

Atomic Physics

PHY 215
Thermodynamics and
Modern Physics

Spring 2026
MSU

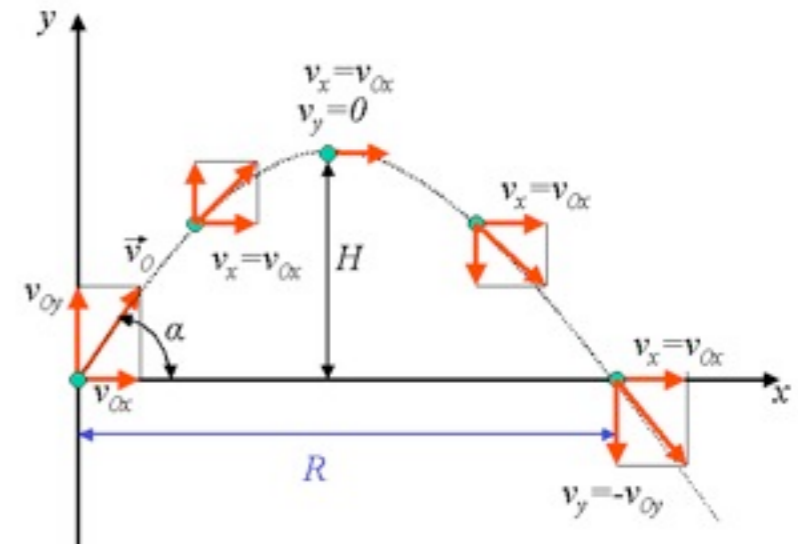
Outline

- Waves vs. Particles
 - Review of Wave Diffraction
- deBroglie Waves
- Davison and Germer
- Electron Double Slit
- **Electrons Behave Like Waves!**
- Complementarity

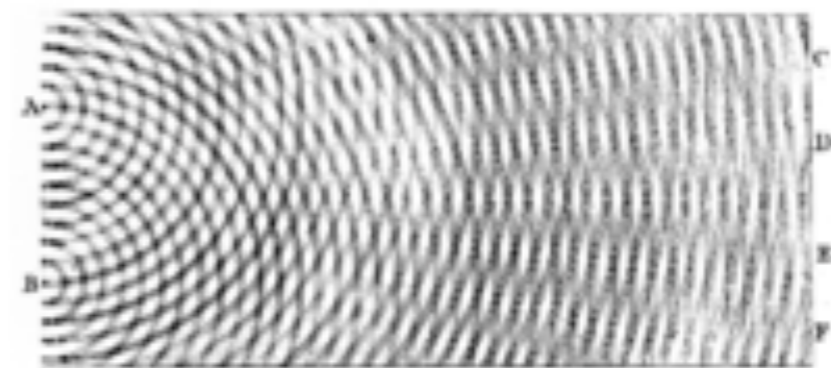
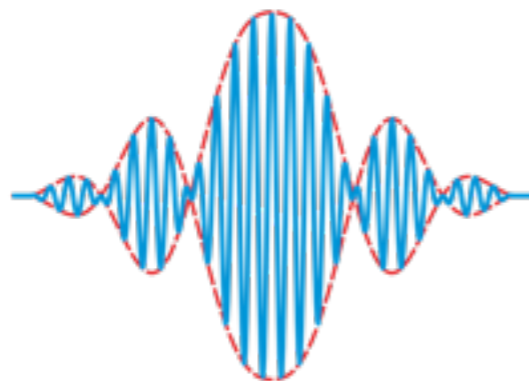
Particles vs. Waves

Property	Particle	Wave
Location	Definite	Indefinite
Momentum	Definite	Indefinite
Interference	No	Yes

Particle



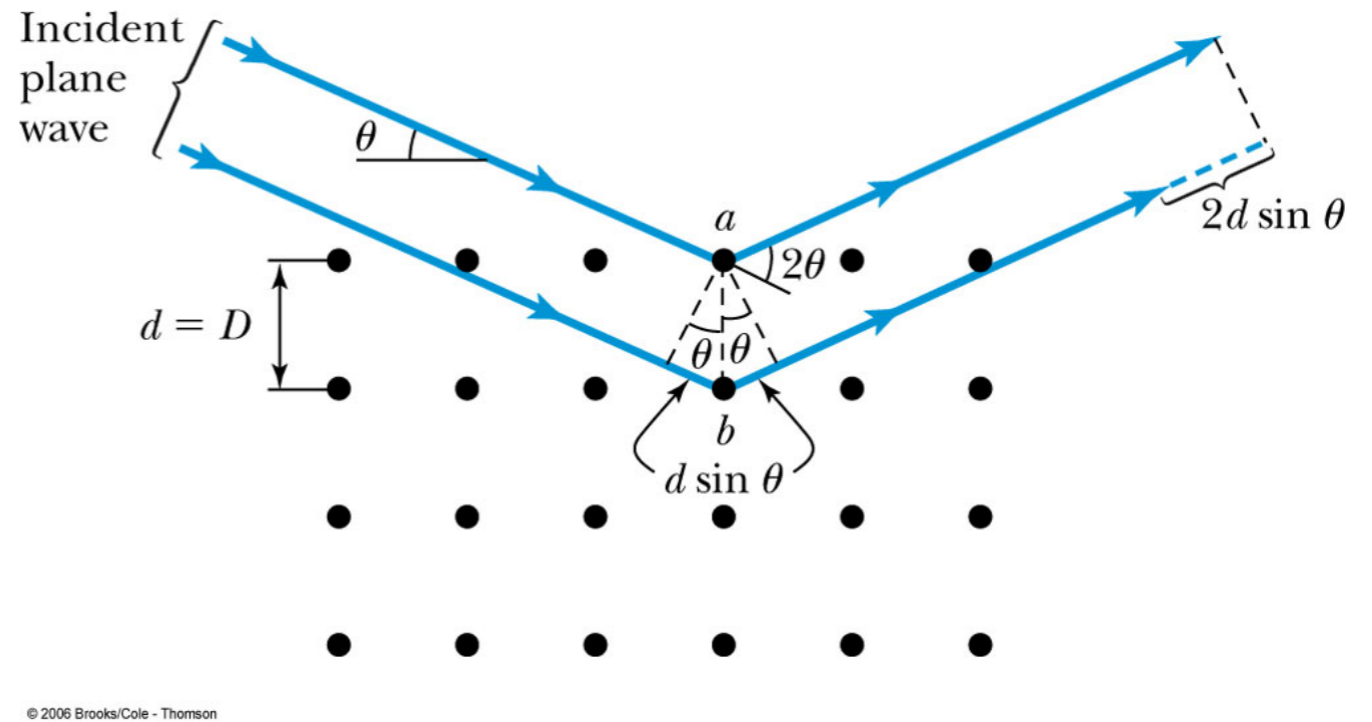
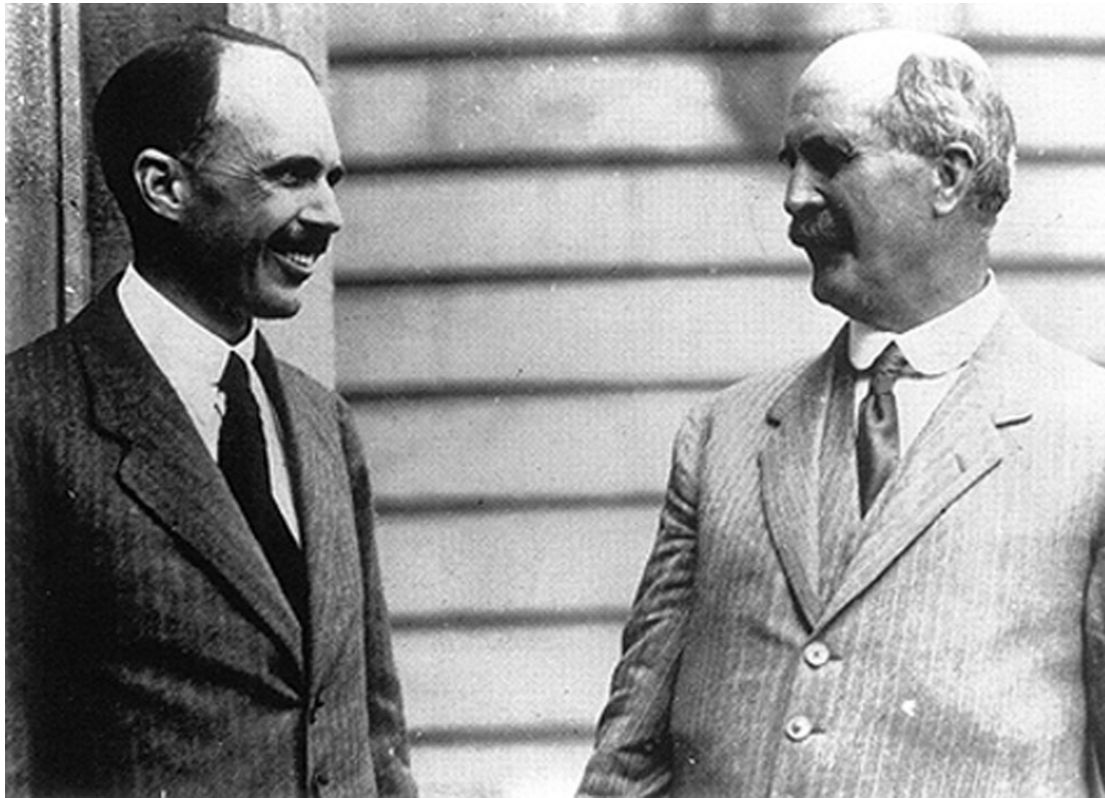
Wave



