

# Thermodynamics

PHY 215  
Thermodynamics and  
Modern Physics

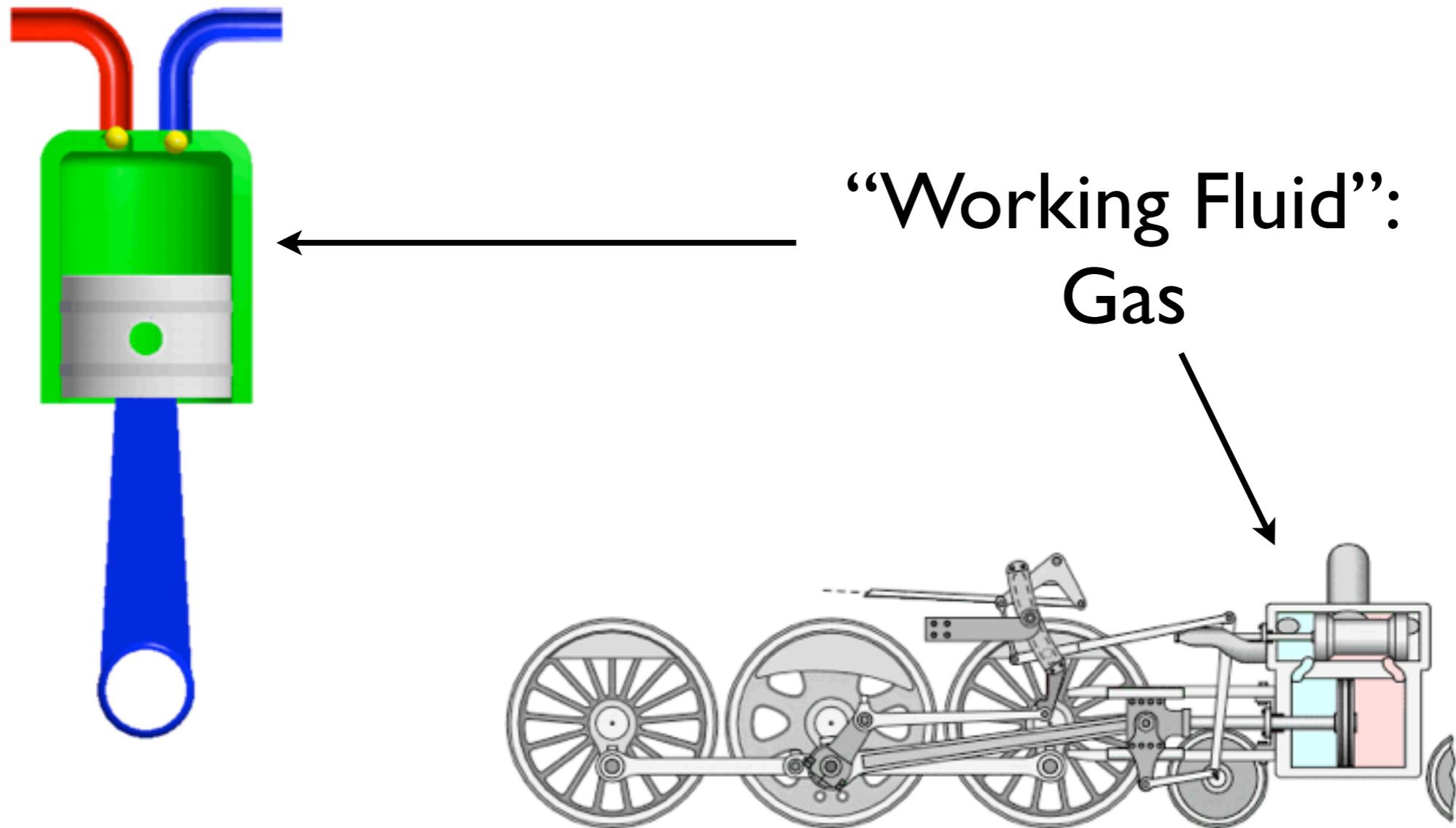
Spring 2026  
MSU

# Outline:

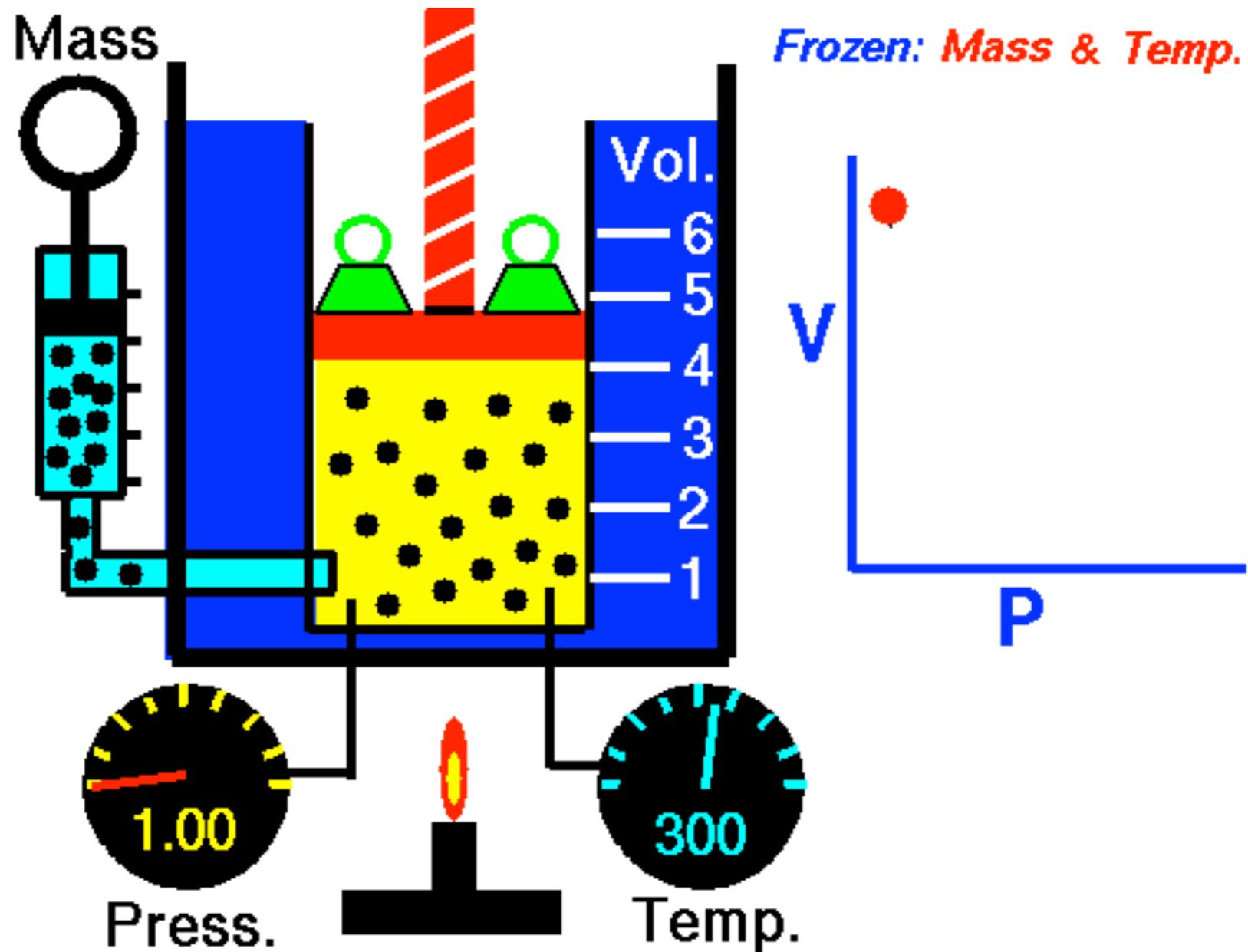
# The Ideal Gas Law

- Systems of interest: Gases
- Ideal Gas Laws
- Kinetic Theory of Gases
- Equipartition
- Molar Specific Heat of Gases

