Special Relativity

$$\underbrace{\beta = \frac{v}{c}}_{\text{speed}} \qquad \underbrace{\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}_{\text{Lorentz gamma factor}}$$

$$L = \frac{L_0}{\gamma}$$

length contraction

L_0 : proper length

$$T = \gamma T_0$$

time dilation

T_0 : proper time

$$E_0 = m_0 c^2$$

m_0 : rest mass

E_0 : rent energy

$E = \gamma E_0$: total relativistic energy

$K = (\gamma - 1) E_0$: relativistic kinetic en.

$\vec{p} = \gamma m_0 \vec{u}$: relativistic momentum

$$E^2 = p^2c^2 + m_0^2 c^4 \qquad \frac{p}{E} = \frac{\beta}{c}$$

Relativistic velocity addition (in the longitudinal direction):

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}} \qquad \beta = \frac{B + \beta'}{1 + B\beta'}$$

Special Relativity 2.

Relativistic Doppler effect
(in the longitudinal direction):

$$f_{\uparrow} = f_0 \sqrt{\frac{c+v}{c-v}} = \sqrt{\frac{1+\beta}{1-\beta}}$$

$$f_{\downarrow} = f_0 \sqrt{\frac{c-v}{c+v}} = \sqrt{\frac{1-\beta}{1+\beta}}$$

$c = 3 \cdot 10^8$ m/s (299,792,458. m/s)

$c = \lambda \cdot f$

Thermodynamics 1.

Temperature scales:

$$T_K = T_{°C} + 273.15$$

$$\left(T_{°C} = \frac{5}{9}(T_{°F} - 32) \; ; \; T_{°F} = \frac{9}{5} T_{°C} + 32 \right)$$

Thermal expansion:

$$\left.\begin{array}{ll} \Delta L = \alpha \cdot L_0 \cdot \Delta T & : \text{linear} \\ \Delta V = \beta \cdot V_0 \cdot \Delta T & : \text{volumetric} \end{array}\right\} \beta = 3\alpha$$

ΔT is in K or °C NOT in °F

Ideal gas law:

$$\boxed{\begin{array}{c} pV = nRT \\ \text{or} \\ pV = Nk_BT \end{array}}$$

$R = 8.31 \; J/(mol\,K)$

$k_B = 1.38 \times 10^{-23} \; J/K$

$N_A = 6.02 \times 10^{23} \; 1/mol$

$nR = Nk_B$ therefore $\boxed{R = N_A \cdot k_B}$

n: amount of substance in mol

N: number of atoms or molecules in 1

m: mass of one particle in kg

M: mass of one mole substance
or molar mass in kg/mol

$$\boxed{m \cdot N = n \cdot M} \Big\} \text{ mass of the system}$$

Thermodynamics 2.

STP: standard temperature and pressure:

$T = 0°C = 273K$

$p = 1atm = 101.3\,kPa = 1.013 \times 10^5\,Pa$

If $n = 1mol$, then $pV = nRT$ gives

$V = 0.0224\,m^3 = 22.4\,l$: molar volume

Kinetic theory:

$$v_{RMS} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{M}}$$ } this formula

has always 3 in it, because:

$$\bar{E}_{translation} = \underset{\text{x, y and z directions}}{3} \cdot \frac{1}{2}k_BT = \frac{3}{2}k_BT$$

But:

$\bar{E} = \frac{f}{2}k_BT$; $f =$ 3 : monoatomic; 5 : diatomic or linear; 6 : polyatomic

$U = E_{th} = N \cdot \bar{E} = \frac{f}{2}Nk_BT = \frac{f}{2}nRT$: internal or thermal energy of the gas containing N atoms or molecules.

Thermodynamics 3.