Diffraction, Thomas Young (1803)

Images: <http://en.wikipedia.org>
<http://www.staff.amu.edu.pl>
<http://micro.magnet.fsu.edu>

Bragg Diffraction



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William Lawrence Bragg

1890-1972

William Henry Bragg

1862-1942

Nobel Prize 1915

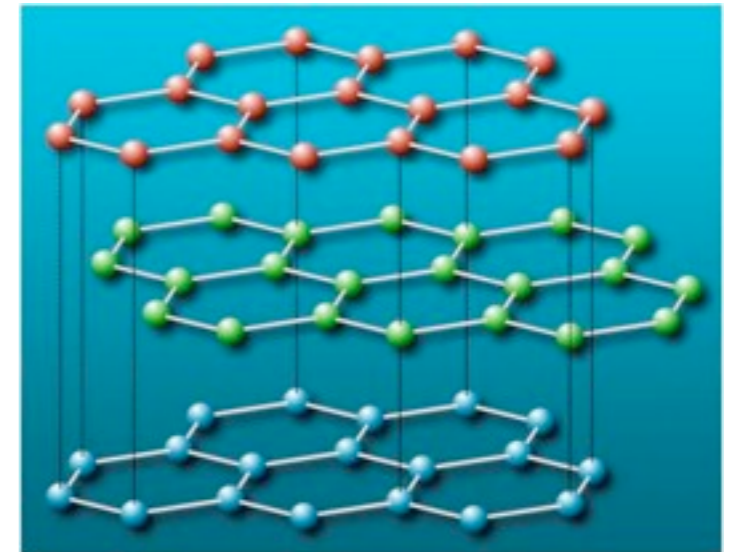
**Classically, light
behaves as a wave!**

Bragg's Law:
$$n\lambda = 2d \sin \theta$$

Images: Thornton and Rex

Electrons

- Are parts of an atom.
- Atoms are particles. —————→
- Ergo, electrons are particles.
- Or are they...



Graphite

Concept Test

- For a light wave with wavelength λ and frequency ν , the following relation is always true (in a vacuum):

- $\lambda\nu=c$ ← $[c] = \frac{\text{m}}{\text{s}} = [\lambda] \cdot [\nu]$

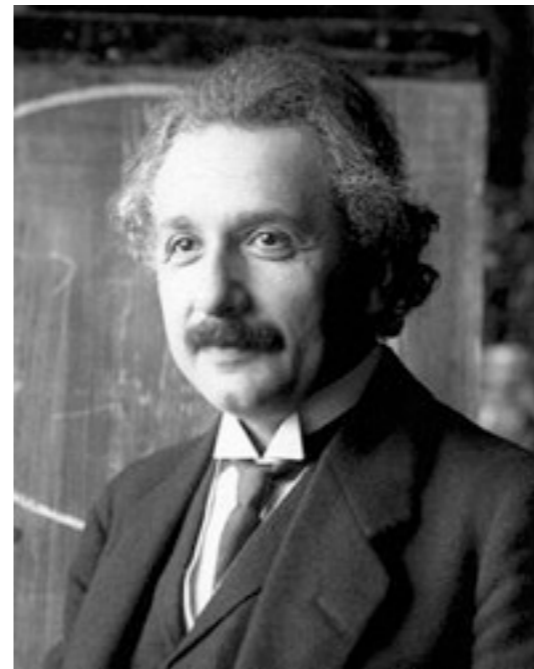
- $\lambda c=\nu$

- $\nu c=\lambda$

where c is the speed of light.

For Light

- $\lambda\nu=c$
- $E=pc$ (Einstein)
- $E=h\nu$ (Planck)
- $pc=h\nu=hc/\lambda$
- $\therefore \lambda=h/p$



A Crazy Idea...

Matter waves

An electron with momentum p
“has” a wavelength

$$\lambda = h/p \quad !$$

Quantum “fuzziness”: a wave is hard to
localize within a size of order λ

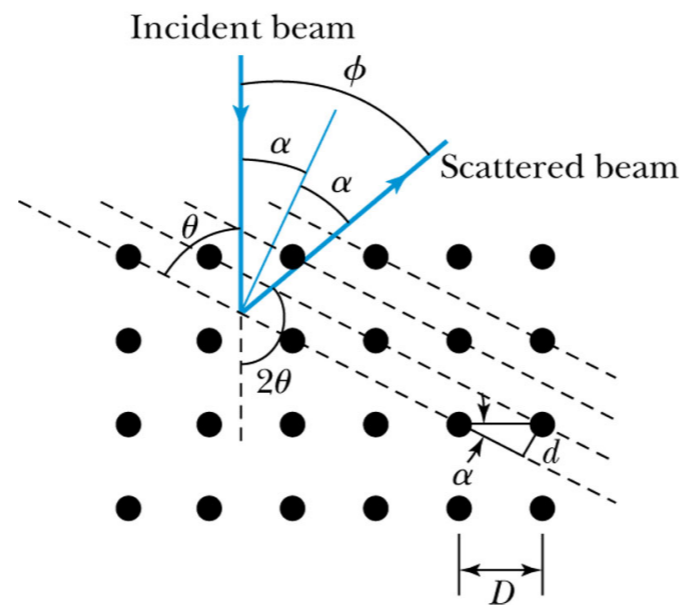
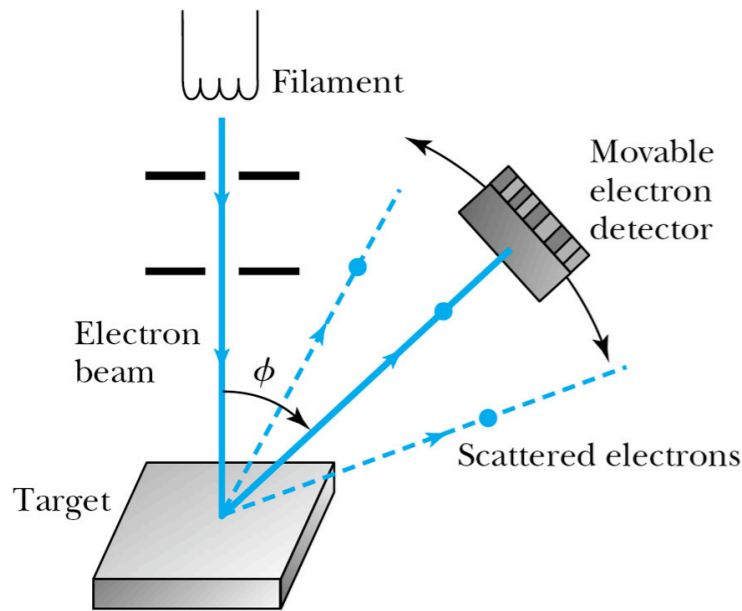
Heisenberg will make this more precise!