# Systems of Interest

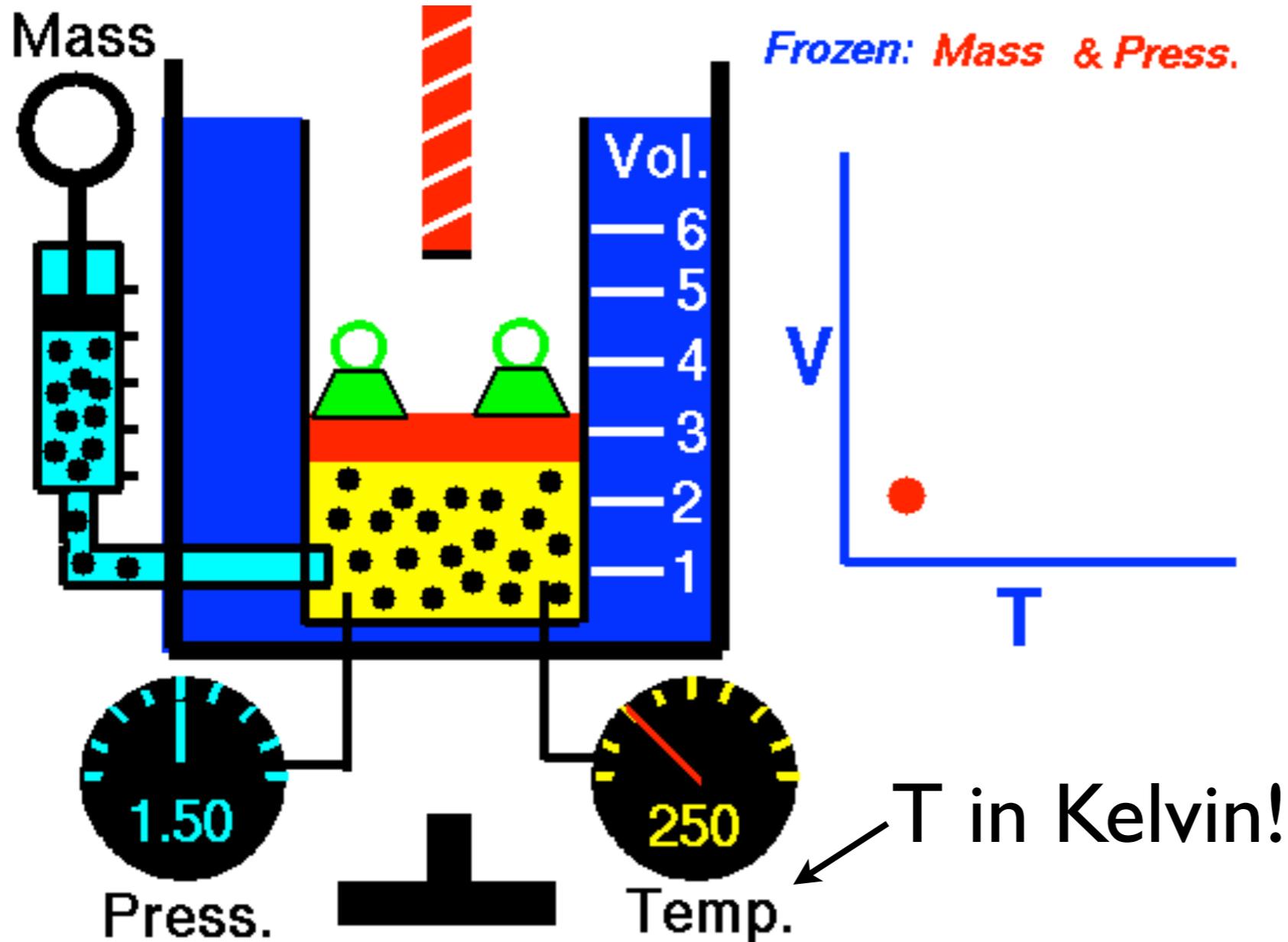


# Boyle's Law



$$pV = \text{const, at fixed } T \text{ and } n$$

# Charles's Law



$V \propto T$ , at fixed  $P$  and  $n$

$P \propto T$  at fixed  $V$   
Guy-Lussac's Law

# Combined

## THE IDEAL GAS LAW

$$PV = nRT \leftarrow \text{“Equation of State”}$$

where  $n$  is the number of moles of gas  
and  $R$  is the gas constant,  
 $R = 8.31 \text{ J}/(\text{mol} \cdot \text{K})$

One mole of an ideal gas at  
standard temperature ( $0^\circ\text{C}$ ) and  
pressure (1 atm) (“STP”)  
occupies approx. **22.4 liters**.

$$\begin{aligned} [p] &= 1 \text{ Pa} = 1 \text{ N}/\text{m}^2 \\ [V] &= 1 \text{ m}^3 = 10^3 \text{ l} \\ [T] &= ^\circ \text{K} \\ [n] &= \text{moles} \end{aligned}$$

# Molecular Version of Ideal Gas Law

Alternate form:

