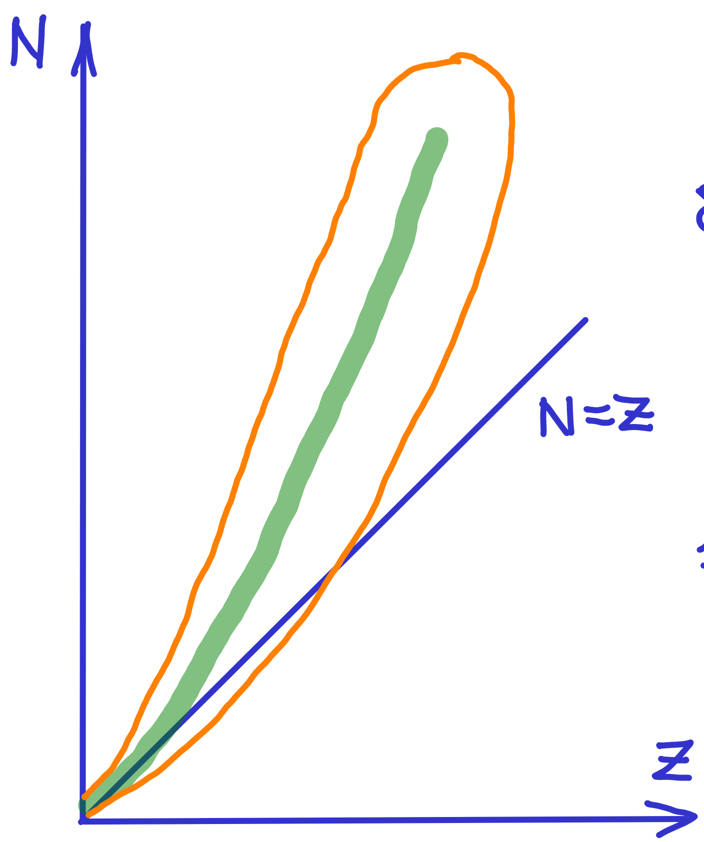
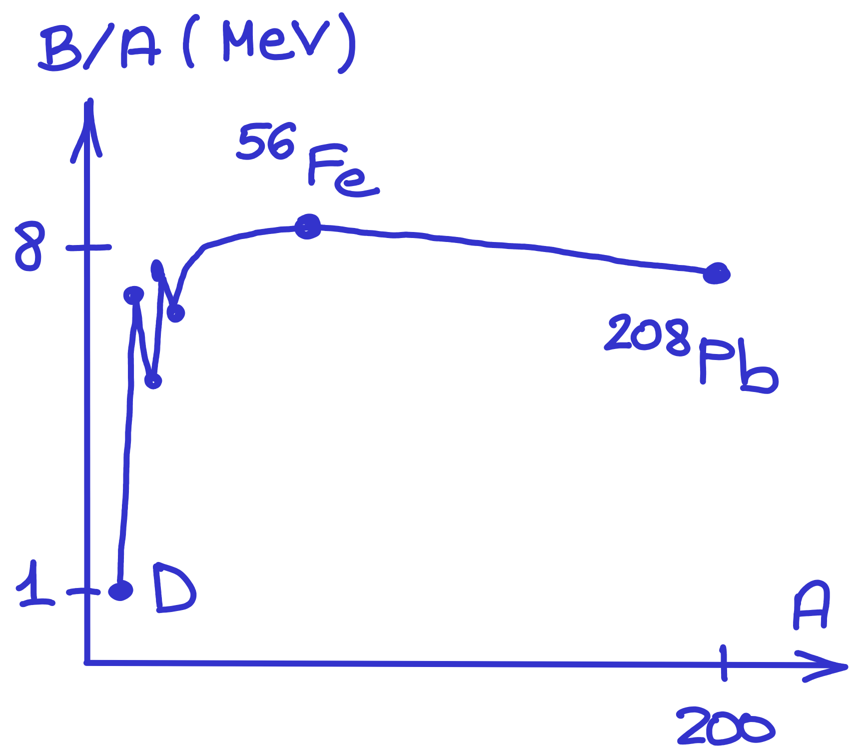
Binding energy and stability

Binding energy:

$$B({}^{A}_{Z}X_{N}) = [\underbrace{N m_n + Z M({}^{1}H)}_{\text{parts}} - \underbrace{M({}^{A}_{Z}X_{N})}_{\text{whole}}]c^2$$

the energy required to separate the nucleus into free neutrons and protons.





