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1 Goals

2 A Little Bit of Minkowski

You’ve probably never heard of Hermann Minkowski (1864-1909), but his influence on 20th century physics was imaginative and fundamental. He invented a language—plus a brand new kind of geometry—that actually simplified the physics of relativity before Einstein became known to the rest of Europe. While still anonymous, though, Einstein and Minkowski were well-known to one another and it’s arguable as to who was more surprised at Einstein’s breakthroughs. Einstein never expected to rely on a mathematician and Minkowski certainly thought that Einstein would never amount to anything. A match not made in heaven.

Minkowski was a child mathematical prodigy. His parents emigrated to Germany in 1872, when Hermann was 8 years old—they settled in the university town of Königsberg which provided ample opportunity for his unexpected talents to become apparent and be nurtured. He entered the University of Köningsberg at the age of 16 and received his doctorate in mathematics at 21. As a student he won a prestigious French mathematics competition and then moved up through the German and Swiss university systems as a specialist in the connections between geometry and number theory. The important time for our story is the period between 1896 and 1902. Minkowski began teaching at the Swiss Federal Polytechnic in Zürich in 1896 at the age of 32, the same year that Einstein began his studies there at the age of 17. Einstein registered in many of Minkowski’s classes in the next four years but didn’t endear himself to his mathematics instructor because of his habit of habitually skipping his classes. In 1949 he later wrote, “...the most fascinating subject at the time that I was a student was Maxwell’s theory. . . ” conceding later that

> “I had excellent teachers (for example, [Adolf] Hurwitz, Minkowski), so that I should have been able to obtain a mathematical training in depth. I worked most of the time in physical laboratory, however, fascinated by the direct contact with experience. The balance of the time I used, in the main, in order to study at home the works of Kirchhoff, Helmholtz, Hertz, etc.”

That is, he skipped classes in order to study electromagnetism on his own.

Minkowski was shocked by Einstein’s relativity paper. He’d thought along similar lines, but never quite got over the conceptual difficulties. But his surprise was only partly about the physics. Famously, he wrote to one of his students, Max Born (whom we’ll meet later):

> ”For me it came as a tremendous surprise, for in his student days Einstein had been a real lazybones. He never bothered about mathematics at all.”

And he was right. Einstein had little patience with the over-mathematization of physics. Later when he was wrestling with his general theory of relativity, he lamented to Arnold Sommerfeld, a leading senior physicist from Munich,

> "But one thing is certain, never before in my life have I troubled myself over anything so much, and that I have gained great respect for mathematics, whose more subtle parts I considered until now, in my ignorance, as pure luxury! Compared with this problem, the original theory of relativity is childish."

Minkowski married while in Zurich and he and his wife eventually had a family of two daughters. By 1907-1908 he had come to grips with relativity, having politely written to his former "lazy bones,"

> "Dear Doctor Einstein,
>
> At our seminar in the W.S. we also wish to discuss your interesting papers on electrodynamics. If you still have available reprints of your article in the *Ann. d. Phys. u.* Ch., Vol. 17, I would be grateful if you would send us a copy. I was in Zurich recently and was pleased to hear from different quarters about the great interest being shown in your scientific successes.
>
> With best regards, yours sincerely,
>
> H. Minlowski"

From the work that grew of this reacquaintance came one of the more poetic and attention-getting physics talks ever given. On September 21, 1908, in the 80th annual general meeting of the German Society of Scientists and Physicians, in Cologne Minkowski presented a talk entitled "*Raum und Zeit*," Space and Time. Born wrote later,

"...I went to Cologne, met Minkowski and heard his celebrated lecture 'Space and Time'...He told me later that it came to him as a great shock when Einstein published his paper in which the equivalence of the different local times of observers moving relative to each other was pronounced; for he had reached the same conclusions independently but did not publish them because he wished first to work out the mathematical structure in all its splendor. He never made a priority claim and always gave Einstein his full share in the great discovery.

After having heard Minkowski speak about his ideas, my mind was made up at once, I would go to Göttingen to help him in his work."

Planck had urged Einstein to attend, but he failed to do so. What an amazing event that would have been for the still patent clerk from Bern. The opening paragraph is famous among all physicists today,

> "M. H.! [ladies and gentleman!] The views of space and time, which I would like develop, have sprung from the experimental-physical soil. Therein lies their strength. They tend to be radical. Henceforth space by itself and time by itself, fade away completely into shadow, and only a kind of union of the two will preserve independent permanency."

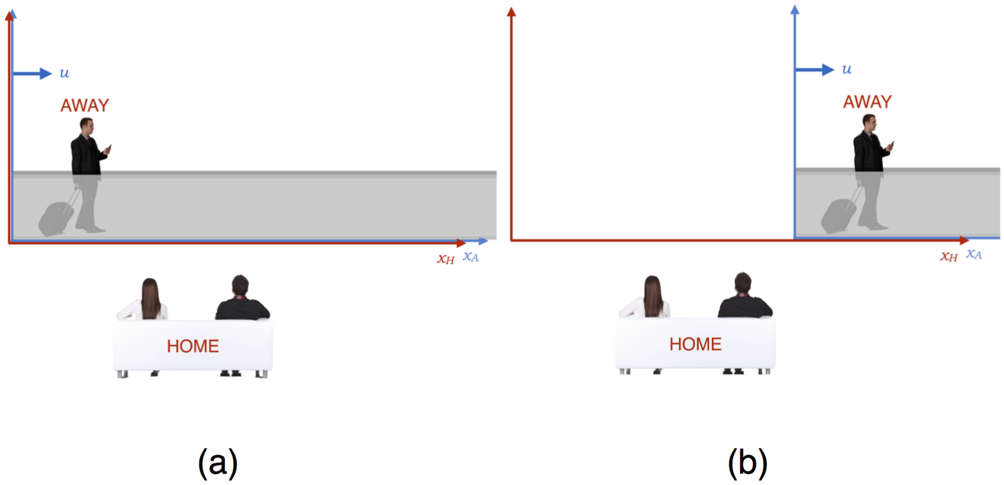

Figure 1: sidewalkintro

3 The Second Postulate

In the last chapter, we developed Einstein's two postulates of relativity. The first postulate was a throwing down of the philosophical gauntlet: no phenomenon—neither mechanical (like Galileo's) nor electromagnetic—can be used to distinguish motion in a frame of reference moving at a constant velocity (an inertial frame) relative. But the fun is all in the second populate: that the speed of light is a constant, an *invariant*. Let's go to the airport.