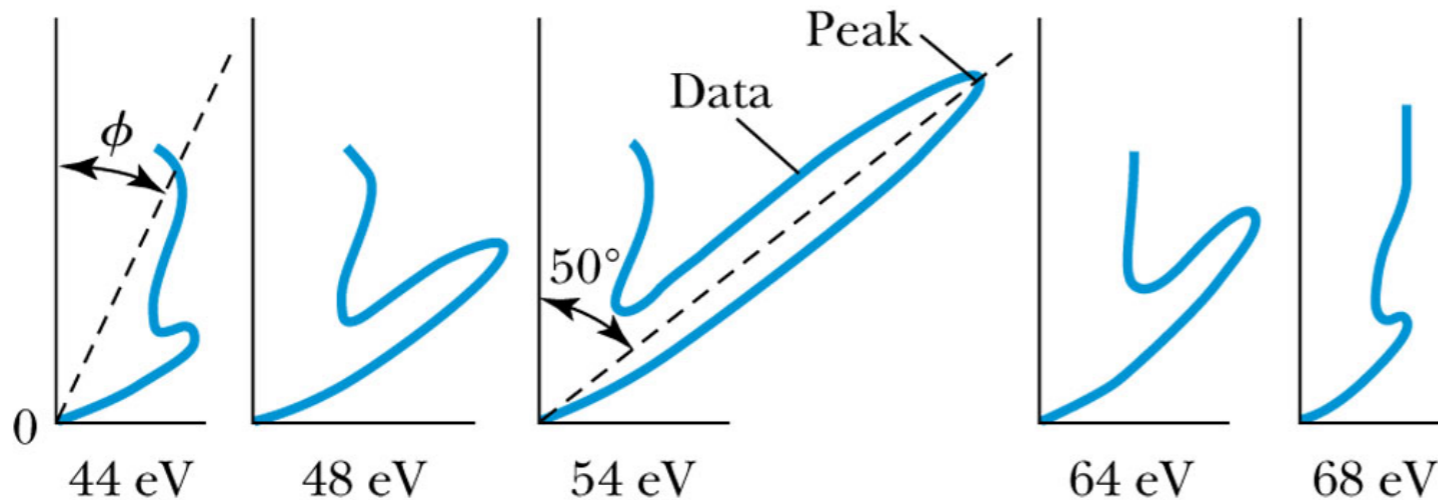
Louis de Broglie
1892-1987
Nobel Prize: 1929

Experimental Proof!

1925: Diffraction of Electrons by a Nickel Crystal




Intensity = radial distance along dashed line to data at angle ϕ



Clinton Davission (R)
(1881-1958)
Nobel Prize 1937
Lester Germer
(1896-1971)

Images: Thornton and Rex

What is the wavelength of a tennis ball?

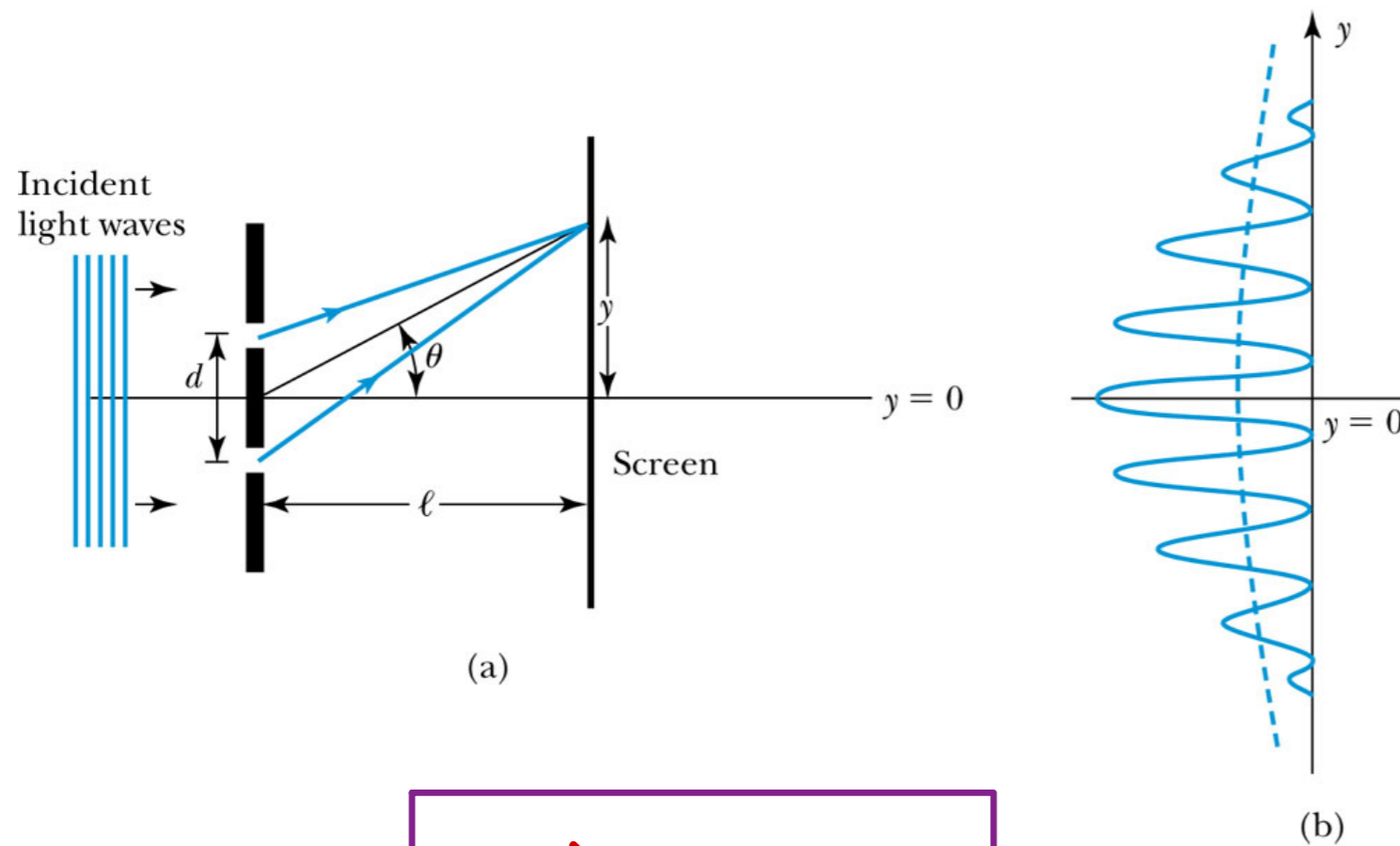

$$\begin{aligned} \lambda &= \frac{6.63 \times 10^{-34} \text{ J s}}{60 \frac{\text{mi}}{\text{hr}} \cdot 1600 \frac{\text{m}}{\text{mi}} \cdot \frac{1}{3600} \frac{\text{hr}}{\text{s}} \cdot 0.1 \text{ kg}} \\ &= 2.5 \times 10^{-34} \text{ m} \\ &= \mathcal{O}(10^{-24}) \times \text{size of atom} \end{aligned}$$

Quantum “fuzziness” of tennis ball is *very* small!

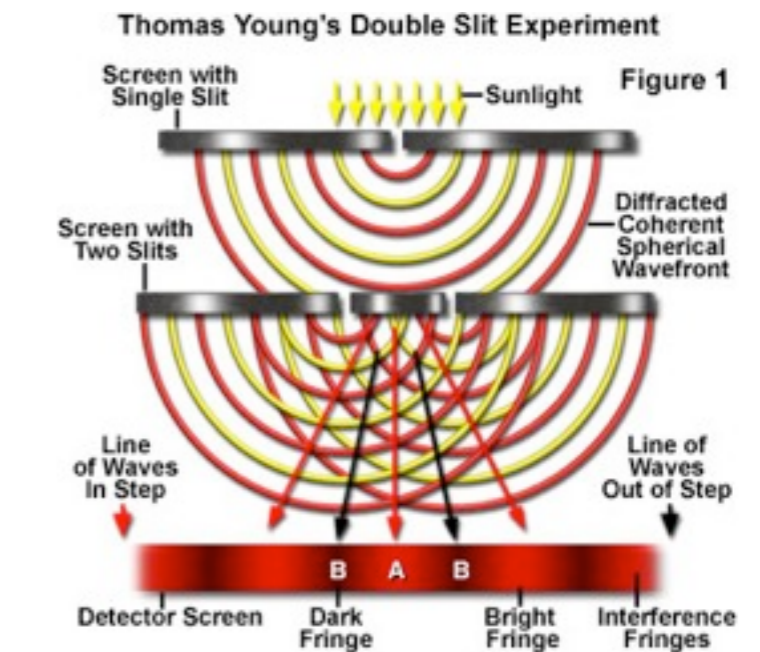
(I can't blame my inability to hit a tennis ball to quantum effects.)