The von Weizsäcker formula

$$B({}^{A}_{Z}X_N) = a_V \cdot A - a_A \cdot A^{2/3} - \frac{0.72 \cdot Z(Z-1)}{A^{1/3}} - a_S \frac{(N-Z)^2}{A} + \delta$$

$a_V = 15.8$ MeV : volume

$a_A = 18.3$ MeV : surface

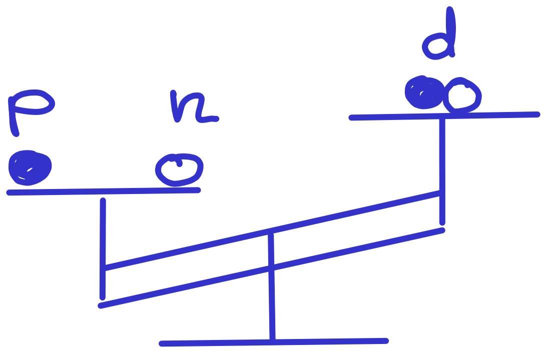
$a_S = 23.2$ MeV : symmetry

δ : pairing

$$\delta = \begin{cases} +\Delta & : \text{even-even} \\ 0 & : \text{even-odd, odd-even} \\ -\Delta & : \text{odd-odd} \end{cases}$$

$\Delta = 33\text{ MeV} \cdot A^{-3/4}$

The deuteron



$\rightsquigarrow \gamma : B_d$

B_d : binding energy of the deuteron

$$m_p + m_n = m_d + \frac{B_d}{c^2} \qquad \Big/ + m_e$$

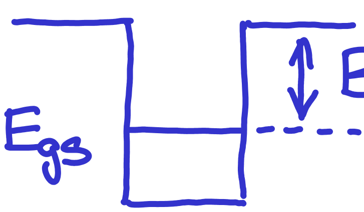
$$\underbrace{(m_p + m_e)}_{{}^1\text{H atom}} + m_n = \underbrace{(m_d + m_e)}_{{}^2\text{H} = \text{D atom}} + \frac{B_d}{c^2}$$

$$M({}^1H) + m_n = M({}^2H) + \frac{B_d}{c^2}$$

m_p, m_n, m_d : nuclear masses

$M({}^1H), M({}^2H)$: atomic masses

(Electron binding energies are negligible compared to nuclear energies.)



B_d : 2.224 MeV

The deuteron doesn't have excited states.

If $E_\gamma = hf > B_d$, then $\gamma + d = p + n$.

Disintegration energy

$${}^{A}_{Z}X_{N} \rightarrow D + y + Q$$

- ${}^{A}_{Z}X_{N}$: parent
- D : daughter
- y : lighter product
- Q : energy

$$Q = \left[M({}^{A}_{Z}X_{N}) - M(D) - M(y) \right] c^2$$

Q : initial minus final mass-energy

B : final minus initial mass-energy

$$\boxed{Q = -B}$$

If $B > 0$, then the nucleus is stable.

If $Q > 0$, then the nucleus is unstable, and it may decay.

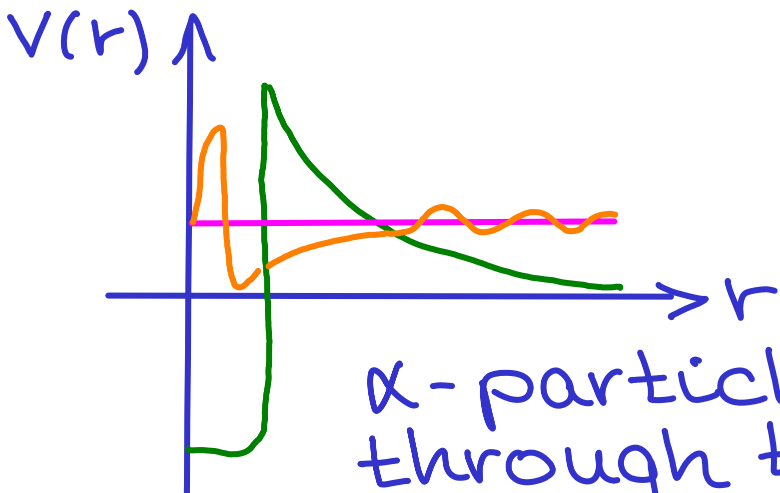
Alpha decay

$$^{A}_{Z}X_{N} \rightarrow \, ^{A-4}_{Z-2}D_{N-2} + \alpha \qquad (\alpha = \, ^{4}_{2}He_{2})$$

$$Q = \left[M(^{A}_{Z}X_{N}) - M(^{A-4}_{Z-2}D_{N-2}) - M(^{4}_{2}He_{2}) \right] c^2$$