3.1 Time and Space Increments

As a million-miler on a U.S. airline, I spend way too much time in airports. "As" a physicist in an airport? Well, there's just too much fun to be had. We all enjoy the moving sidewalks as a visual example of relatively moving inertial frames.[1] So let me have my fun and allow me to use the moving sidewalk in our examples. Stay with me.

Figure 1(a) and (b) show CouchPeople and WearyTraveler at the airport. CouchPeople have a long lay-over and they're people-watching. WearyTraveler is on the moving sidewalk. Figures 1 (a) and (b) show two different times for his journey across the terminal.[2]

If we're in the airport with CouchPeople, we'd label it and them as in the Home Frame (HF) while the WearyTraveler is in the Away Frame (AF). We'll use this for all kinds of examples. Here's the first one.

[1] Okay. You might enjoy them as a way to get from one end of the terminal to another.
[2] Don't you hate the people who just stand on the moving sidewalks?

Figure 2:

3.2 The Second Postulate's First Surprise

Figure 3 is a fake electronic device set up on a test bench that we'll pretend measures the time differences of light passing through the two photodetectors, A and D. What happens is that light shined on the apparatus from the left would go through A and register a photo-signal that would pass through the cable to the oscilloscope input, B. Its time of arrival is then registered on the screen, C. There is an identical unit next to it with another photocell, D with an identical cable that sends its signal to input E for time calculation and display at F. Internal to both oscilloscopes is a crystal clock that keeps regular time and displays it at C and E as clock hands representing the number of pulses that each internal clock registers. In this case, the two devices have been calibrated so that they share a common start time.

Notice too that there are three rulers on the table, each a foot long. So the photo detectors are exactly 3 feet apart. So knowing the distance and then measuring the times, we could determine the speed of light if a beam shines from left to right.

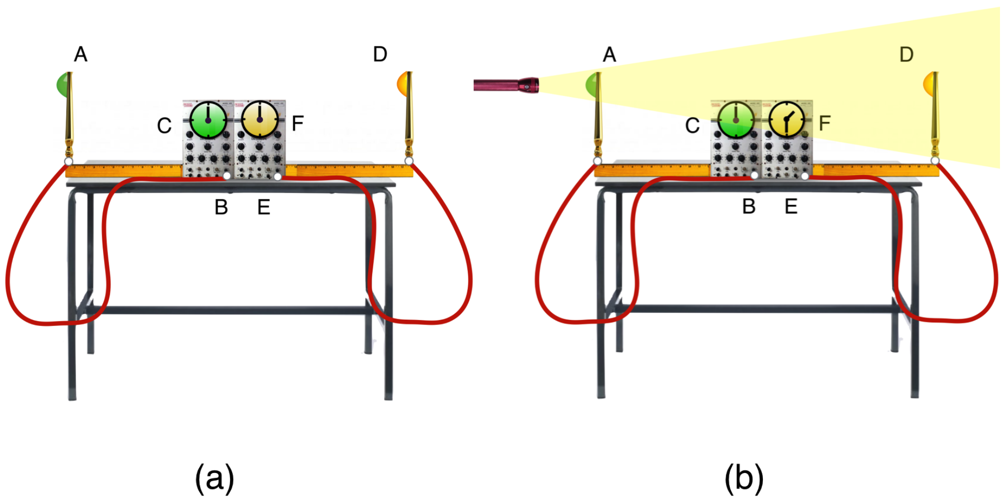

Figure 3: timedevice

And indeed, that's what we'll do. In Fig. 3 (a) the devices are ready for a signal and in (b) a common light beam has been shined on them both from left to right. Notice that they now show different times, representing the time that it takes for the beam to go from A to F. Since light travels about a foot in a nanosecond,[3] the time difference that will be registered between our two cartoon devices would be about 3 ns, which is easily discernible with modern electronics.

What the Second Postulate says is rather astounding, which we can illustrate with our sidewalk and our fake device. Let's assume that both the CouchPeople and the WearyTraveler have built identical devices. CouchPeople set theirs up in the HF next to the sidewalk, while WearyTraveler sets his up on the sidewalk with him. Then, just like on our test bench above, a single beam of light is directed along the sidewalk, from left to right so that it passes through both sets of apparatus.

> **Wait.** *Might the beam be slowed down or somehow affected by passing through one device before it gets to the other?*
>
> **Glad you asked.** *Good question! In principle it might. But our airport people are good scientists and so they first set up their experiment in the airport...one after the other...and shined a light through them both. What they measured was that there is no affect—they measure the same speed for both.*

Figure 4 shows our equipment loaded up (a) and with the sidewalk having moved a bit to the right, (b). The experiment comes from shining the light, which we see in Fig. 5. Here's where the fun comes in. Let's ask three questions of our travelers:

1. What is the speed of light as measured by the CouchPeople for the HF apparatus?
2. What is the speed of light as measured by the WearyTraveler for his device in the AF?
3. What is the speed of light as measured by the CouchPeople... using the AF device... the one on the sidewalk in the AF?

[3] That's just a rule-of-thumb that one learns in a physics laboratory.