Double Slit Experiment

Light



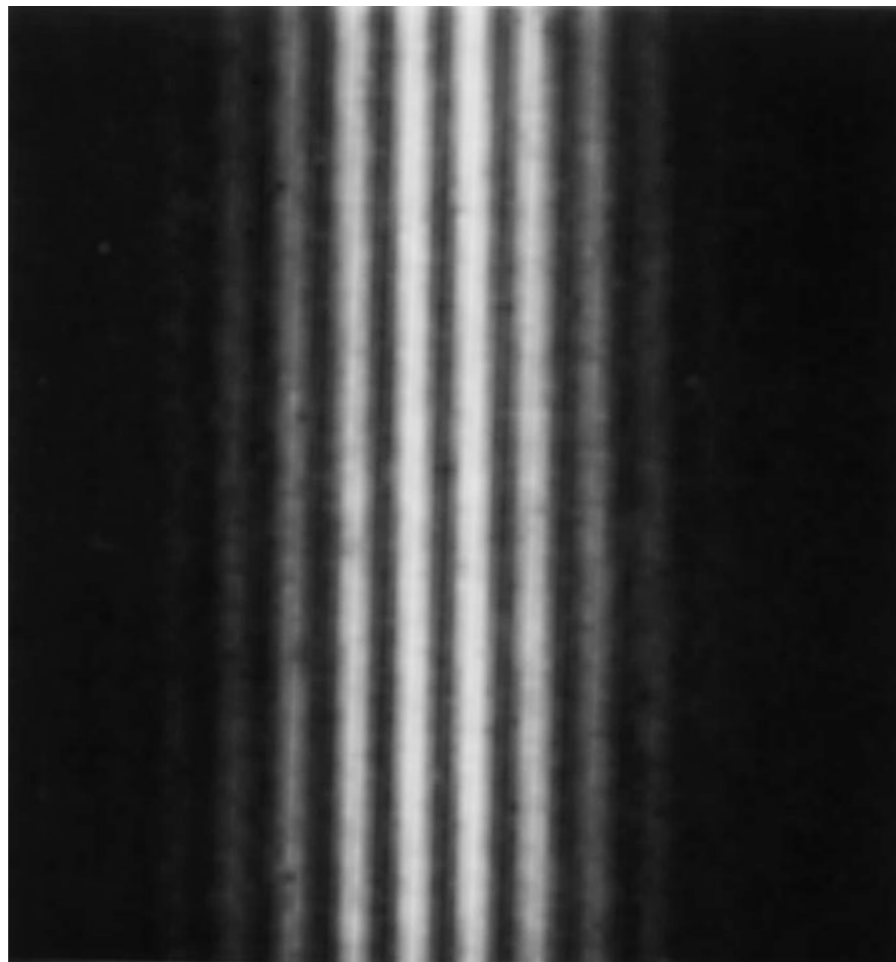
$$2d \sin\theta = n\lambda$$



Diffraction, Thomas Young (1803)

Images: Thornon and Rex
<http://micro.magnet.fsu.edu>

Double Slit Experiment: Electrons!



© 2006 Brooks/Cole - Thomson

C. Jönsson, 1961

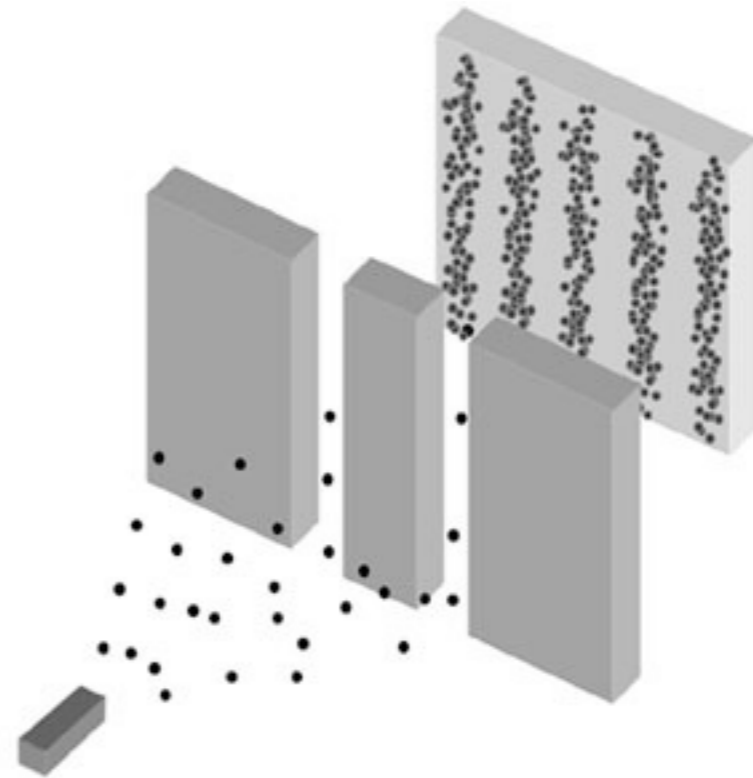
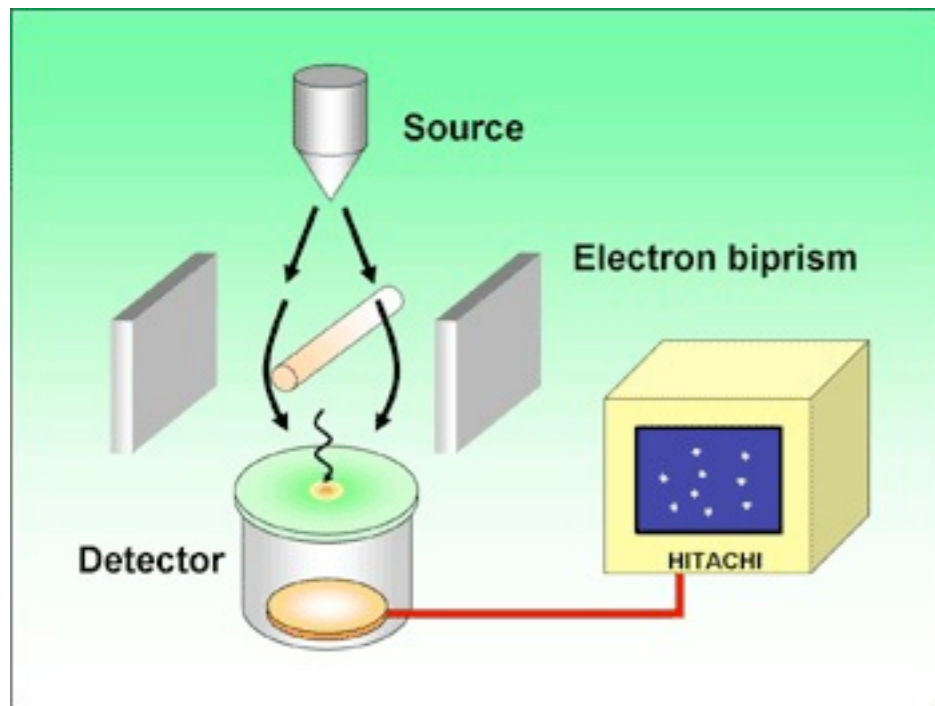


Image: Thornon and Rex
<http://stephenwhitt.files.wordpress.com>

One electron at a time!



Electrons, even one at a time, behave like waves!