Heat:

$Q = C \cdot \Delta T$; C: heat capacity
$Q = c \cdot m \cdot \Delta T$; c: specific heat

$$c = \frac{C}{m}$$

$1\,\text{cal} = 4.1868\,\text{J}$ $\quad$ $1\,\text{Cal} = 1\,\text{kcal} = 4186.8\,\text{J}$

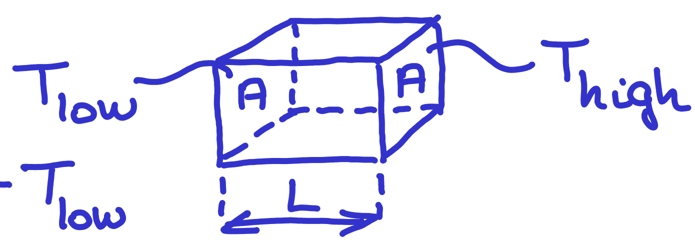
Phase transitions or transformations:

solid-liquid: $Q = L_F \cdot m$; L_F = heat of fusion
liquid-gas: $Q = L_V \cdot m$; L_V = heat of vaporization

Heat conduction:

$Q = k \cdot \frac{A}{L} \cdot \Delta T \cdot t$;

$P = Q/t$; $\Delta T = T_{high} - T_{low}$



Heat radiation:

$P = \sigma \varepsilon A T^4$; $\sigma = 5.67 \times 10^{-8}\ W/(m^2 K^4)$

$\lambda_{max} \cdot T = b$; $b = 2.898 \times 10^{-3}\ m \cdot K$

black-body: $a = 1$ and $\varepsilon = 1$

(Heat convection: way too complicated
Hot fluid rises.)

Thermodynamics 4.

First Law: $\Delta U = Q + W$ $\Big\}$ $W_{gas} = -W$, $Q_{gas} = -Q$

W is the work done ON the system (gas).
Q is the heat transferred To the system (gas).

Second Law: $\frac{Q}{T} \leq \Delta S$: the entropy of a closed system cannot decrease.

Molar heats for ideal gases:

$Q = C_v \cdot n \Delta T$: at constant volume

$Q = C_p \cdot n \Delta T$: at constant pressure

$C_v = \frac{f}{2} R$; $C_p = \frac{f+2}{2} R = C_v + R$

Ideal Gas processes:

→ isothermal : constant T $\longleftrightarrow$ $\Delta U = 0$

$$p_1 V_1 = p_2 V_2$$

→ isobaric : constant p

$$V_1 / T_1 = V_2 / T_2$$

→ isochoric : constant V $\longleftrightarrow$ $W = 0$

$$p_1 / T_1 = p_2 / T_2$$

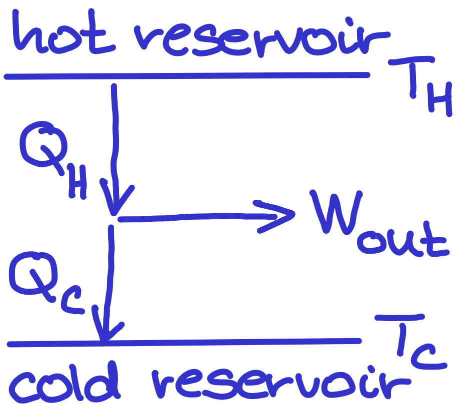
→ adiabatic : $Q = 0$ $\longleftrightarrow$ constant S

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma} \; ; \; \gamma = C_p / C_v = \frac{f+2}{f}$$

(isocaloric, iso-entropic or isentropic)

Thermodynamics 5.

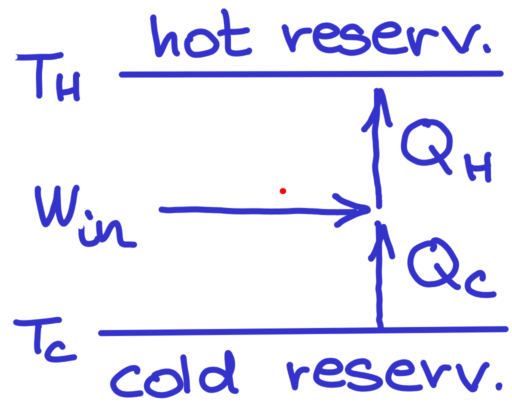
Heat engines



$$\boxed{Q_H = W + Q_c}$$

conservation of energy

refrigerators heat pump



The efficiency:

Engine:

$$\eta = \frac{W_{out}}{Q_H} = \frac{T_H - T_c}{T_H}$$

$$0 \le \eta < 1$$

Carnot

The coefficient of performance:

Refrigerator:

$$\kappa = \frac{Q_c}{W_{in}} = \frac{T_c}{T_H - T_c}$$

Heat pump:

$$\kappa = \frac{Q_H}{W_{in}} = \frac{T_H}{T_H - T_c}$$

Quantum Physics 1.