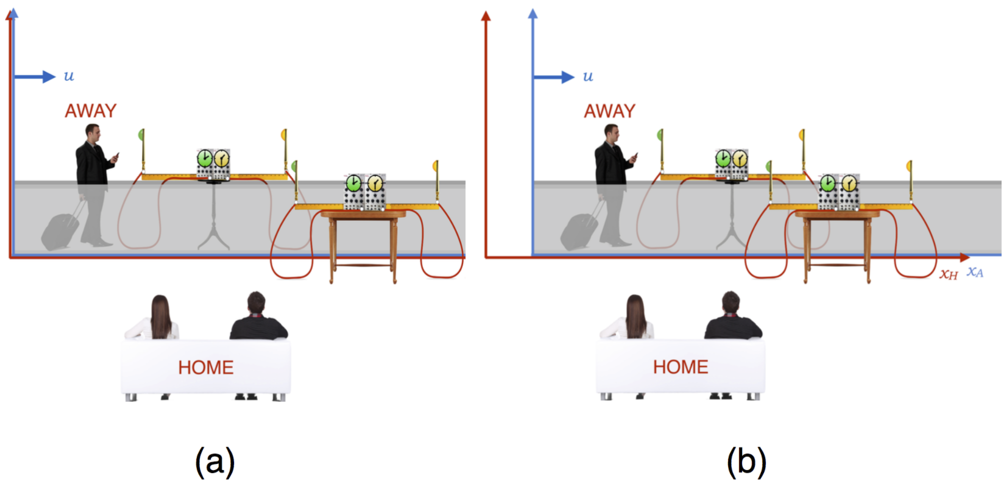

Figure 4: sidewalksetup1

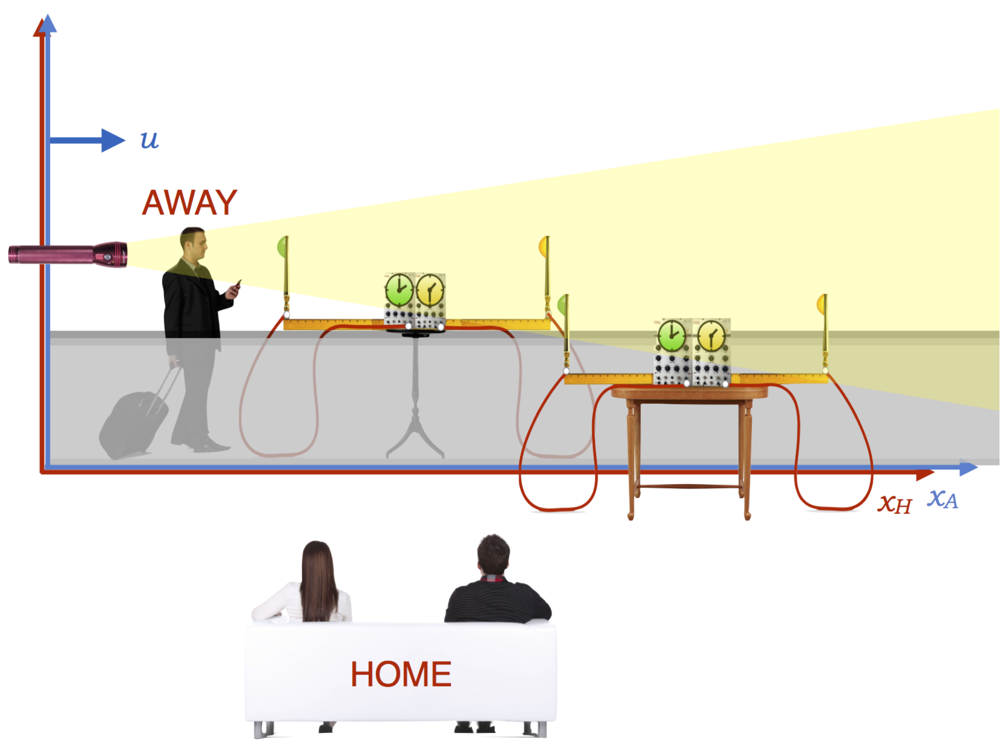

Figure 5: sidewalksetup2

Question 1 is easy. We've already done it in the setup. The HF people measure the speed of light to be 3.0×10^8 m/s. What we all know and love as c.

Question 2 leads to a surprise. Even though WearyTraveler and CouchPeople are each sampling the same beam and even though WearyTraveler is moving away from the source of the light, they he measures the speed of light to also be 3.0×10^8 m/s! You might think that it would somehow have to go faster in order for him to get that same speed. But that's what Einstein's Second Postulate requires. But we're not done with Strange.

What about Question 3? The CouchPeople would measure the speed of light for the machine in the sidewalk's frame to be...3.0×10^8 m/s. Now that's really disturbing and our second surprise coming from the Second Postulate.

> **Wait.** *That's crazy! The sidewalk has no affect on the speed of light even though it's moving away from the source of the ight?*
>
> **Glad you asked.** *Yup. That indeed, is one of the strange things about Special Relativity. Somehow we have to explain this.*

Yes, the Second Postulate suggests strange things about the world.

> **Wait.** *I'm not done with you yet. The Second Postulate was put forward as a testable assertion, not a statement of experimental fact. A postulate is only a proposition. A suggestion. Why behave like it's true?*
>
> **Glad you asked.** *What we'll see is that if we assume the postulate we can derive measurable facts about nature which are a consequence of the postulate and check them. If they work, then we should accept the postulate. If they don't, then it was an interesting try, Albert, but no dice.*

But let's build a clock.

A Light Clock

Back to the sidewalk. Figure 6 shows the raw materials for another fake measuring device. A bathroom mirror like on the left side of the figure. Okay, two of them, mounted horizontally as shown on the right side. The mirrors are separated by a distance L and a little hole is drilled, H, to admit a burst of light from the laser pointer. The hole is quickly plugged and the light beam, B, just bounces up and down. We mount it up on the sidewalk as in Fig. 7 and WearyTraveler counts the round-trips of the light. Up-down, up-down, Tick-Tock. . . so, yes, it's a light-clock. The round trip time that the light pulse takes go up and down we'll call the increment of time measured *from within the Away Frame (the sidewalk)*, t_A.

What do CouchPeople see as the contraption moves by them on the sidewalk? The HF view of the ("moving") clock is different. The light pulse certainly has the same vertical up and down motion, but as it goes up and then comes down the sidewalk has moved *horizontally* and so we see a kind of triangular path as shown in Fig. 8. So for those of us on the ground, the pulse travels further than for the AF observer and the time that it takes to make a complete trip *as observed from the airport Home Frame*, up and down, we'll call t_H.

The distances traveled are different, so how do the two times relate to one another if the speed of light along the two different trajectories is actually the same in the two frames—which is what the Second Postulate says? Let's calculate it.

