PHY215-05: Atomic Model

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1 Periodic Table

The periodic table is organized by atomic number (Z) rather than atomic mass (A) due to a critical discovery in the early 20th century that revolutionized our understanding of elements. Here's the breakdown:

1.1 Mendeleev's Original Approach (Atomic Mass)

- When Dmitri Mendeleev created the first periodic table in 1869, he ordered elements by atomic mass (A). This worked well for most elements, as similar masses often correlated with similar chemical properties.

- However, inconsistencies arose. For example:

- Tellurium (Te) has a higher atomic mass than iodine (I), but Mendeleev placed Te before I because their chemical properties aligned better with the table's structure. This contradicted the "atomic mass" rule.

- Isotopes (atoms of the same element with different masses) later complicated this approach, as they showed that atomic mass alone isn't unique to an element.

1.2 The Discovery of Atomic Number (Z)

- In 1913, Henry Moseley conducted experiments using X-ray spectroscopy and found that each element emits X-rays at a unique frequency. This frequency corresponds to the number of protons in the nucleus (Z), which determines an element's identity.

- Moseley's work proved that atomic number (Z) is the fundamental property that defines an element, not atomic mass. This resolved Mendeleev's inconsistencies:

- Te (Z=52) comes before I (Z=53), matching their chemical behavior.

- Isotopes (e.g., carbon-12 and carbon-14) share the same Z but different A, so they belong in the same position on the table.

3. Why Z Works Better Than A

- Chemical Properties: The arrangement of electrons (determined by Z) dictates an element's chemical behavior. Elements in the same group (column) have similar electron configurations, explaining their shared properties.

- Periodic Law: The modern periodic law states that elements' properties repeat periodically when ordered by Z. This reflects the filling of electron shells, which drives periodic trends (e.g., ionization energy, electronegativity).

- Predictive Power: Organizing by Z allows accurate predictions of missing elements (e.g., germanium, discovered after Mendeleev's time) and their properties.

1.3 Why Mendeleev Used Atomic Mass

- In Mendeleev's era, techniques to measure nuclear charge (Z) didn't exist. Atomic mass was the most reliable measurable property available.

- His table's success despite atomic mass limitations was a testament to his insight into chemical patterns, even before the discovery of subatomic particles.

1.4 Summary

The periodic table evolved from atomic mass to atomic number because Z directly reflects an element's nuclear charge and electron structure, which govern its chemical behavior. This shift resolved inconsistencies in Mendeleev's original table and laid the foundation for our modern understanding of chemistry.

2 Speed of light

In Maxwell's equations, the relationship between the speed of light c and the vacuum properties (such as permittivity and permeability) is given by:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}.$$

2.1 Derivation Process:

1. Maxwell's Equations: By combining the equations of electric and magnetic fields (Gauss's Law, Faraday's Law of Electromagnetic Induction, and Ampère-Maxwell Law), a wave equation can be derived.

2. Wave Equation: After eliminating the electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$, the resulting wave equation takes the form

$$\nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}.$$

3. Wave Speed: The solution to this wave equation describes electromagnetic waves propagating at speed $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$. Maxwell observed that this speed matches the experimentally measured speed of light, leading to the prediction that light is an electromagnetic wave.

2.2 Physical Significance:

- Vacuum Permittivity ε_0: Quantifies how electric fields interact with charges in a vacuum.

$$\varepsilon_0 \approx 8.854 \times 10^{-12}\,\mathrm{F/m} \approx 8.854 \times 10^{-12} C^2/(N \cdot m^2).$$

Note: $1\,\mathrm{F/m} = 1\,\mathrm{C}^2 \cdot \mathrm{N}^{-1} \cdot \mathrm{m}^{-2}$, for Farad per meter.

- Vacuum Permeability μ_0: Quantifies how magnetic fields interact with currents in a vacuum.

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m} = 4\pi \times 10^{-7} N/A^2.$$

(exact by definition in SI units). Note: $1\,\mathrm{H/m} = 1\,\mathrm{N} \cdot \mathrm{A}^{-2}$, for Henry per meter.

Their units and dimensions reflect their roles in Coulomb's law (ε_0) and Ampère's law (μ_0).

- Speed of Light c: Determined entirely by the electromagnetic properties of the vacuum, reflecting the propagation speed of electromagnetic interactions.

When we calculate $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$, we get $c \approx 2.998 \times 10^8 m/s$

2.3 Experimental Verification

- The electric force between two point charges q_1 and q_2, separated by a distance r, is

$$\mathbf{F} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{r},$$

attractive if opposite charges, repulsive if same charges.

- The magnetic force between two parallel wires, carrying current I with length L and separated by a distance d, is

$$\mathbf{F} = \frac{\mu_0 I^2 L}{2\pi d},$$

attractive if currents are parallel, repulsive if antiparallel.

Independent measurements of ε_0 and μ_0, when substituted into the formula, yield a value of c that closely matches the actual speed of light, confirming the correctness of Maxwell's theory.

This relationship unifies electromagnetic phenomena with optics and stands as a core achievement of classical electromagnetism.