$$PV = NkT$$

where  $N$  is the number of molecules in  
the gas

and  $k$  is the Boltzmann's constant,

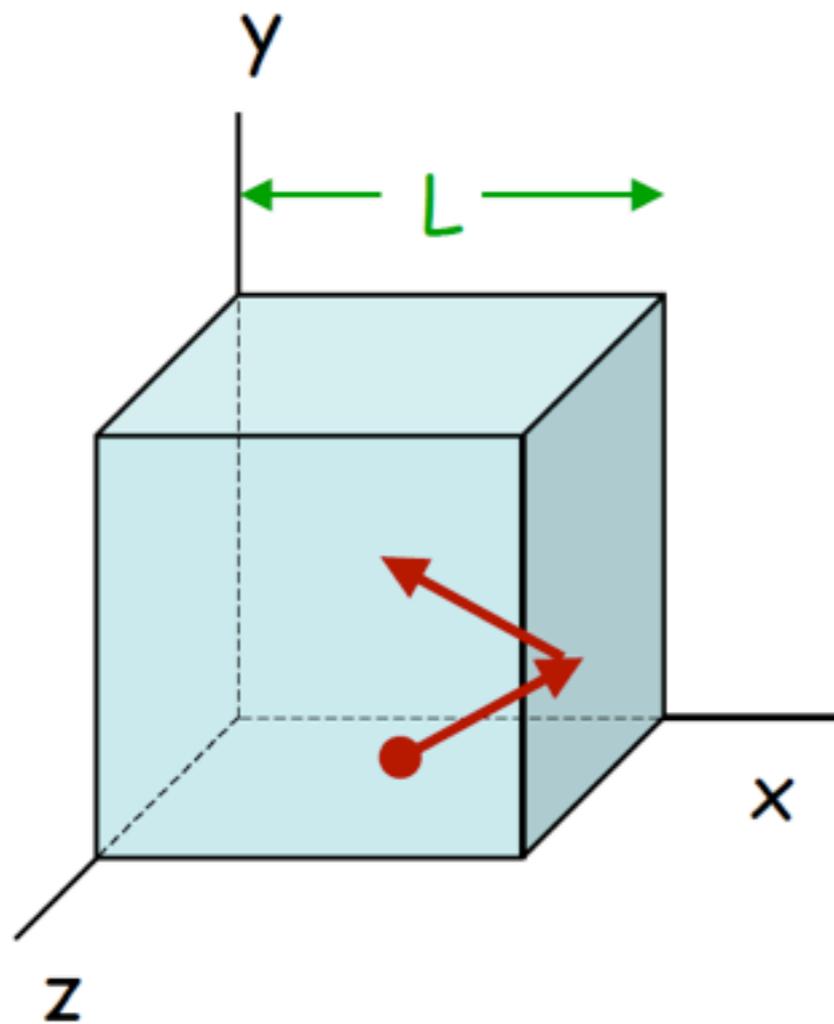
$$k = 1.38 \times 10^{-23} \text{ J/K}$$

$$N_A = \text{Avagadro's Number} \\ = 6.022 \times 10^{23}$$

(Comparing the two forms gives  $R = N_A k$ .)

# Kinetic Theory of Gases

Consider  $N$  molecules ( $n$  moles with  $n = N/N_A$ ) in a cubical box of side  $L$ , i.e.  
Volume =  $L^3$ .



Change in momentum  
at the x-wall is

$$\Delta p_x = 2 m v_x$$

Time between  
collisions with the  
x-wall is

$$\Delta t = 2 L / v_x$$

# Origin of Pressure

Average rate of change of momentum in x-direction:

$$\Delta p_x / \Delta t = (2mv_x) / (2L/v_x) = m v_x^2 / L$$

This is force exerted by the molecule.