Hitachi Laboratories

Dr. Tonomura Akiro

Movie: www.hitachi.com

Duality and Complementarity

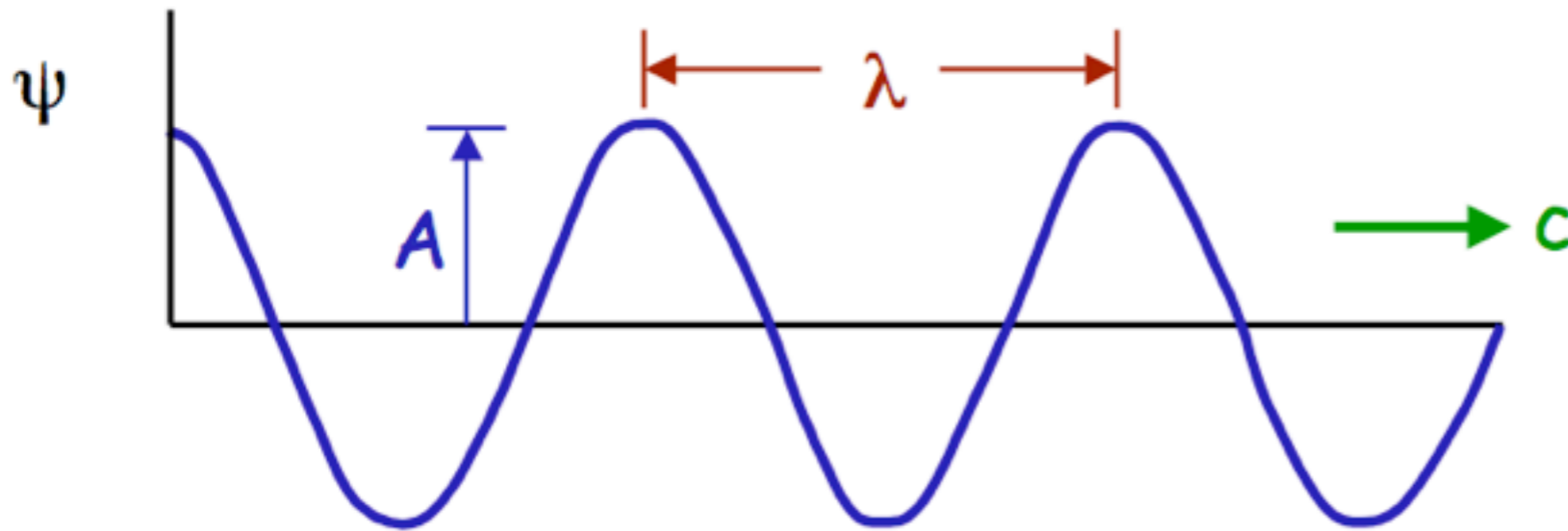
- Wave-Particle Duality: all matter and energy exhibits *both* wave and particle like properties!
- Complementarity: a single quantum-mechanical system can behave like a wave or a particle, *but not both simultaneously!*
(Bohr)

Summary, so far

- Light is classically a wave, but can behave like a particle (e.g. the photoelectric effect).
- Electrons are classically described as particles, but can behave like waves (e.g. the Davisson-Germer experiment).
- We need a description that unifies the particle and wave aspects of natural systems!

Waves

A plane wave: (light)



Amplitude: A
Wavelength: λ
Speed: c
Frequency: $\nu = c/\lambda$

$$\psi(x,t) = A \cos[2\pi (x - ct) / \lambda] \quad (\text{right-moving wave})$$

It is convenient to rewrite:

$$\psi(x,t) = A \cos(kx - \omega t)$$

Wave number: $k = 2\pi/\lambda$

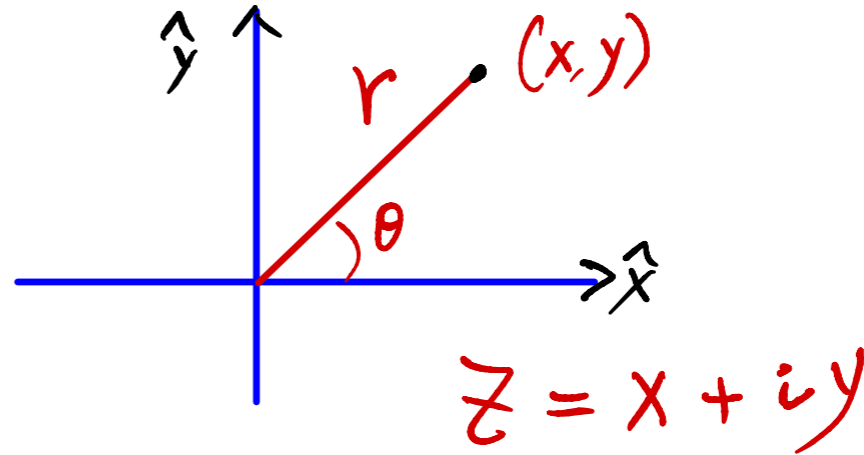
Angular frequency: $\omega = 2\pi \nu$

$$\text{Wave relation: } c = \lambda \nu = \omega/k$$

All light waves have same speed c in vacuum, independent of wave number k .

Complex Exponentials

- $i = \sqrt{-1}$
- $e^{i\theta} = \cos\theta + i \sin\theta$
- All complex numbers have a polar form:
 $z = x + iy = re^{i\theta}$, $x = r\cos\theta$, $y = r\sin\theta$
- $z^* = x - iy$, $zz^* = x^2 + y^2 = r^2$
- Plane wave: $\psi = Ae^{i(kx - \omega t)}$
- Classical Physics: use only real or imaginary part



Matter Waves

$$\psi = A e^{\frac{i}{\hbar}(px - Et)}$$

- Quantum Mechanics: **Complex waves**

- $\psi = A e^{i(kx - \omega t)}$

$$E = \frac{p^2}{2m} \quad \text{free particle!}$$

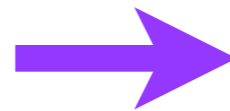
$$\omega = \frac{\hbar k^2}{2m} \quad \text{dispersion relation}$$

$$v = \frac{p}{m} = \frac{d\omega}{dk}$$

$$\neq \frac{\omega}{k}$$

Planck: $E = h\nu = \hbar\omega$

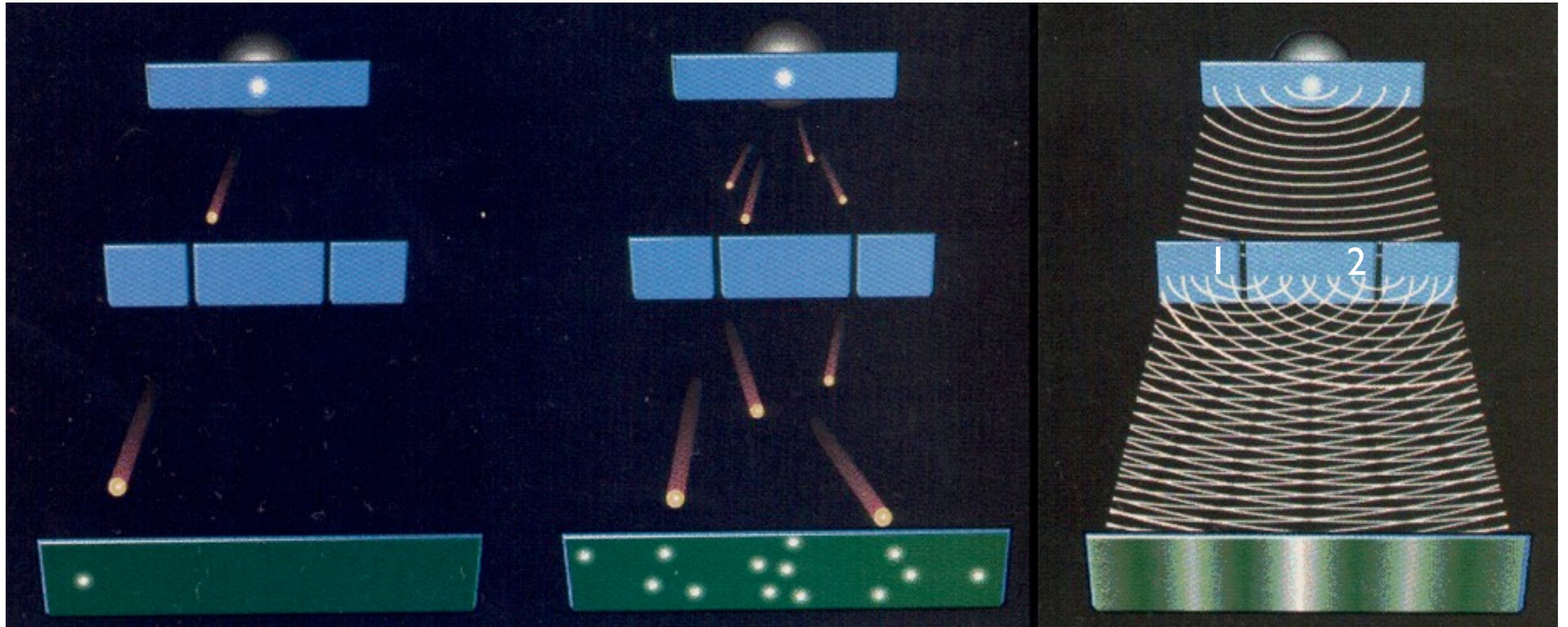
deBroglie: $p = \frac{h}{\lambda} = \hbar k$



How do we interpret $\psi(x,t)$?

