $\Delta x \, \Delta p \geq \hbar/2$

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http://www.staff.amu.edu.pl
http://micro.magnet.fsu.edu

Energy-Time Uncertainty

- For a free particle
  - $E=\hbar^2k^2/2m$, $\Delta E=(\hbar^2k/m)\Delta k$
  - $\Delta x=(\hbar k/m)\Delta t$
  - $\therefore$ $\boxed{\Delta E\,\Delta t \geq \hbar/2}$
- Einstein & Bohr: a state that exists for only a short time cannot have a definite energy.
- Other "conjugate" pairs exist in different cases: (L,θ), etc.

“Copenhagen” Interpretation

- Bohr and collaborators developed an interpretation of quantum mechanics, stating that quantities like the “position” and “momentum” of a particle only have meaning to the extent that they are measured.
  - Complementarity: One cannot describe a physical observable simultaneously in terms of both particles and waves.
  - Uncertainty: Conjugate variables cannot be simultaneously measured.
  - Born: $|\Psi(x)|^2$ measure how likely a particle is to be found at position x, within volume d^3x.

Bohr & Copenhagen

1921

Now

Blegdamsvej 17

Niels Bohr
1885-1962
Nobel Prize 1922

Images: wikipedia.org, Niels Bohr Institute, flickr

Where are we now?

- Experimental evidence shows that
  - light has particle-like properties
  - and matter has wave-like properties.
- The wave-like properties of matter imply
  - *quantization* of energy levels in the atom
  - position-momentum uncertainty
  - existence of a “wavefunction” $\Psi(x,t)$.

What remains to be done?

- How do we systematically
  - determine $\Psi(x,t)$ and
  - extract the values of measurements from $\Psi(x,t)$?

Werner Heisenberg
1901-1976
Nobel Prize 1932

Erwin Schrödinger
1887-1961
Nobel Prize 1933