$$\text{Total Force} = \sum_{i=1}^N (m v_x^2)_i / L$$

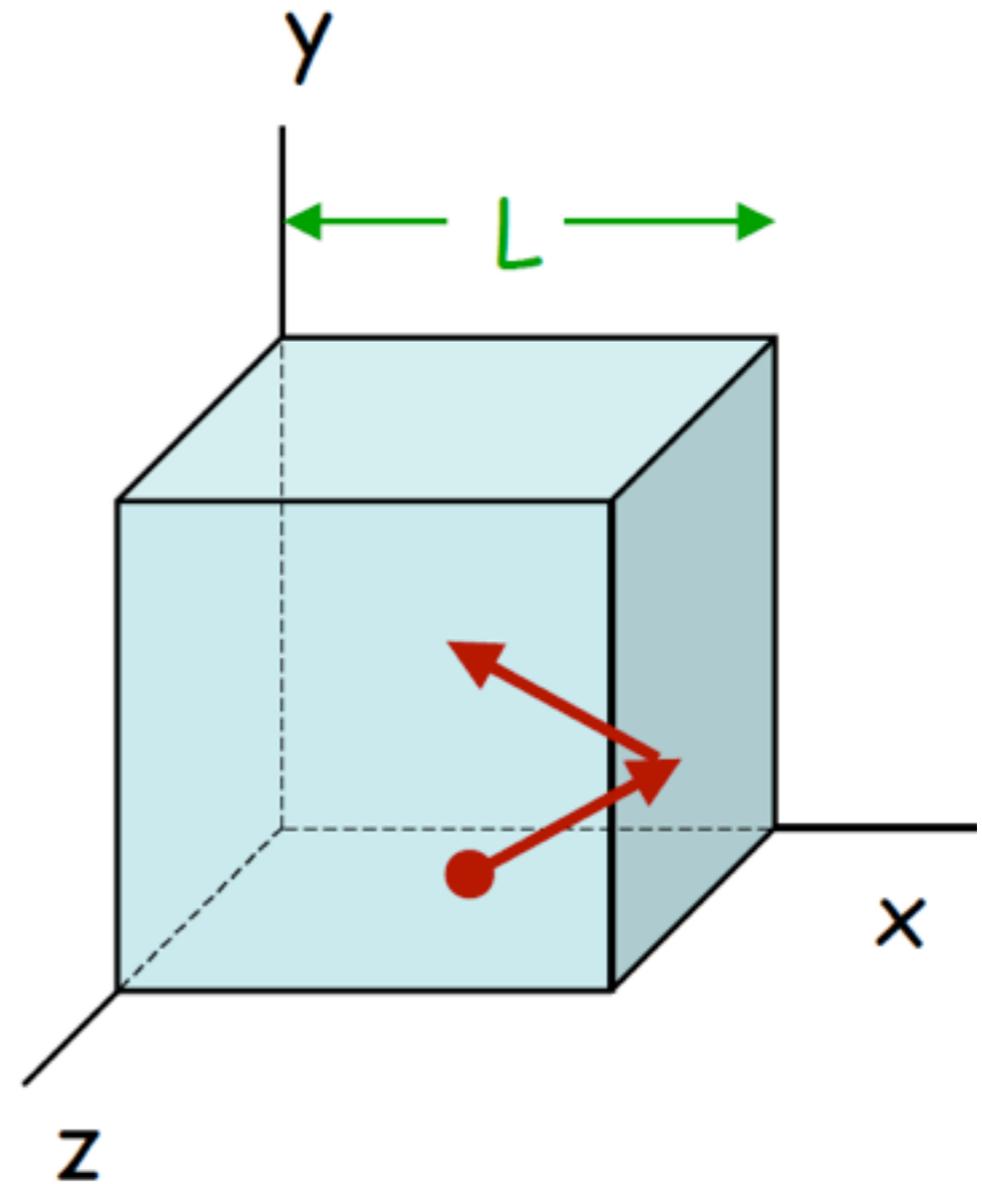
$$\begin{aligned} \text{Pressure } P &= \text{Force/Area} = F/L^2 \\ &= (m/L^3) \sum (v_x^2)_i \end{aligned}$$

$$\Rightarrow P = (m/L^3) N \langle v_x^2 \rangle$$

average

$mN = nM$  is the total mass.

where  $n = \#$  of moles  
 $M =$  molar mass



$$P = (nM/V) \langle v_x^2 \rangle$$

# Relation to Gas Law

For any molecule:  $v^2 = v_x^2 + v_y^2 + v_z^2$

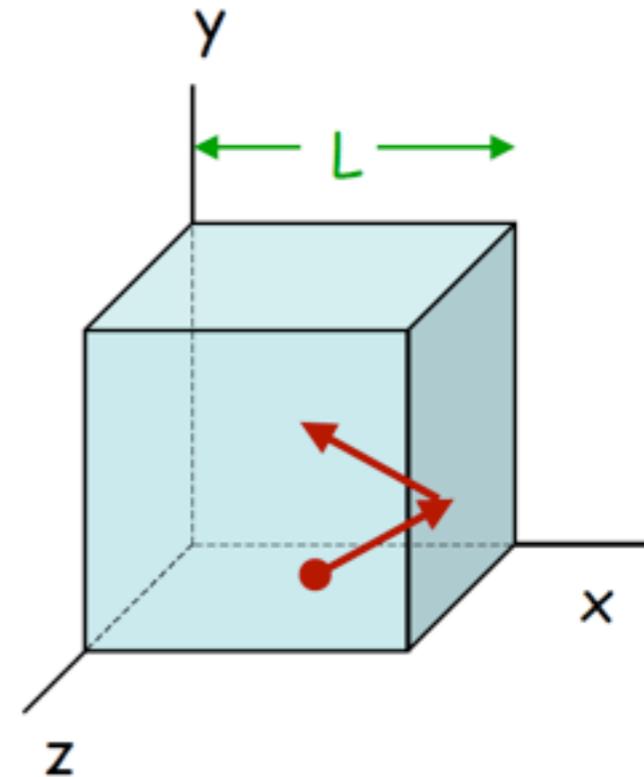
$$\Rightarrow \langle v_x^2 \rangle = (1/3) \langle v^2 \rangle$$

$$\Rightarrow P = (nM/3V) \langle v^2 \rangle$$

Define *root-mean-square speed*  $v_{rms}$ :