Example: $^{230}_{92}U \rightarrow \, ^{226}_{90}Th + \alpha$

$$Q = 6 \text{ MeV}$$

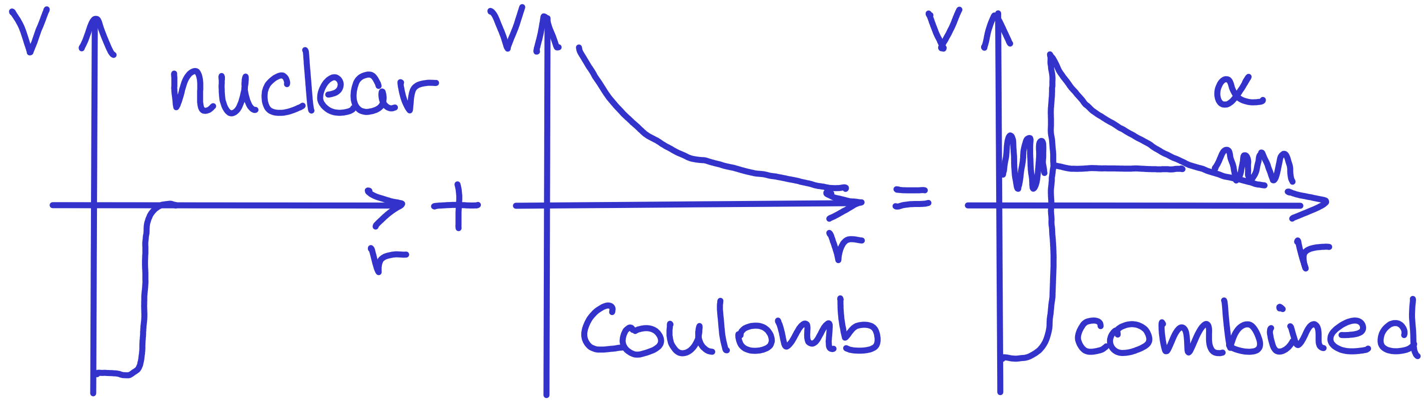


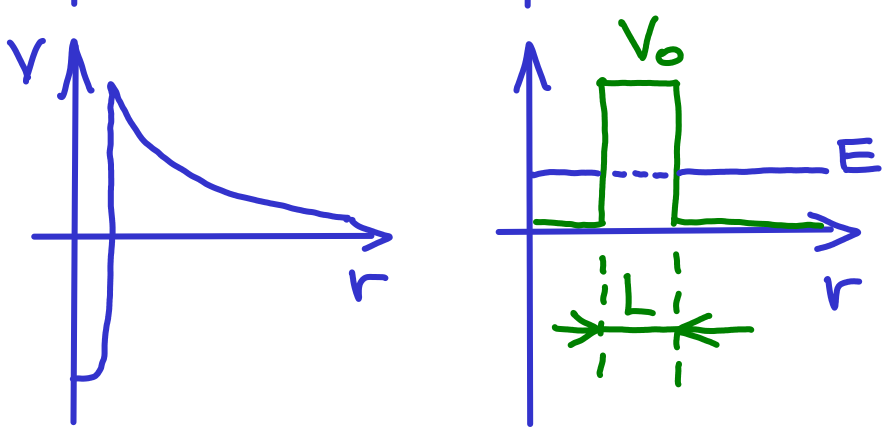
α-particle tunneling through the Coulomb barrier.

Large α energy $\leftrightarrow$ short half life.

Small α energy $\leftrightarrow$ long half life.

Alpha particle tunneling





$V_0 = 13.7\,\text{MeV}$

$E = 4.10\,\text{MeV}$

$L = 12.2\,\text{fm}$

$$m_\alpha = 3727.38\ \text{MeV}/c^2$$

$$\hbar c = 197.33\ \text{eV nm}$$

$$T = 16 \cdot \frac{E}{V_0}\left(1 - \frac{E}{V_0}\right) \cdot e^{-2\kappa L}$$

$$\kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar} = \frac{\sqrt{2mc^2(V_0 - E)}}{\hbar c} =$$

$$= \frac{\sqrt{2 \cdot 3727.38 \cdot (13.7 - 4.10)(\text{MeV})^2}}{197.33\ \text{eV nm}} =$$

$$= \frac{267.5\ \text{MeV}}{197.33\ \text{eV nm}} = \frac{267.5\ \text{eV}}{197.33\ \text{eV fm}} =$$

Alpha particle tunneling (2)

$= 1.356 \frac{1}{fm}$

$2kL = 33.08$

$e^{-2kL} = 4.306 \cdot 10^{-15}$

$16 \cdot \frac{E}{V_0}\left(1 - \frac{E}{V_0}\right) = 16 \cdot \frac{4.10}{13.7}\left(1 - \frac{4.10}{13.7}\right) =$

$= 3.355$

$T = 1.445 \cdot 10^{-14}$