Black-body radiation:

Wien's displacement law:

$$\lambda_{max} \cdot T = b$$

$\hookrightarrow b = 2.898 \cdot 10^{-3}\ m \cdot K$

Stefan-Boltzmann law:

$$P = \sigma \varepsilon A T^4 \; ; \quad 0 \leq \varepsilon \leq 1 \; : \text{emissivity}$$

$\varepsilon = 1$ for black body

$\hookrightarrow \sigma = 5.6705 \cdot 10^{-8} \ \frac{W}{m^2 K^4}$

Planck's radiation curve:

$$I_\lambda = \frac{2\pi c^2 h}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

$$I_f = \frac{2\pi h}{c^2} \cdot \frac{f^3}{e^{\frac{hf}{k_B T}} - 1}$$

Key assumption: energy quantum:

$$\Delta E = h \cdot f$$

$\hookrightarrow h = 6.6261 \cdot 10^{-34}\ Js$

Planck constant

Quantum Physics 2.

Rydberg formula for the H-atom:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n^2} - \frac{1}{k^2} \right) \qquad n = 1, 2, 3, 4, \ldots \quad k > n$$

$$\hookrightarrow R_H = 1.096776 \cdot 10^7 \ 1/m$$

Einstein's formula for the photo-electric effect:

$$hf = \varphi + eV_0 \qquad \varphi: \text{work function}$$

$$hf = \varphi + \frac{1}{2}mv^2 \qquad V_0: \text{stopping pot.}$$

$\frac{1}{2}mv^2$: kinetic energy of the photo-electron, small, only a couple of eV.

X-ray production by bremsstrahlung:

$$eV_0 = h \cdot f_{max} = \frac{hc}{\lambda_{min}} \qquad (c = \lambda \cdot f)$$

Compton-effect:

Energy conservation:

$hf + m_e c^2 = hf' + E_e$; $E_e = m_e c^2 + K$

$f = \frac{c}{\lambda}$

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

Compton wavelength:

$$\lambda_c = \frac{h}{m_e c} = \frac{hc}{m_e c^2} = \frac{1240 \text{ eVnm}}{511{,}000 \text{ eV}} = 2.4263 \text{ pm}$$

Quantum Physics 3.

Photon:

$$\left.\begin{array}{l} E = hf \\ c = \lambda f \end{array}\right\} \Rightarrow E = \frac{hc}{\lambda}$$

$$E^2 = p^2c^2 \Rightarrow E = pc \Rightarrow p = \frac{E}{c}$$

$$\Rightarrow p = \frac{h}{\lambda}$$

De Broglie: electron wave/matter wave:

$\lambda = \frac{h}{p}$: de Broglie wavelength

Bragg's law:

$$n \cdot \lambda = 2d \cdot \sin\theta \qquad n = 1, 2, 3, 4, \ldots$$

Bohr model:

$$E_{n,Z} = -\frac{Z^2}{n^2} \cdot E_0 \quad ; \quad E_0 = 13.6\,\text{eV}$$

$$r_{n,Z} = \frac{n^2}{Z} \cdot a_0 \quad ; \quad a_0 = 0.529\,\text{Å}$$

$$\beta_{n,Z} = \frac{Z}{n} \cdot \alpha \quad ; \quad \alpha \cong \frac{1}{137}$$

E_0 : ionization energy of the Hydrogen atom

a_0 : Bohr radius

α : fine structure constant

Wave mechanics 1.

Schrödinger equation (time indep.):

$$-\frac{\hbar^2}{2m} \cdot \frac{d^2\Psi(x)}{dx^2} + V(x)\cdot\Psi(x) = E\cdot\Psi(x)$$

$E \in \mathbb{R}$: energy of the system

Normalization:

$$1 = \int_{-\infty}^{\infty} \rho(x)\,dx = \int_{-\infty}^{\infty} |\Psi|^2\,dx$$

Probability density: $\rho(x) = |\Psi(x)|^2$

Heisenberg Uncertainty Principle:

$$\Delta x \cdot \Delta p_x \geq \hbar/2$$

$$\Delta y \cdot \Delta p_y \geq \hbar/2$$

$$\Delta z \cdot \Delta p_z \geq \hbar/2$$

$$\Delta t \cdot \Delta E \geq \hbar/2$$

$$\hbar = \frac{h}{2\pi}$$

Wave mechanics 2.