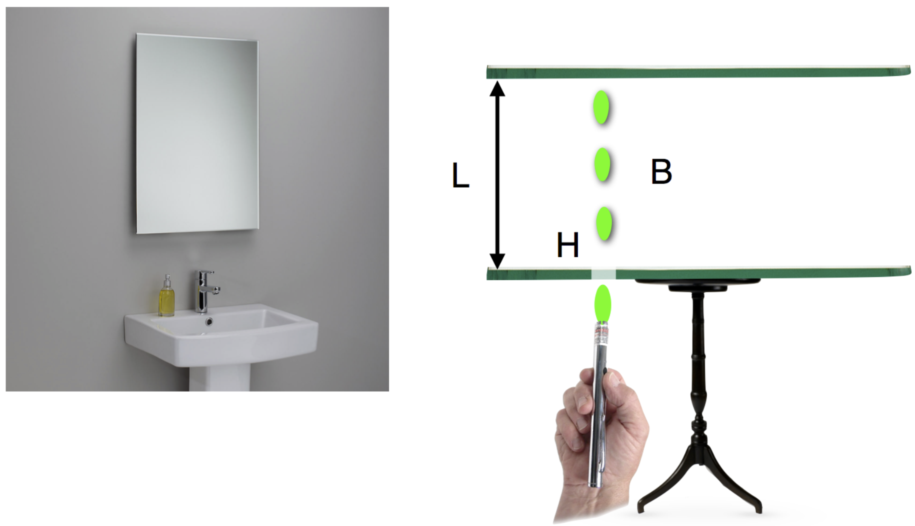

Figure 6: mirrors

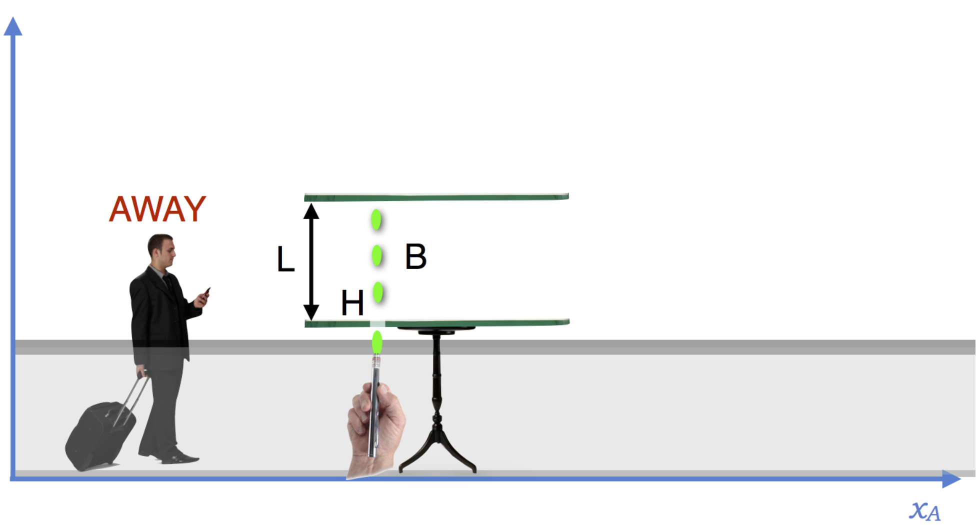

Figure 7: mirrorAF

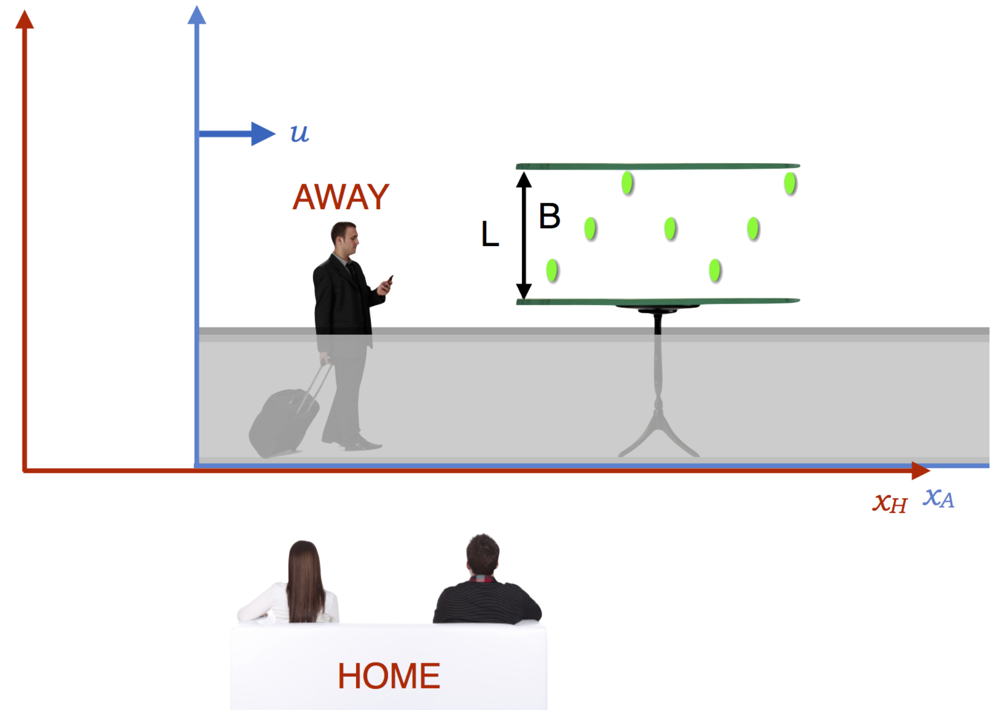

Figure 8: mirrorHF

Time Dilation

the derivation of the light clock would go here

This is an amusing result. It means that a clock in an inertial frame of reference as observed from another inertial frame of reference would appear to keep different time by this factor:

$$t_H = \left(\frac{1}{\sqrt{1 - (\frac{u}{c})^2}} \right) t_A \tag{1}$$

$$t_H = \gamma t_A. \tag{2}$$

I've introduced a new quantity, γ, which is an important function in relativity called the "relativistic gamma function." (Say "gamma" to a physicist, and she'll know it to be this thing.) This situation in which time intervals would be measured by two observers to be different is called **Time Dilation**. It seems a crazy thing, except that it's truly the way nature works.[4]

[4]...which we can now declare is the beginning of "crazy" in modern physics.

definition, time dilation

Let's also define a second useful quantity that we'll need it a lot, and that's the ratio of the velocity of a reference frame, u, to that of the speed of light. We call that "beta," β:

$$\beta = u/c \tag{3}$$

Since nothing can go faster than the speed of light (we'll see why in a bit), β can only be less than or equal to 1, or $\beta \leq 1$. So that the gamma function can be compactly written:

$$\gamma = \left(\frac{1}{\sqrt{1 - (\frac{u}{c})^2}} \right)$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{4}$$

Now while Eq. 4 looks complicated, we don't need to evaluate it for our purposes. Let's graph it and then we can refer to that plot for the whole story. Fig. 9 shows γ as a function of β. (Figure 10 shows it more precisely in the region $\beta < 0.6$ which might be useful for you.)