$$v_{rms} = \sqrt{\langle v^2 \rangle}$$

$$\Rightarrow PV = (nM/3) v_{rms}^2$$



From ideal gas law:  $PV = nRT$

$$\Rightarrow (nM/3) v_{rms}^2 = nRT$$

$$\Rightarrow v_{rms} = \sqrt{3RT/M}$$

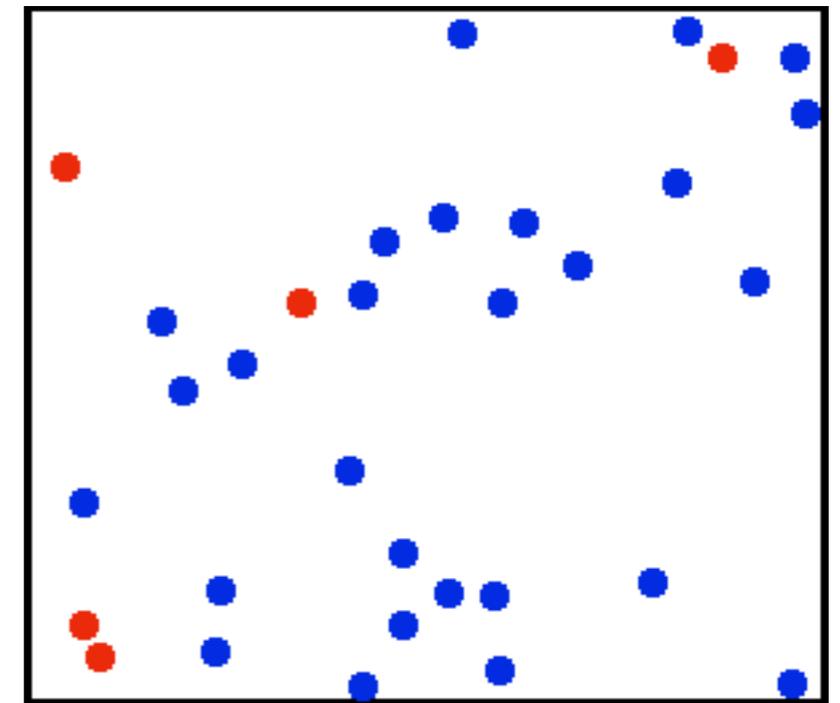
# Internal Kinetic Energy

Average (translational) kinetic energy per molecule

$$= (1/2) m \langle v^2 \rangle = (1/2) m (3RT/M)$$

Using  $M/m = N_A$ ,

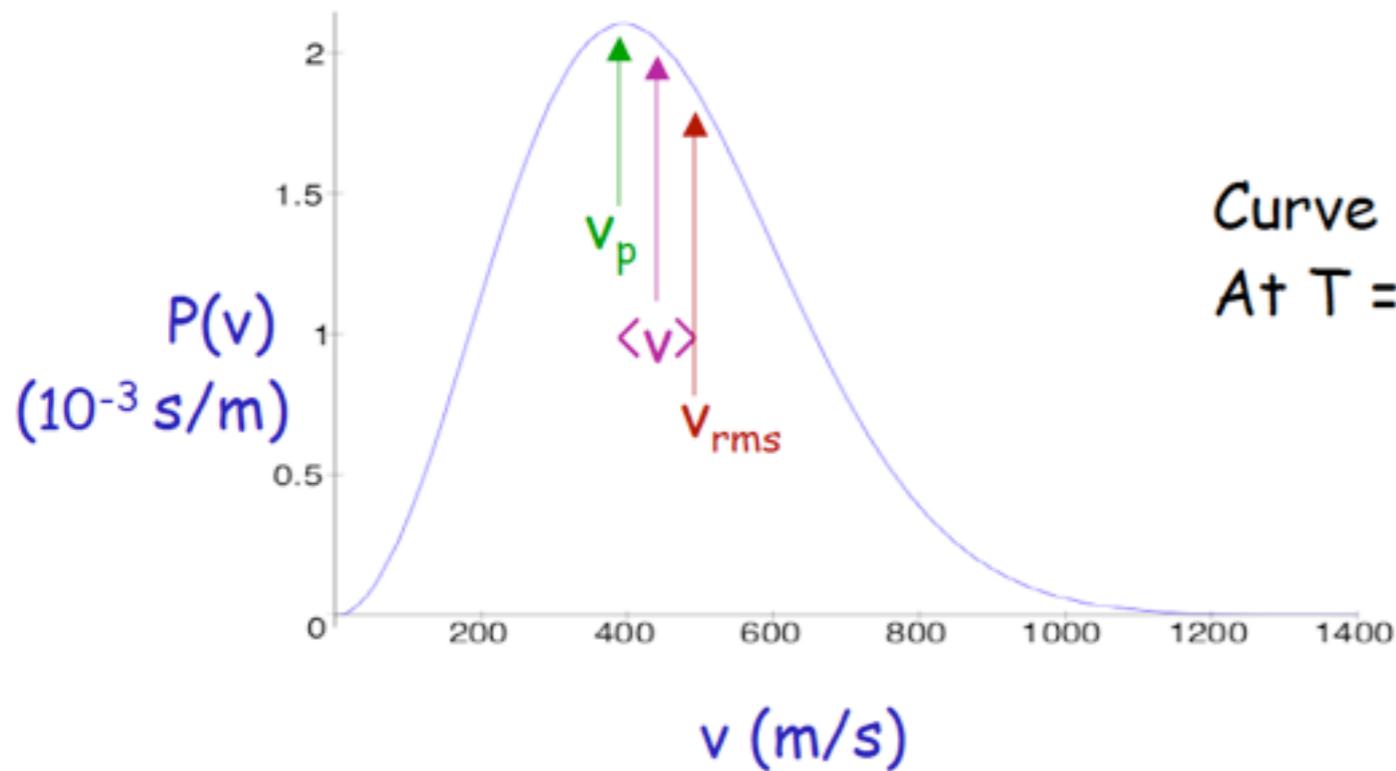
$$\langle K \rangle = 3RT/(2N_A) = (3/2) k T$$