How do we calculate it in general?

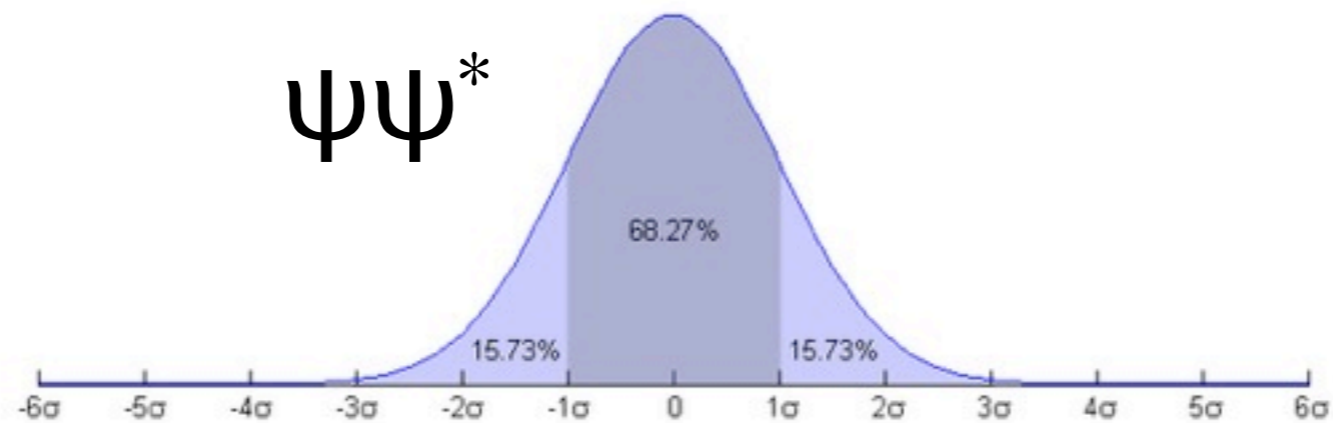
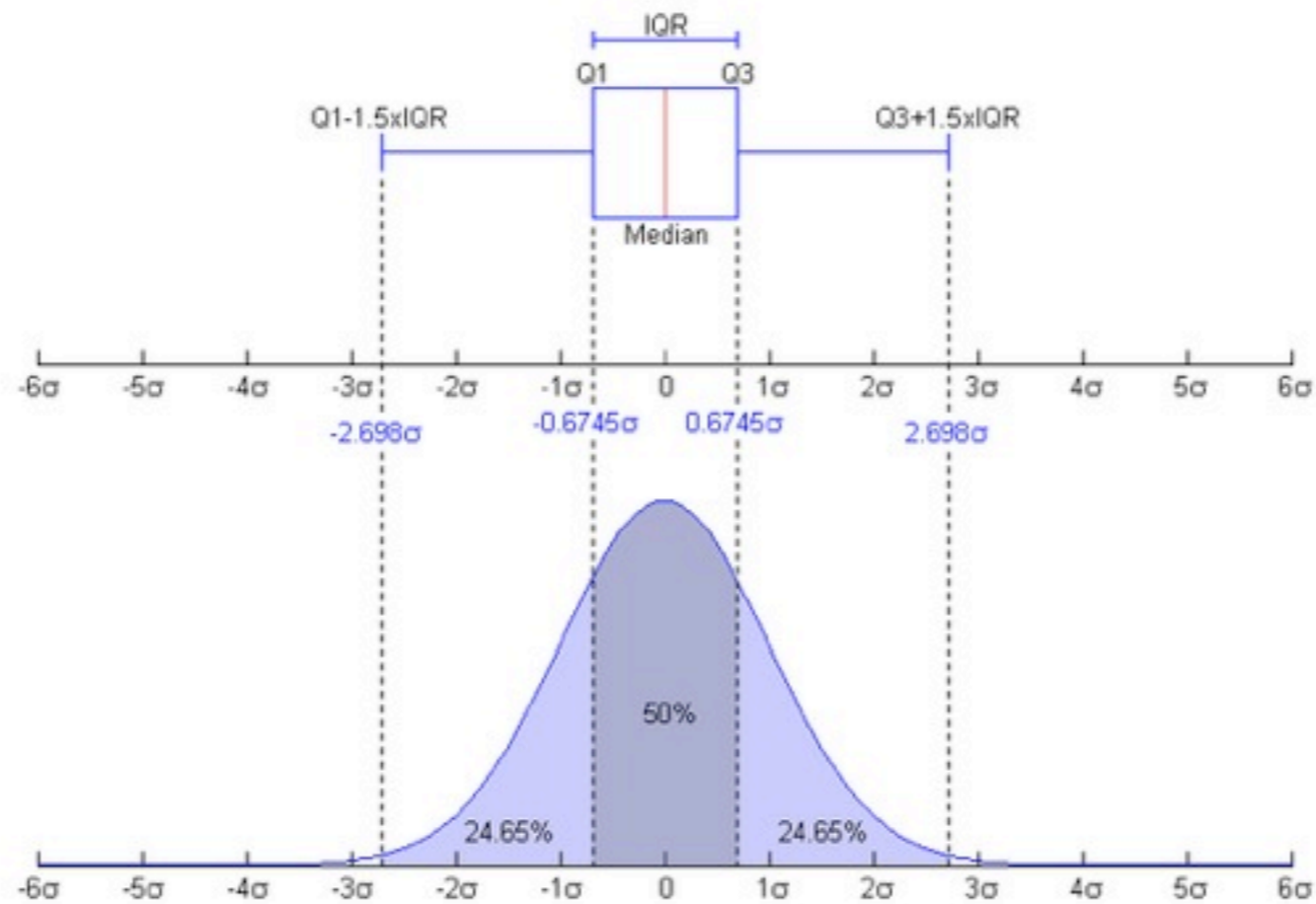
Interference & Superposition



Electron diffraction due to interference of matter waves coming from the two slits:

$$\psi_{\text{screen}} = \psi_1 + \psi_2$$

Probability Density



$\psi\psi^*$

Gaussian PDF

Image: wikipedia.org

Summary

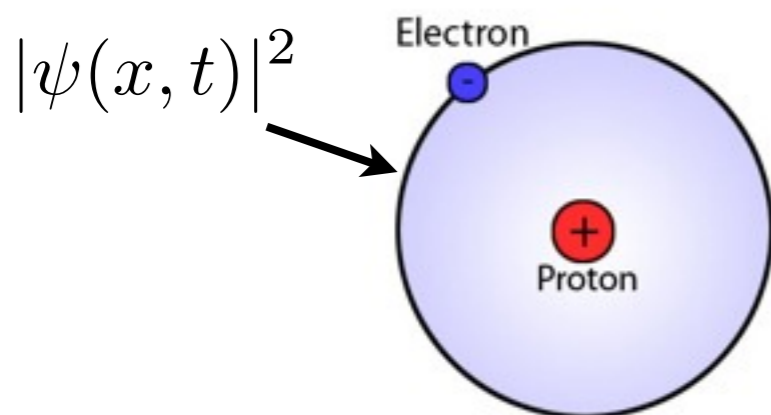
- The Bohr model reproduces the Rydberg formula for Hydrogen spectra.
- The “shell hypothesis” explains peaks in X-ray spectra, and implies periodic table should be ordered by Z .
- Matter waves correspond to complex wavefunction $\psi(x,t)$.
- *Probability* of finding particle at position x is proportional to $|\psi(x,t)|^2$.

Outline

- Born's Interpretation of ψ (cont'd)
- Heisenberg Uncertainty Principle
- "Copenhagen Interpretation"
- Complementarity and the single photon
- Where are we now? What remains to be done?

Born's Interpretation of ψ

- Born suggested that the *probability* of finding a particle at position x and time t is proportional to $|\psi(x,t)|^2 = \psi\psi^*$.
- Positions of destructive interference have $\psi=0$, and no *probability* to find the particle at that location.