Infinite square well

One dimension:

$$\psi_n(x) = \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{n\pi x}{L}\right) \; ; \quad n = 1, 2, 3, \ldots$$

$$E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2} = n^2 \cdot \frac{h^2}{8mL^2} = n^2 \cdot E_1$$

Three dimensions:

$$E_{n_1, n_2, n_3} = \frac{h^2}{8m}\left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2}\right)$$

Simple Harmonic Oscillator:

$$V(x) = \frac{1}{2} k x^2 \qquad (k: \text{spring constant})$$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega = \left(n + \frac{1}{2}\right) hf; \quad n = 0, 1, 2, \ldots$$

$$E_0 = \frac{1}{2}\hbar\omega : \text{zero point energy}$$

Hydrogen atom

Shell n	orbits	ℓ	m_ℓ	# of orbitals	# of $\bar{e}$
K $n=1$	1s	0	0	1	2
L $n=2$	2s 2p	0 1	0 $-1,0,1$	1 3 → 4	8
M $n=3$	3s 3p 3d	0 1 2	0 $-1,0,1$ $-2,-1,0,1,2$	1 3 5 → 9	18
N $n=4$	4s 4p 4d 4f	0 1 2 3	0 $-1,0,1$ $-2,-1,0,1,2$ $-3,-2,-1,0,1,2,3$	1 3 5 7 → 16	32
O $n=5$	5s 5p 5d 5f 5g	0 1 2 3 4	0 $-1,0,1$ $-2,-1,0,1,2$ $-3,-2,-1,0,1,2,3$ $-4,-3,-2,-1,0,1,2,3,4$	1 3 5 7 9 → 25	50

$n = 1, 2, 3, \ldots$ (K, L, M, N, O, P...)

$\ell = 0, 1, 2, \ldots (n-1)$ (s, p, d, f, g, h...)

$m_\ell = -\ell, \ldots, 0, \ldots \ell$: # : $(2\ell+1)$

$$m_s = \pm \frac{1}{2}$$

$$L = |\vec{L}| = \sqrt{\ell(\ell+1)} \cdot \hbar$$

Shell n has n different subshells, n^2 orbits and $2n^2$ electrons.

Spectroscopic symbols

$$n\ {}^{M}L_{J}$$

n : principal quantum number (often dropped)

L : orbital angular momentum (S, P, D, F, G, H)

M : multiplicity of the state

$M = 2S+1$; S : spin q. n.

J : total angular momentum quantum number

Examples:

Hydrogen ground state:

$\underset{1s^1}{\boxed{\uparrow}}$: $1\,{}^2S_{1/2}$ or ${}^2S_{1/2}$

Helium ground state:

$\underset{1s^2}{\boxed{\uparrow\downarrow}}$: $1\,{}^1S_0$ or ${}^1S_0$

Nuclear physics

Notation: $^{A}_{Z}X_N$; $A = Z + N$

$\alpha, \beta^+, \beta^-, \gamma, p, n, d, t$

Nuclear radius: $R = r_0 \cdot A^{1/3}$; $r_0 = 1.2\,fm$

Binding energy:

$$B(^{A}_{Z}X_N) = [Z \cdot M(^{1}H) + N \cdot m_N - M(^{A}_{Z}X_N)]\,c^2$$

$$1\,AMU = 1u = \frac{1}{12} M(^{12}_{6}C_6)$$

$$1\,AMU = 931.494061 \ MeV/c^2$$

Radioactivity:

$N = N_0 \cdot e^{-\lambda t}$ $\qquad$ $R = R_0 \cdot e^{-\lambda t}$

$N = N_0 \cdot e^{-t/\tau}$ $\qquad$ $R = R_0 \cdot e^{-t/\tau}$

$N = N_0 \cdot 2^{-t/T_{1/2}}$ $\qquad$ $R = R_0 \cdot 2^{-t/T_{1/2}}$

$R = \lambda \cdot N$ $\qquad$ $R_0 = \lambda \cdot N_0$

$\lambda = \frac{1}{\tau}$; $T_{1/2} = \ln 2 \cdot \tau$; $\ln 2 = 0.6931$

Q-value: initial minus final

mass-energy : $Q = -B$