$\Rightarrow \langle K \rangle = (3/2) k T$   $\leftarrow$   $(1/2)kT$  “per degree of freedom”  
 $(v_x, v_y, v_z)$

Temperature is a measure of the average kinetic energy of gas molecules!

# Maxwell Distribution



$$P(v) = 4\pi \left( \frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-\frac{Mv^2}{2RT}}$$

$e^{-E_{kin}/kT}$

$P(v) dv$  is the probability that a molecule has speed between  $v$  and  $v + dv$ .

# Internal Energy: U

- Monatomic gas - Single atoms:

$$U = N \left(\frac{3}{2}\right) kT = \left(\frac{3}{2}\right) nN_A kT = \left(\frac{3}{2}\right) nRT$$

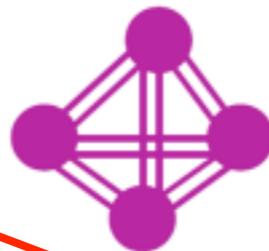
Each atom has 3 Degrees of Freedom.  
(K. E. in x, y, or z directions).

- Diatomic molecule:



Rotates (in two planes)  
 $\Rightarrow$  5 degrees of freedom.

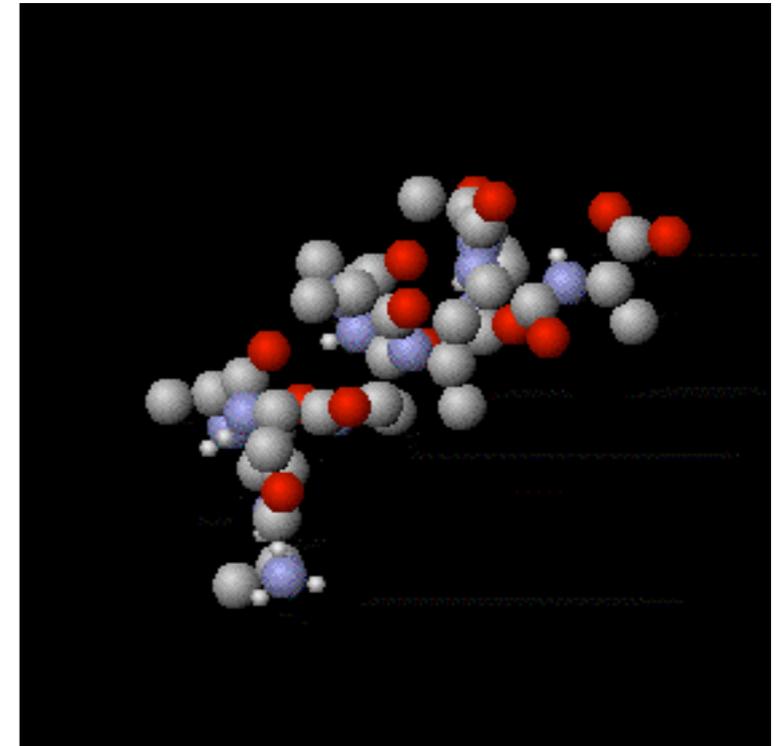
$$U = \left(\frac{5}{2}\right) nRT$$



- Polyatomic molecule:

Rotates in all 3 planes  
 $\Rightarrow$  6 degrees of freedom.  
(3 translational + 3 rotational).

$$U = \left(\frac{6}{2}\right) nRT = 3 nRT$$

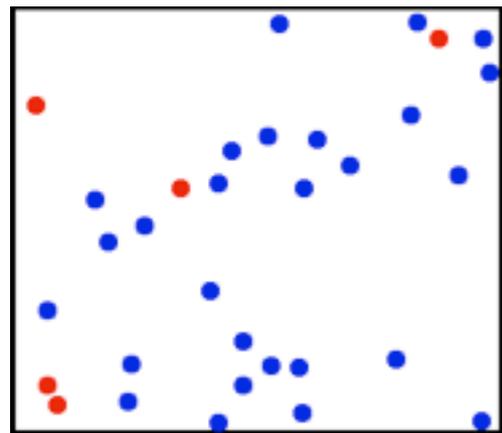


Vibrational  
degrees of freedom!

