PHY215-09: Atomic nucleus

C.-P. Yuan

April 25, 2025

1 Carbon-14 (^{14}C) dating

The key physics principles enabling carbon-14 (^{14}C) dating of a dead tree are rooted in radioactive decay, nuclear physics, and thermodynamics of carbon exchange. Here's a concise breakdown:

1. Radioactive Decay of ^{14}C

- Physics Principle: ^{14}C is an unstable isotope of carbon that undergoes β^- decay with a half-life of $\sim 5,730$ years:

$$^{14}\mathrm{C} \rightarrow ^{14}\mathrm{N} + e^- + \bar{\nu}_e$$

- Key Insight: The decay follows first-order kinetics (exponential decay), described by:

$$N(t) = N_0 e^{-\lambda t},$$

where $N(t)$ is the remaining ^{14}C atoms, N_0 is the initial amount, λ is the decay constant, and t is time since death.

2. Steady-State Production of ^{14}C in the Atmosphere

- Physics Principle: Cosmic rays (high-energy protons from the Sun) collide with atmospheric to produce neutrons which then collide with nitrogen (^{14}N) to produce ^{14}C via the nuclear reaction:

$$n + ^{14}\mathrm{N} \rightarrow ^{14}\mathrm{C} + p$$

- Cosmic rays are high-energy particles that originate from outer space. They mainly consist of protons (about 90%), alpha particles

(helium nuclei), and a small fraction of heavier nuclei. There are also electrons and gamma-ray photons in cosmic rays. When these high-energy cosmic ray particles, especially protons, enter the Earth's atmosphere, they collide with the nuclei of atoms in the atmosphere.
- The abundant molecules in Earth's atmosphere are:

- Nitrogen (N_2) — $\sim 78\%$ by volume.
- Oxygen (O_2) — $\sim 21\%$ by volume.
- Water vapor (H_2O) — Variable ($0.01 - 4\%$), depending on location and weather.
- Carbon dioxide (CO_2) — $\sim 0.04\%$ (420+ ppm) and increasing.

Trace gases like argon (Ar), neon (Ne), and methane (CH_4) are also present in smaller amounts.
- One of the most important processes for neutron production is the spallation reaction. The spallation process can be thought of as a "shattering" of the target nucleus. The incident cosmic ray particle transfers so much energy to the nucleus that it becomes unstable and fragments into smaller nuclei and individual nucleons (protons and neutrons). The neutrons produced in this way can then go on to participate in other nuclear reactions in the atmosphere, such as the production of radioactive isotopes like carbon-14.
- Balance: Production and decay of ^{14}C in the atmosphere maintain a constant ratio of $^{14}C/^{12}C$ ($\approx 1.3 \times 10^{-12}$) in carbon dioxide ($CO_2$).

3. Carbon Exchange in Living Organisms

- Physics/Thermodynamics: Living organisms (e.g., trees) continuously exchange carbon with the environment via photosynthesis (for plants) or metabolism (for animals).
- Equilibrium: While alive, the $^{14}C/^{12}C$ ratio in the organism equals the atmospheric ratio because carbon is constantly replenished.

4. Decay After Death (Closed System)

- Physics Principle: When the tree dies, it ceases carbon exchange with the environment, becoming a closed system for carbon.
- Key Observation: The ^{14}C in the dead wood decays without replenishment, while the stable ^{12}C remains unchanged. The ratio $^{14}C/^{12}C$ decreases exponentially over time.

5. Measurement and Age Calculation

- Physics Tool: By measuring the remaining $^{14}C/^{12}C$ ratio in the dead wood and comparing it to the known atmospheric ratio, the time since death (t) is calculated using the decay law:

$$t = \frac{1}{\lambda} \ln \left(\frac{N_0}{N(t)} \right) = \frac{\text{half-life}}{\ln 2} \ln \left(\frac{\text{initial ratio}}{\text{measured ratio}} \right).$$

Critical Assumptions (Underlying Physics Context):

- The atmospheric $^{14}C/^{12}C$ ratio has been approximately constant over time (adjustments are made for known variations, e.g., nuclear testing or solar activity affecting cosmic ray flux).

- No significant contamination or fractionation (isotope separation) of carbon has altered the sample's ratio after death.

These principles rely on the conservation of mass-energy (via radioactive decay), nuclear reaction dynamics (cosmic ray interactions), and thermodynamic equilibrium (carbon exchange in living systems). Together, they form the basis of radiocarbon dating.