Wait. *So, how in my life does this matter?*

Glad you asked. *It matters in big ways and in small ways. Let's get a feel for just how fast, is fast!*

First let's look at Eq. 4 at two extremes. At very slow velocities, then u would be very small compared to the enormous c, so β would be very small as well. Given where it's located in the form of γ you can see that the denominator would be very close to just 1 and so γ would be close to 1. That's the first extreme: a non-relativistic limit is one in which $\gamma is \sim 1$.

The other limit is when a frame is moving with a very high relative velocity, so that $\beta = u/c$ is a number close to 1. The closer to 1. The closer β is to 1, the smaller the difference in the denominator is and the larger γ is. So that's the second extreme: a highly relativistic limit is one in which γ is very large. From its shape in Fig. 9 you can see that it grows very quickly as β passes the 0.8 mark. When $\beta = 1$? Well that's not physically possible—and mathematically uncomfortable—as we'll see. Let's look at some representative speeds.

One of the fastest man-made objects might be a rocket with enough speed to reach the "escape velocity" necessary to break free of Earth's gravitational pull. That speed is 7.9 km/s which is about 20 times the speed of sound. What's the β for such a rocket?

$$\beta = \frac{v_{\text{escape}}}{c} = \frac{7.9 \times 10^3 \text{ m/s}}{3 \times 10^8 \text{ m/s}} = 0.00003$$

So look at Fig. 9 or even more usefully, Fig. 10. A β of 0.0003 is about the size of a single pixel at the most left-hand portion of that curve, so the γ associated with such a rocket is effectively just

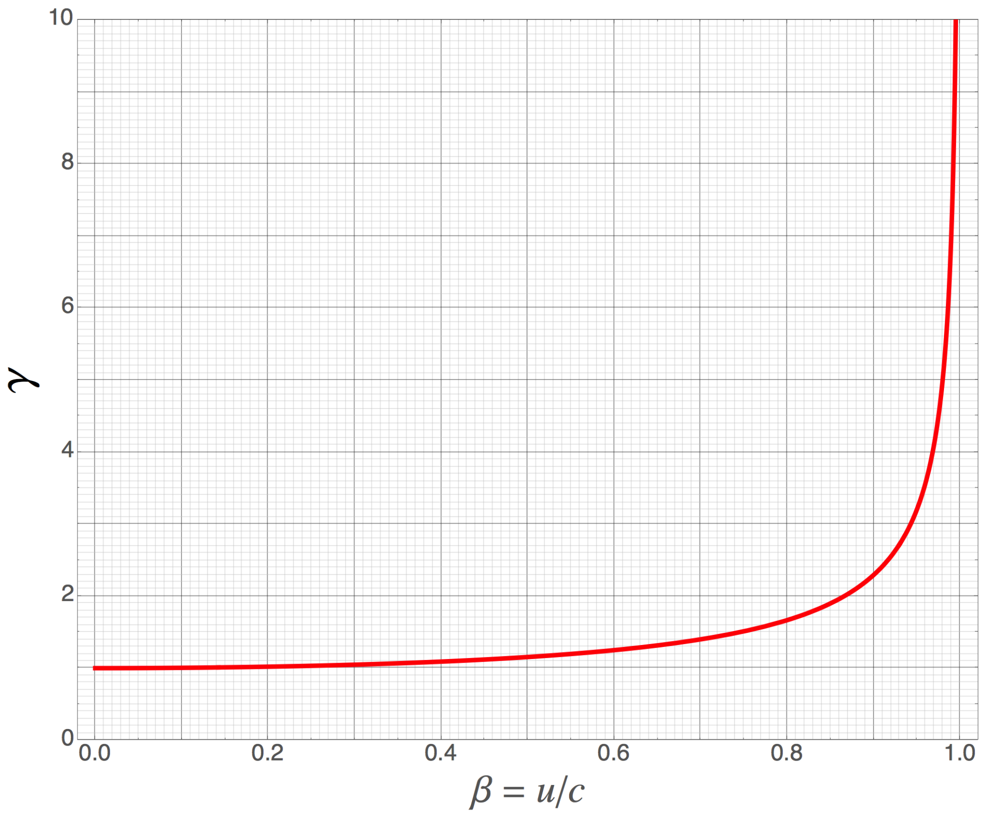

Figure 9: The "gamma function," γ shown over the entire range of β.

1.0, in fact, it's $\gamma = 1.00000000045$. This means that a clock on the rocket would essentially keep the same time as a clock on Earth which is one of the examples of how relativity is not an everyday concern. Even if you're a rocket scientist.

But what about the electrons in your parent's old TV set? For them, β is closer to 0.5. Those electrons needed to be precisely aimed at the TV screen from the back and precisely scanned across it, But they move so fast from the electron gun at the back of the set that electrical engineers need to take into relativity into account or Lucy and Desi would have looked funny. So, sure, small things move fast.

What about an object bigger than the whole solar system?

Quasars as an example...

So the relationship between two inertial frames, measuring a time interval is:

