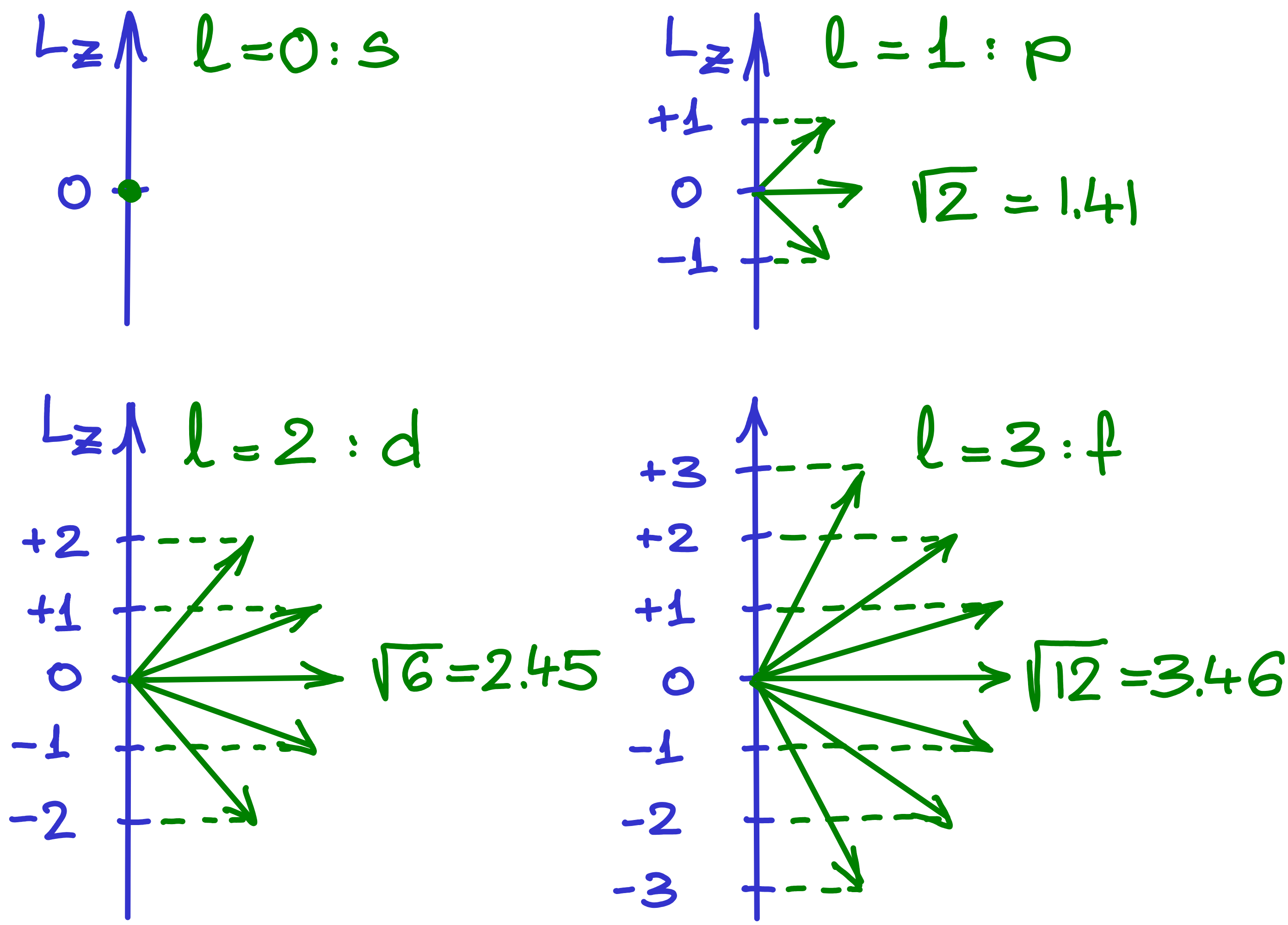
Orbital angular momentum

$$L = \sqrt{\ell(\ell+1)} \cdot \hbar \quad ; \quad 0 \le \ell < n$$

$$L_z = m_\ell \cdot \hbar \quad ; \quad -\ell \le m_\ell \le \ell$$



Bohr condition : $L = n \cdot \hbar$
According to Bohr : $L = \hbar$ when $n = 1$.
According to Schrödinger : $L = 0$ when $n = 1$ or when $\ell = 0$ in higher states. But how can you have $L = 0$ in a spherically symmetric s state?