3 Fine structure constant

The fine-structure constant (α) is a dimensionless fundamental constant that characterizes the strength of electromagnetic interactions. Its expression in terms of Planck's constant (h) and the speed of light (c) is:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0} \cdot \frac{1}{\hbar c},$$

where:
- e is the elementary charge,
- ε_0 is the vacuum permittivity,
- $\hbar = \frac{h}{2\pi}$ is the reduced Planck constant.
- Its current Value:

The latest CODATA (2018) recommended value of α is:

$$\alpha \approx 0.0072973525693 \quad (\text{approximately } \frac{1}{137.035999206}).$$

3.1 Running coupling constant

The fine-structure constant depends on energy in the context of quantum field theory. This phenomenon is known as "running coupling constants" and arises due to quantum corrections (virtual particle-antiparticle pairs) modifying the effective strength of electromagnetic interactions at different energy scales.

In quantum electrodynamics (QED), the effective value of α increases slightly with energy. This is because higher-energy interactions probe closer to the bare electron charge, where the screening effect of virtual electron-positron pairs is reduced.

$\alpha \sim \frac{1}{137}$ at low energy scale (about zero), and it runs to about $\frac{1}{128}$ at high energy scale (about 100 GeV).

Note that in terms of the absolute temperature Kelvin (K), $1\,\mathrm{eV} \sim 10^4 K$, so $1\,\mathrm{GeV} \sim 10^{13} K$.

4 Maxwell Equations

4.1 Differential form of Maxwell's equations

- **Gauss's law for electricity:**

$$\nabla \cdot \vec{D} = \rho$$

- Here, $\vec{D}$ is the electric displacement field, ρ is the free electric charge density. In a vacuum, $\vec{D} = \epsilon_0 \vec{E}$, where ϵ_0 is the permittivity of free space and $\vec{E}$ is the electric field. This equation relates the divergence of the electric displacement field to the electric charge density. It implies that the source of the electric displacement field is the electric charge.

- **Gauss's law for magnetism:**

$$\nabla \cdot \vec{B} = 0$$

- $\vec{B}$ is the magnetic flux density. This equation states that the divergence of the magnetic field is zero. It implies that there are no magnetic monopoles. Magnetic field lines always form closed loops.

- **Faraday's law of electromagnetic induction:**

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

- It shows that a time-varying magnetic field induces an electric field. The curl of the electric field is related to the negative rate of change of the magnetic field. This is the basis for the operation of generators and many other electromagnetic devices.

- **Ampere-Maxwell law:**

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

- $\vec{H}$ is the magnetic field intensity and $\vec{J}$ is the free current density. In a vacuum, $\vec{B} = \mu_0 \vec{H}$, where μ_0 is the permeability of free space. This equation shows that a magnetic field can be generated by a current (the $\vec{J}$ term) and a time-varying electric field (the $\frac{\partial \vec{D}}{\partial t}$ term).

4.2 Integral form of Maxwell's equations

- **Gauss's law for electricity:**

$$\oint_S \vec{D} \cdot d\vec{S} = \int_V \rho dV$$

- The surface integral of the electric displacement field over a closed surface S is equal to the total charge enclosed within the volume V bounded by the surface S.

- **Gauss's law for magnetism:**

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

- The surface integral of the magnetic flux density over any closed surface is zero, indicating that the magnetic field lines do not begin or end at a point (no magnetic monopoles).

- **Faraday's law of electromagnetic induction:**

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{S}$$

- The line integral of the electric field around a closed loop C is equal to the negative rate of change of the magnetic flux through the surface S bounded by the loop C.

- **Ampere - Maxwell law:**

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{S} + \frac{d}{dt} \int_S \vec{D} \cdot d\vec{S}$$

- The line integral of the magnetic field intensity around a closed loop C is equal to the sum of the current passing through the surface S bounded by the loop C and the rate of change of the electric displacement flux through the surface S.

4.3 Maxwell's wave equations in vacuum

In vacuum, where the charge density $\rho = 0$ and the current density $\vec{J} = 0$, the electric field $\vec{E}$ and the magnetic field $\vec{B}$ satisfy the following wave equations: - For the electric field:

$$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

- For the magnetic field:

$$\nabla^2 \vec{B} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

They can be derived from Maxwell's equations. The wave equations for $\vec{E}$ and $\vec{B}$ show that electromagnetic waves can propagate in vacuum with a speed $v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$, which is equal to the speed of light c.

5 Why green leaves appear green in sunlight

Here's an explanation of why green leaves appear green in sunlight using the concept of spectral lines.

5.1 Absorption and reflection of light

- Sunlight contains a continuous spectrum of wavelengths, including all the colors of the rainbow. When sunlight shines on a green leaf, the leaf absorbs most of the light in the red and blue regions of the spectrum. This absorption occurs because the chlorophyll and other pigments in the leaf have specific energy levels that match the energy of photons in these wavelengths. The absorbed light energy is used in photosynthesis.

- However, the leaf reflects light in the green region of the spectrum. The spectral lines corresponding to green wavelengths are not absorbed by the leaf's pigments but are instead scattered and reflected back to our eyes. As a result, we perceive the leaf as green.

5.2 Role of chlorophyll

- Chlorophyll, the main pigment in leaves, has a molecular structure that determines its absorption and reflection properties. It has a complex ring - like structure with conjugated double bonds, which gives it the ability to interact with light in a specific way. The energy differences between the electronic states of chlorophyll molecules match the energy of red and blue light photons, causing these wavelengths to be absorbed. Green light photons, on the other hand, do not match the energy differences as well, so they are not absorbed and are reflected.

5.3 Summary

In summary, the green color of leaves under sunlight is due to the selective absorption of certain wavelengths of light by the pigments in the leaf, particularly chlorophyll, and the reflection of green - wavelength light, which is what our eyes detect.