$$T_H = \gamma T_A \tag{5}$$

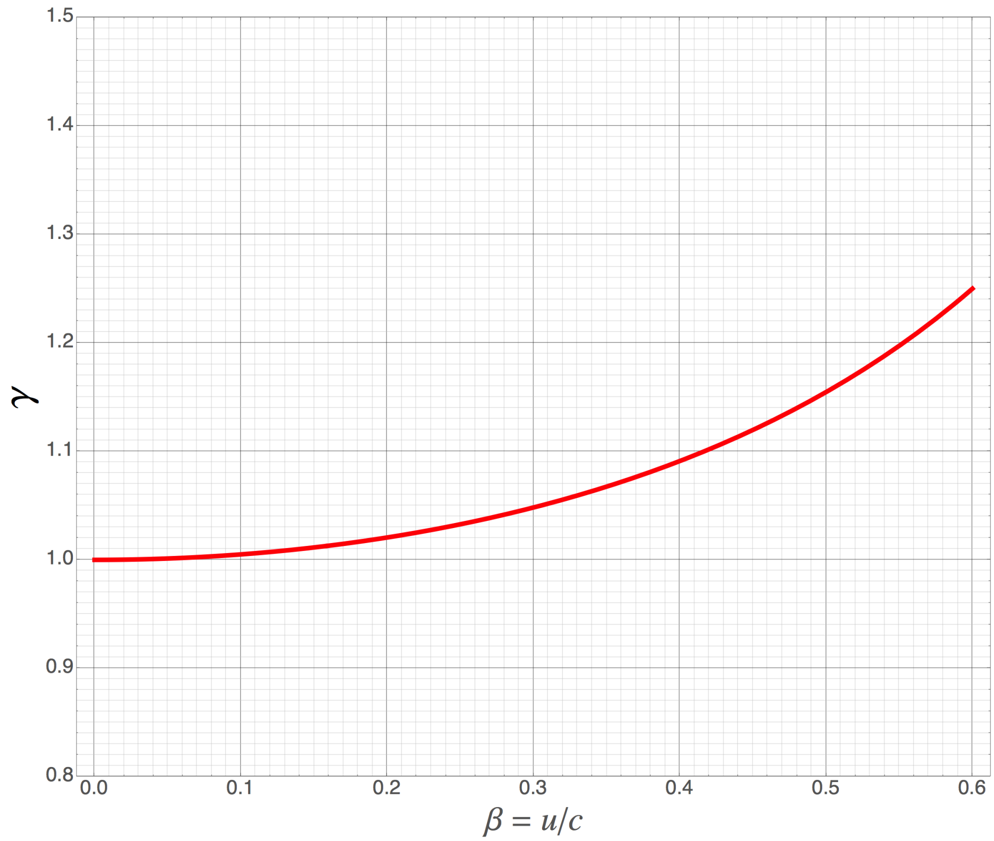

Figure 10: The γ function at lower speeds, for $\beta < 0.6$. Notice that the vertical axis starts at 1.0.

Wait. *So a clock in a high-speed frame would run nearly infinitely slowly?*

Glad you asked. *Two responses. First, it's more correct to say that a clock in the AF would appear to run infinitely slowly as viewed from the HF. And second, well, yes, but with a big caveat. We'll see that an object with a mass can never reach the speed of light. But if infinity is a prediction about Nature, absurdity is the cause. That rings true given the paradoxes that we just talked about!*

The velocity of light is obviously special. Remember how our CoachPeople measured the speed of light to be the conventional c for the WearyTraveler's machine? We just saw that time becomes a function of speed—it's warped as viewed across inertial reference frames: clocks appear to move slower. Since c is a speed, so it's a ratio of space and time, if time is warped in order to keep the speed of light constant, something must happen to space also! Hey, you're pretty smart! That's exactly what happens.

Rather than writing out the derivation, which is a little complicated, let me just report the result. For WearyTraveler's three rulers, which amount to 3 feet in his frame, CouchPeople would not see them to be 3 feet, but something shorter. Space also appears to be warped across inertial frames to the tune of:

$$L_H = \frac{L_A}{\gamma} \tag{6}$$

Here, as in Eq. 5, the Home and Away frames—this time for lengths—are related through the same γ function, but in the opposite way from Time Dilation. A HF observer would see that a length in an AF would appear to be shorter than that same length would be in the frame in which it's at rest. Said another way, if we have a meter stick on the moving sidewalk—which would have the length of, well, 1 meter there—as viewed from the ground, that meter stick would appear to be *shorter* than 1 meter. This phenomenon of observers in co-moving inertial rest frames measuring lengths to be different by virtue of their relative velocities is called **Length Contraction**.

Wait. *Why "dilation" and why "contraction"?*

Glad you asked. *Clocks in the AF appear to run slower as measured from the HF, and so the interval between tick-tock is longer, hence "dilation." You have your eyes "dilated" at the eye-doctor and pupils get large. A length in the AF appears to be shorter when measured from the HF, hence "contraction." You can see this in the two formulae since $\gamma > 1$.*

definition, LC and TD

examples, all using the sidewalk with explicit use of rulers

So there we have it. What does our c-measuring device actually do? First, we use 3 rulers as a length and then we use the internal crystal clocks of the oscilloscopes to measure time. But each of these quantities are messed with from the perspective of the airport frame as compared with the sidewalk frame. So if we fashion a ratio of

$$\begin{aligned}\frac{\text{(distance in the sidewalk, measured by HF)}}{\text{(time in the sidewalk, measured by HF)}} &= \frac{L_H}{T_H} \\ &= \frac{L_A\gamma}{\gamma T_A} \\ &= \frac{L_A}{T_A} = c.\end{aligned}$$

That last line comes from WearyTraveler's measurement. So the Second Postulate implies a consistency, if not a troubling one.

4 Coordinate Transformations, 2

When Einstein forced his two postulates onto Maxwell's Electromagnetism, the outcome was a new set of coordinate transformations, which by now you'd not be surprised to learn treat time in a new way. Let's reprise the Galilean Transformations, Eqns. **??** and **??**:

✎

$$x_H = x_A + ut \tag{7}$$

$$t_H = t_A \tag{8}$$

Einstein found a set of transformation equations for space and time that had been previously found by Hendrik Lorentz (1853–1928)[5] who had been manipulating Maxwell's Equations also, but with a very different intention and with the firmly held belief that the ether was an absolutely stationary frame of reference. So traditionally these equations are called the Lorentz Transformations. We'll not use them explicitly, but we can learn a lot by just looking at them. These are the two Lorentz Transformation equations, one for space and now one for time:

$$x_H = \gamma\,(x_A + ut_A) \tag{9}$$

$$t_H = \gamma\left(t_A + \frac{ux_A}{c^2}\right) \tag{10}$$

Look at Eq. 9. It looks familiar and indeed, but for the factor of γ, it's identical to Eq. 7. Either from Fig. 4 or (and?) from the definition in Eq. 4, we see that if the relative speed of the AF as compared to the HF is very much slower than c, then γ is for all practical purposes, extremely close to 1:

[5] We've already "met" Lorentz when we worked on the forces that **E** and **B** fields apply to electrical charges.

$$\gamma(u << c) \to 1.$$

If that's the case, then we recover the Galilean Transformation for space coordinates. How about the time transformation?