Max Born
1882-1970
Nobel Prize 1954



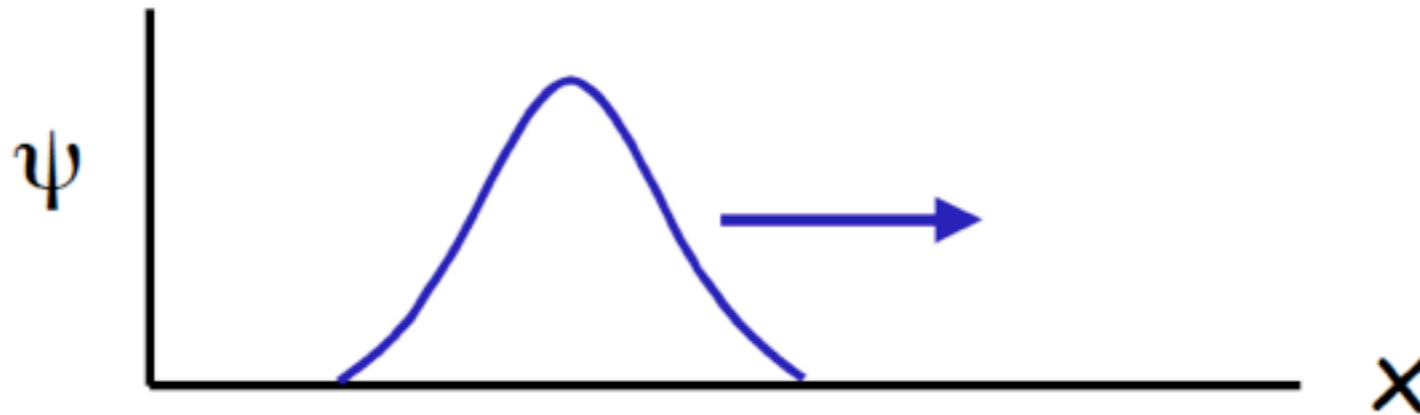
Concept Test

- Born's interpretation of the wavefunction implies that $|\psi(\mathbf{x},t)|^2 = \psi\psi^*$ is the *probability density* of finding a particle in an infinitesimal volume d^3x . All of the following are true except
- $\psi\psi^*$ is a real number
- $\psi\psi^*$ is positive
- $\psi\psi^*$ must be between 0 and 1 ←
- $\int \psi\psi^* d^3x = 1$

“Localized Particle”

A wave packet can be constructed as a continuous sum (integral) of plane waves.

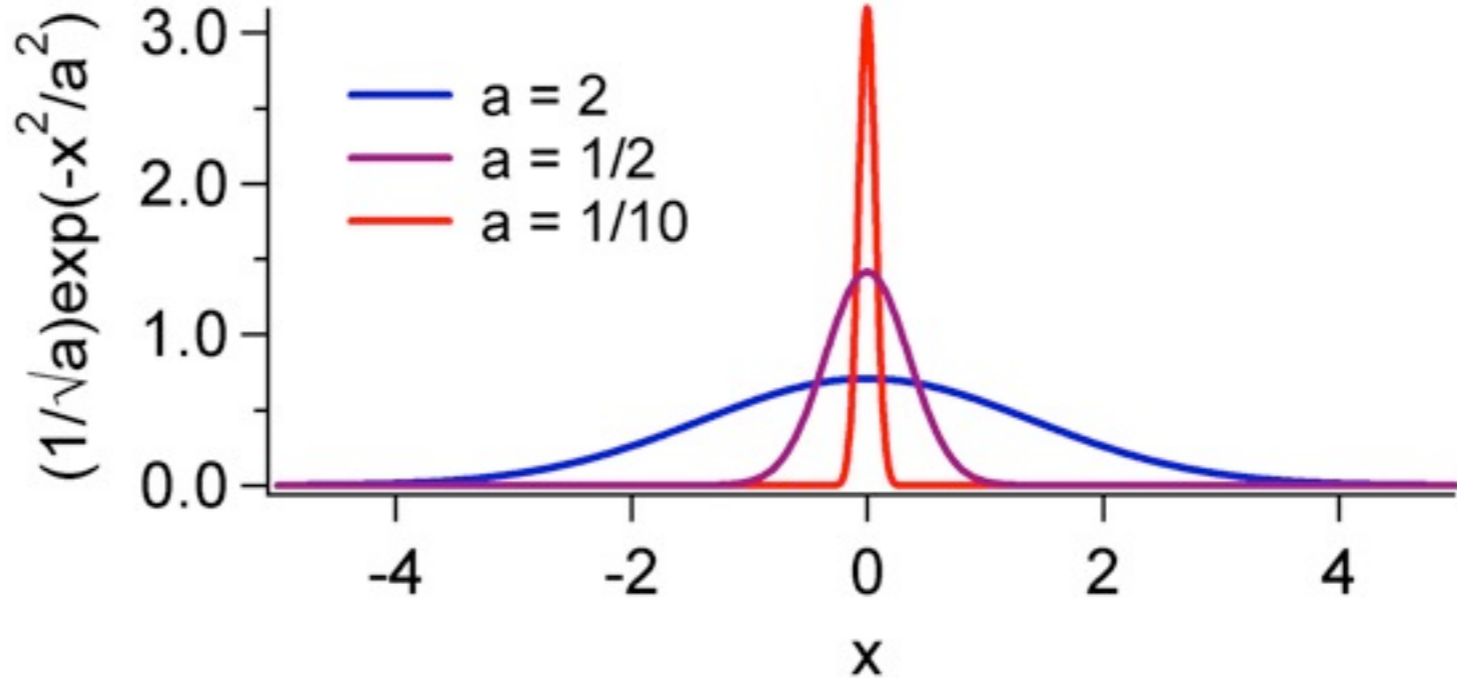
⇒ Fourier Transform



$$\psi(x, t) = \int_{-\infty}^{+\infty} A(k) e^{i(kx - \omega t)} dk$$

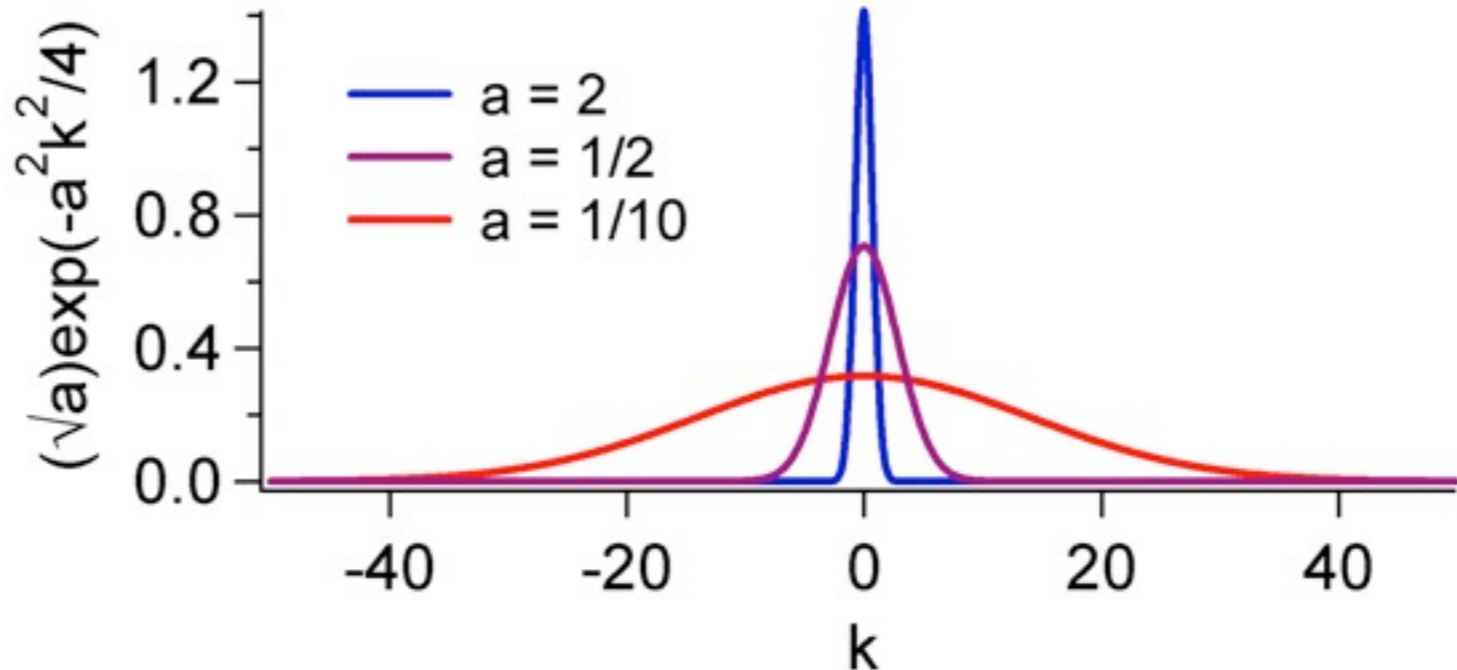
Position vs. Momentum

$$\sigma_x = \Delta x = a/2 \rightarrow$$



Position localization \Leftrightarrow
 Momentum delocalization
 Momentum localization \Leftrightarrow
 Position delocalization

$$\sigma_k = \Delta k = 1/a$$



Heisenberg Uncertainty Relation

$$\Delta x \Delta k \geq 1/2$$

Multiplying by \hbar and using $p = \hbar k$ gives:

$$\Delta x \Delta p \geq \hbar/2$$

No measurement can simultaneously measure position and momentum to an accuracy which violates the uncertainty principle!