Ideal Gas: U is a  
Function of T only!

# Molar Specific Heat

at constant volume



←  $\Delta U$



←  $\Delta Q$

Monatomic Gas

$$\Delta Q = \Delta U = (3/2) n R \Delta T$$

$$\Delta Q = n C_V \Delta T$$

← molar specific heat

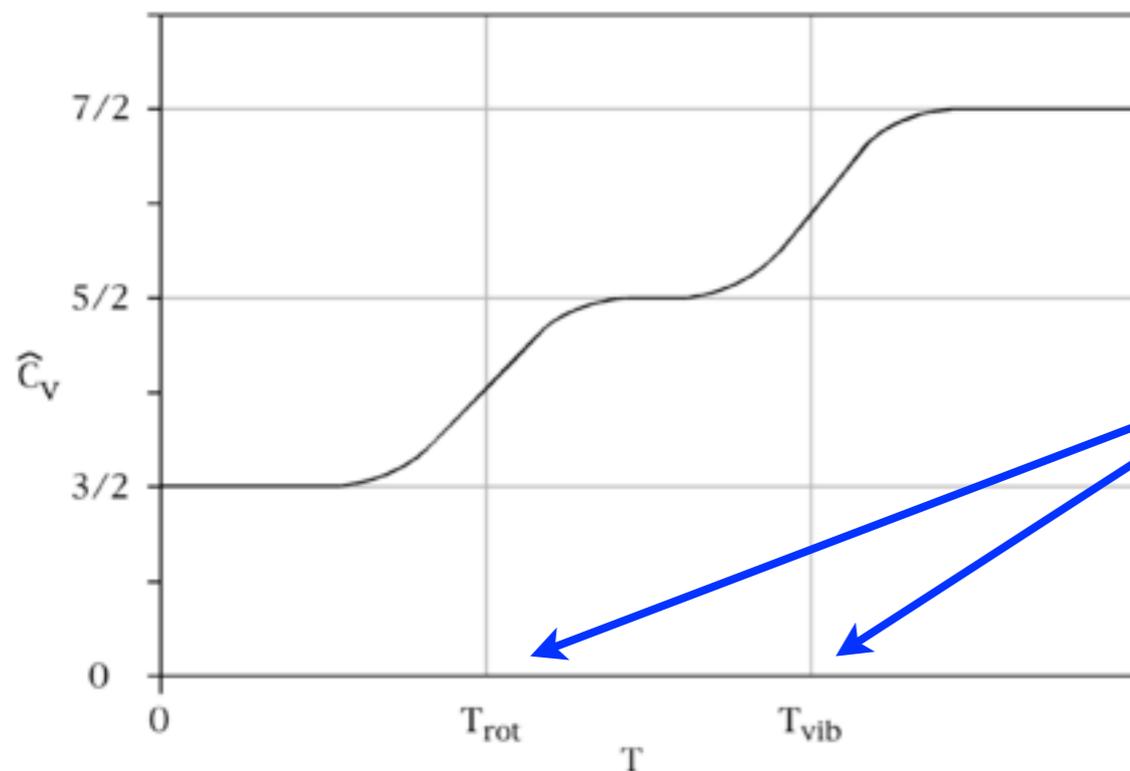
$$C_V = (3/2) R = 12.5 \text{ J}/(\text{mol} \cdot \text{K})$$

# Equipartition

General Case:  $C_v = (\# \text{ degrees of freedom}/2)R$

$$E = (\# \text{ degrees of freedom})(kT/2)$$

Diatomic Molecule



**Quantum Mechanics:**  
Energy Levels Quantized.  
Different modes “turn on”  
at different temperature.

# Summary

- Equation of state for a gas: the ideal gas law.
- Kinetic Theory of Gases: pressure and temperature are manifestations of the kinetic motion of gas molecules.
- Maxwell velocity distribution
- Internal energy of a gas: translational, rotational, and vibrational.
- (Molar) specific heat ( $C_v$ ):  $(1/2)R$  per “active” degree of freedom (dof).
- Internal Energy  $U$ :  $(1/2) nRT$  per dof.