Equation 10 is strange at first since it depends on the space coordinates. It says that the time intervals as measured between two inertial frames would be different, but since we've already gotten our heads around Time Dilation, perhaps this is not too surprising. Further, I'll bet at this point you know what would happen if again, the relative speed between frames is very slow. In that case, $\gamma \to 1$, but also the second term in Eq. 10 has the quantity $\frac{u}{c^2}$ in front of the x_A, which when $u << c$ is very close to zero, so we get back that $t_H = t_A$, that unquestioned presumption in the Galilean transformations.

4.1 Maxwell's Equations, 20th Century Edition

The transformations of space and time were what Einstein needed in order to make good on his Postulate 2 promise. But now let's think more specifically about electric and magnetic fields in relatively moving frames of reference. Instead of a ruler or a clock on our sidewalk, let's load up a magnet and ask how an AF observer (riding with the magnet) and a HF observer (on the ground) would describe its magnetic field. They would both rely on Maxwell's equations which include x and t variables. But in order to separately apply them, the HF observer would take the field equations and transform the space and time variables according to the Lorentz transformations. Remember this would be required in order to maintain the constant speed of light between the two reference frames. Upon making that transformation, something remarkable happens.

Let me show off for a minute. Please? I want to write one of Maxwell's four equations for just one direction in space. Afficianaos will write this slightly fancier. But I want to make a point. Remember the fact that changing a magnetic field in time creates an electric field? (The magnet moving through a coil of wire, setting up a current?)

Here is a simplified version of one of the equations that describes this phenomenon.

$$\frac{\Delta E}{\Delta y} - \frac{\Delta E}{\Delta z} = -\frac{\Delta B}{\Delta t}$$

There. **E** and **B** are functions of space and time. What you see is on the right hand side how if a magnetic field, B, changes in time (the Δ's mean "change of" remember?), then the result is an electric field that's changing in the by the amount of y and z. So there are space and time coordinates all over Maxwell's description of light.

If we're observing some electromagnetic phenomenon on the sidewalk from the airport, the constancy of the speed of light forces us to modify those space and time coordinates and this has physical consequences. Figure 11 is a simple example. WearyTraveler has a magnet with him in his reference frame. The field lines drawn on the picture are those of a bar magnet and that's what *he sees*! What do CouchPeople see? Remember the x's, y's, z's, and t's in Maxwell's equation above? We have to transform them in order to describe what the CouchPeople see. And it's weird and wonderful.

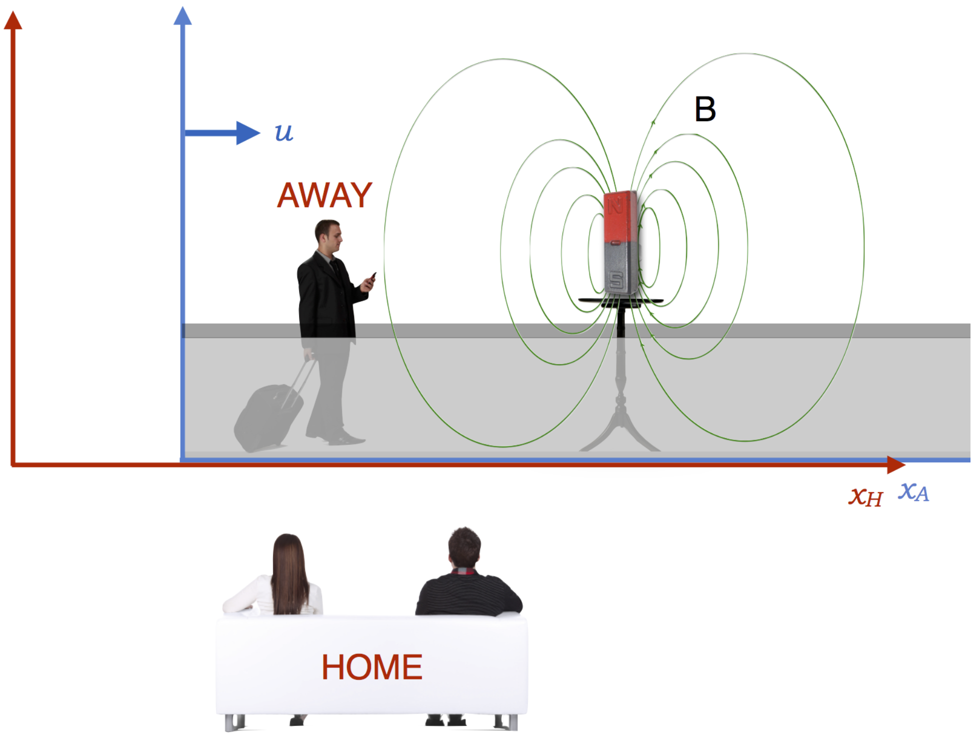

Figure 11: sidewalkmagnet

A **B** field, say from our magnet in the AF looks to the HF to be *a mixture of an E and B field*! An electric field in an AF, **E** (like that emeging from a stationary electric charge) when transformed into the HF appears as a *mixture of both an E field and a B field.* So while our AF, WearyTraveler observer sees only a magnetic field from his magnet, our CouchPeople observers would say, "No!" They would see both a magnetic *and* an electric field!

Relativity does it again. Just like it has taught us to merge space and time from separate concepts to a single spacetime. Space and time have no separate meanings any more. It also forces us to conclude that there is really no such thing as an absolute, permanent electric field or an absolute, permanent magnetic field since relatively moving observers will disagree about their natures. What's "relativistically" appropriate is the **Electromagnetic** Field—a single entity—which will manifest itself in different mixtures of **E** and **B** depending on the frame of reference from which it's observed. Spacetime and Electromagnetism as combined things is what has meaningful existence.

What's further a surprise—and indicative of Einstein's Postulate #1—is that Maxwell's Equations themselves turned out to be perfectly invariant with respect to co-moving inertial frames... but *Newton's Second Law* is not. Maxwell wins, and Newton loses in Special Relativity! A different, actually pretty complicated relationship needs to be substituted for good old $F = ma$ in order to be relativistically correct. As you might expect, for low speeds of co-moving inertial frames of reference, that more complicated relationship reduces to regular, old $F = ma$ when $\beta << 1$.