Werner Heisenberg

1901-1976

Nobel Prize 1932

Concept Test

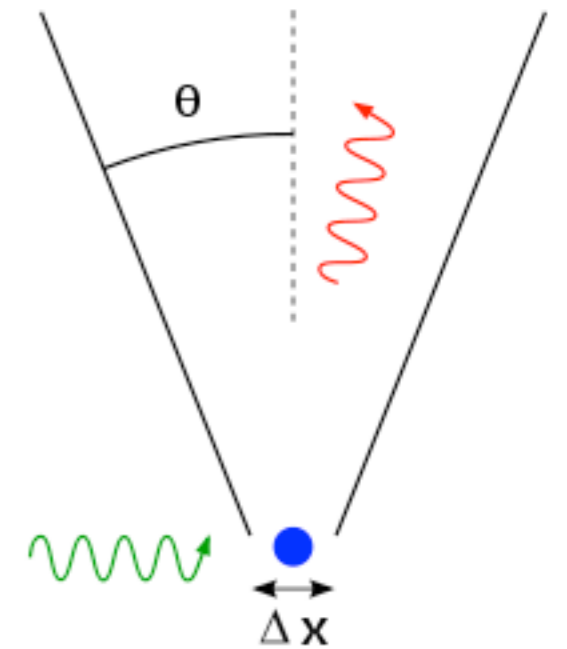
- The de Broglie wavelength of a tennis ball, $\lambda = h/p$, goes to ∞ (the “fuzziness” of the ball) as $p \rightarrow 0$. Why doesn't this prevent us from picking up tennis balls left on the court?
- de Broglie's formula doesn't apply
- $h \rightarrow 0$ when the ball isn't moving
- p is never exactly 0 

Concept Test

- Particle A is confined to a (small) region of size L , while particle B is confined to a region of size $2L$. From the uncertainty relation, we expect that the kinetic energies are related by
 - $E_A < E_B$
 - $E_A = E_B$
 - $E_A > E_B$ ← $\Delta p_A \approx 2 \Delta p_B$

Heisenberg's Microscope

- How can we understand the uncertainty relation physically?
- Suppose we want to locate an electron: we must scatter light off of it!
- To do so precisely, we must use light with a short wavelength ($\Delta x \approx \lambda$)- but such light also has large energy!
- The light must scatter off the electron, but this imparts large momentum ($\Delta p \approx \hbar/\lambda$)!

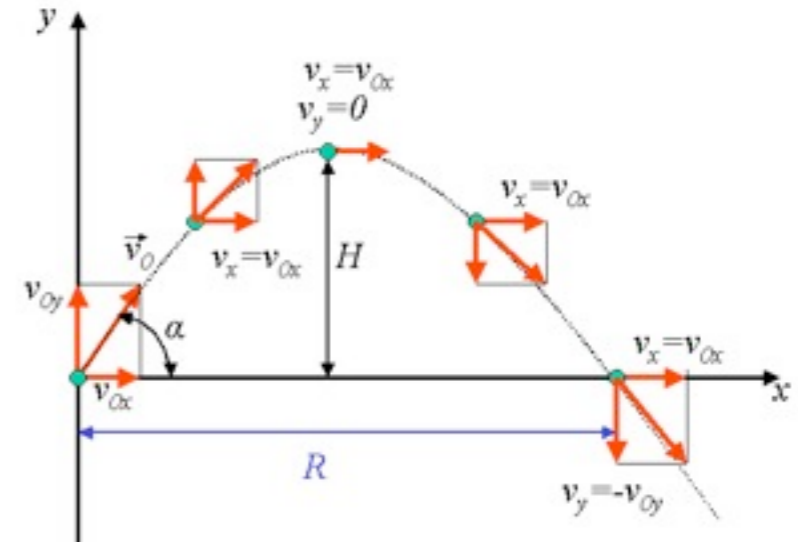


QM crucial to this  argument!

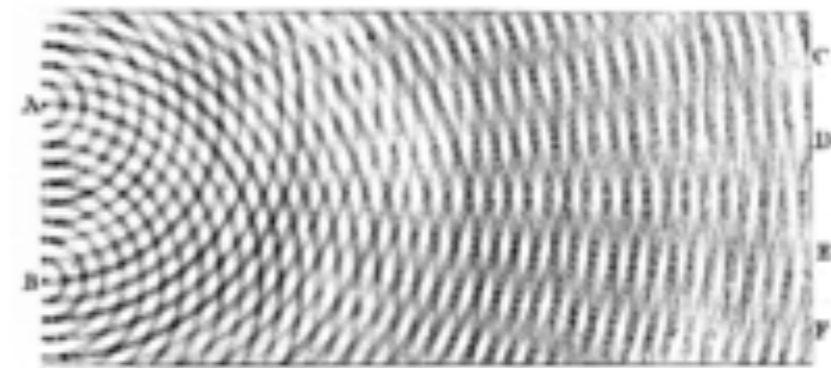
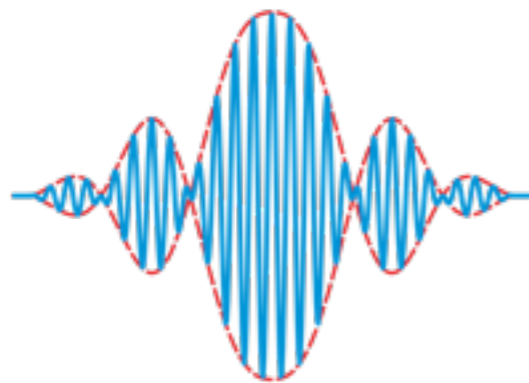
Classical vs. Quantum

Property	Classical	Quantum
Location	Definite	Indefinite
Momentum	Definite	Indefinite
Interference	No	Yes

Particle



Wave



Diffraction, Thomas Young (1803)

Matter Waves:

$$\Delta x \Delta p \geq \hbar/2$$

Images: <http://en.wikipedia.org>
<http://www.staff.amu.edu.pl>
<http://micro.magnet.fsu.edu>

Energy-Time Uncertainty

- For a free particle

- $E = \hbar^2 k^2 / 2m$, $\Delta E = (\hbar^2 k / m) \Delta k$

- $\Delta x = (\hbar k / m) \Delta t$

- $\therefore \Delta E \Delta t \geq \hbar/2$

$$\psi \sim e^{\frac{i}{\hbar}(Px - Et)}$$

- Einstein & Bohr: a state that exists for only a short time cannot have a definite energy.
- Other “conjugate” pairs exist in different cases: (L, θ) , etc.

“Copenhagen” Interpretation

- Bohr and collaborators developed an interpretation of quantum mechanics, stating that quantities like the “position” and “momentum” of a particle only have meaning to the extent that they are measured.
- Complementarity: One cannot describe a physical observable simultaneously in terms of both particles and waves.
- Uncertainty: Conjugate variables cannot be simultaneously measured.
- Born: $|\Psi(\mathbf{x})|^2$ measure how likely a particle is to be found at position \mathbf{x} , within volume $d^3\mathbf{x}$.

Bohr & Copenhagen



1921



Niels Bohr
1885-1962
Nobel Prize 1922



Now

Blegdamsvej 17

Where are we now?

- Experimental evidence shows that
 - light has particle-like properties
 - and matter has wave-like properties.
- The wave-like properties of matter imply
 - *quantization* of energy levels in the atom
 - position-momentum uncertainty
 - existence of a “wavefunction” $\Psi(x,t)$.

What remains to be done?

- How do we systematically
 - determine $\Psi(x,t)$ and
 - extract the values of measurements from $\Psi(x,t)$?



Werner Heisenberg
1901-1976
Nobel Prize 1932



Erwin Schrödinger
1887-1961
Nobel Prize 1933