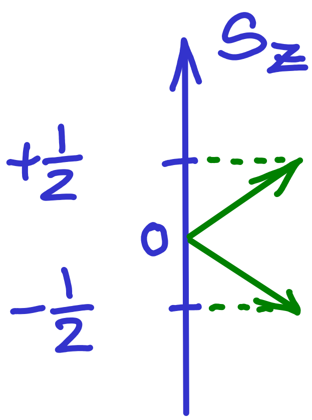
Spin

(Intrinsic angular momentum)

Pauli, Stern-Gerlach
Goudsmit-Uhlenbeck, Dirac

$s = \frac{1}{2}$ $\quad S = \sqrt{s(s+1)}\cdot\hbar = \frac{\sqrt{3}}{2}\hbar$

$m_s = +\frac{1}{2}$ and $-\frac{1}{2}$

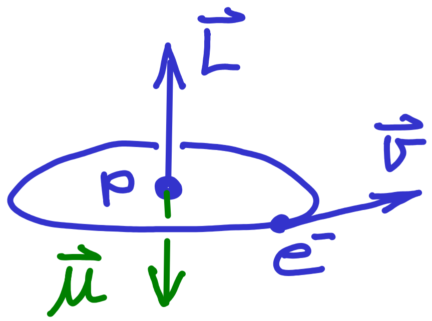


$\frac{\sqrt{3}}{2} = 0.866$

In general, $m_s = -s, -s+1, \cdots, s$

E.g. for $s = \frac{3}{2}$, $m_s = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$

Magnetic moment



$\vec{L}$: orbital angular momentum

$\vec{\mu}$ ($\vec{m}$) : magnetic moment

$\vec{L}$ is due to orbiting mass : $\vec{L} = \vec{r} \times \vec{p}$

$\vec{\mu}$ is due to orbiting charge

$$\mu = I \cdot A = \frac{q}{T} \cdot \pi r^2 = \frac{-e}{\frac{2\pi r}{v}} \cdot \pi r^2 = \frac{(-e) r v}{2}$$

$$\mu = -\frac{e r m v}{2m} = -\frac{e r p}{2m} = -\frac{e}{2m} \cdot L$$

$$\boxed{\vec{\mu} = -\frac{e}{2m} \vec{L}} \qquad \frac{\vec{\mu}}{e} = -\frac{1}{2} \frac{\vec{L}}{m}$$

$$L = \sqrt{\ell(\ell+1)} \cdot \hbar$$

$$L_z = m_\ell \cdot \hbar \longrightarrow \mu_z = -\frac{e}{2m} L_z$$

$$-\ell \le m_\ell \le \ell$$

$$\mu_z = -\frac{e}{2m} \cdot m_\ell \hbar$$

$$\boxed{\mu_z = -\frac{e\hbar}{2m} \cdot m_\ell = -\mu_B \cdot m_\ell}$$

$\mu_B = \dfrac{e\hbar}{2m}$: Bohr magneton

Magnetic moment 2.

$$\mu_z = -\frac{e\hbar}{2m} \cdot m_\ell = -\mu_B \cdot m_\ell$$

$$\vec{\mu} = -\frac{e}{2m}\vec{L} = -\frac{e\hbar}{2m\hbar}\cdot\vec{L} = -\frac{\mu_B}{\hbar}\vec{L}$$

$$\vec{\mu}_\ell = -\frac{\mu_B}{\hbar}\cdot\vec{L} = -g_\ell \frac{\mu_B}{\hbar}\cdot\vec{L} \; ; \; g_\ell = 1$$

$$\vec{\mu}_s = -g_s \frac{\mu_B}{\hbar}\cdot\vec{S} \; ; \; g_s = 2 \quad (\text{P.A.M. Dirac})$$

g_ℓ and g_s : gyromagnetic ratios for angular momentum and spin: how much magnetic moment an orbiting (ℓ) or a spinning (s) charge can produce.

$g_s = 2.002{,}319{,}304{,}361{,}82 \pm 26$

Polykarp Kusch (Nobel, 1955)

Richard Feynman (Nobel, 1965)