This mixture of the individual electric and magnetic field vectors *solves all of the original paradoxes* that we met in the previous chapter. All is well with Maxwell's equations and light, but mechanics turns out to be subtly odd.

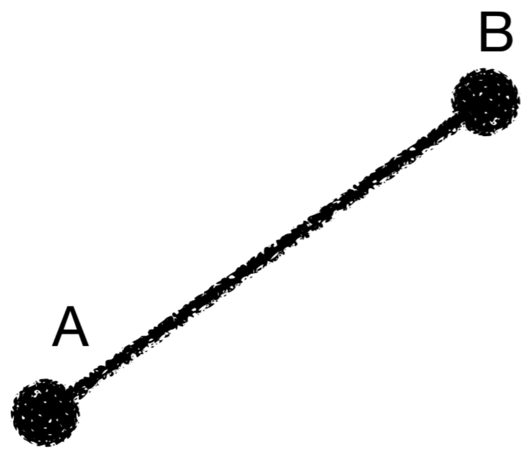

Figure 12: lineAB

5 Invariant Intervals

Now you can imagine why this theory is called "relativity." You've heard it said all the time: "Everything is relative." But it's not true! And even Einstein himself disliked the name "special relativity." He wanted to call his theory "Invariant Theory" because for him, what was most important was what stays the same between two relatively moving observers: the laws of physics and the speed of light. But "relativity" stuck. Let's think harder about this.

5.1 Space Invariants

Let's do some geometry and take something that's simple, and find out that it's also pretty.

Go to your wall with a pen and draw a straight, diagonal line of length L, from point A to B, as in Fig. 12. Mom won't mind, since you're probably not at home. Now take the pink coordinate axes in Fig. 13 and place it with the origin at A and the x axis horizontal. From Pythagoras' Theorem, you can calculate the length of your line as

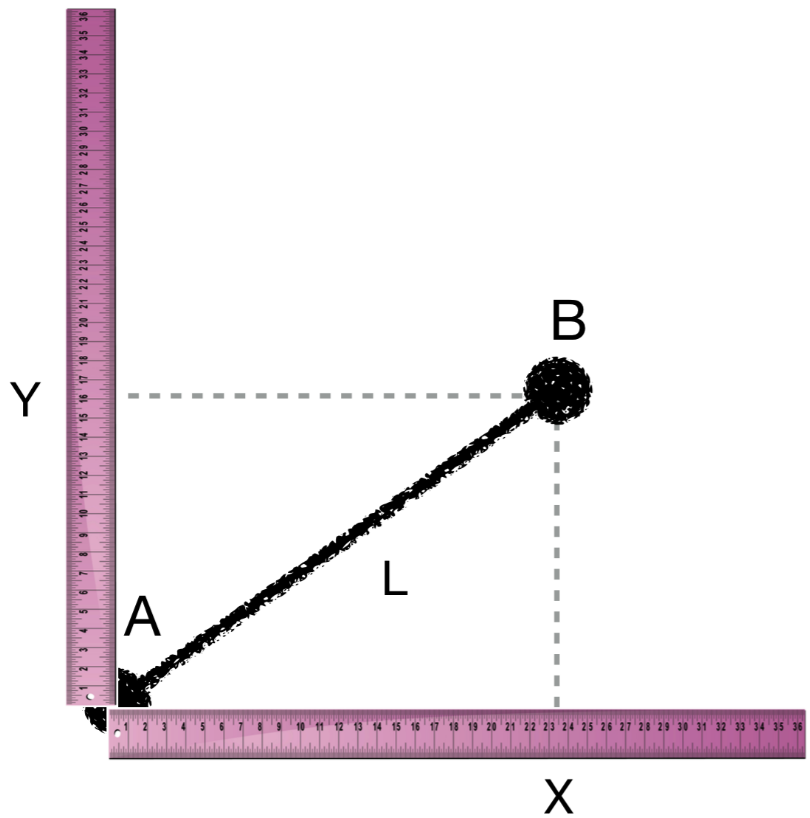

Figure 13: line1

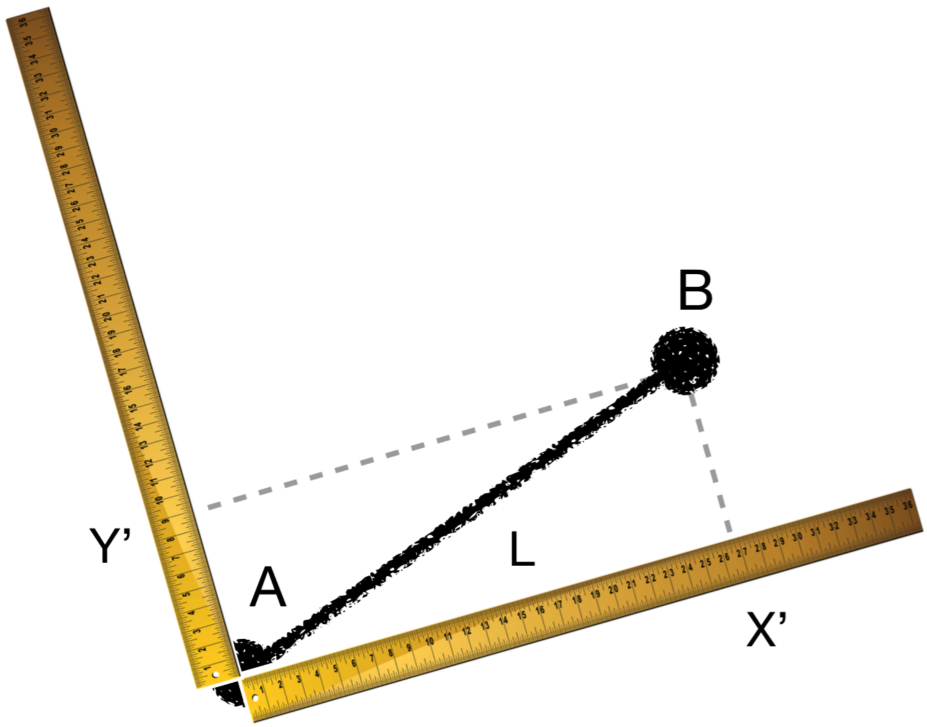

Figure 14: line2

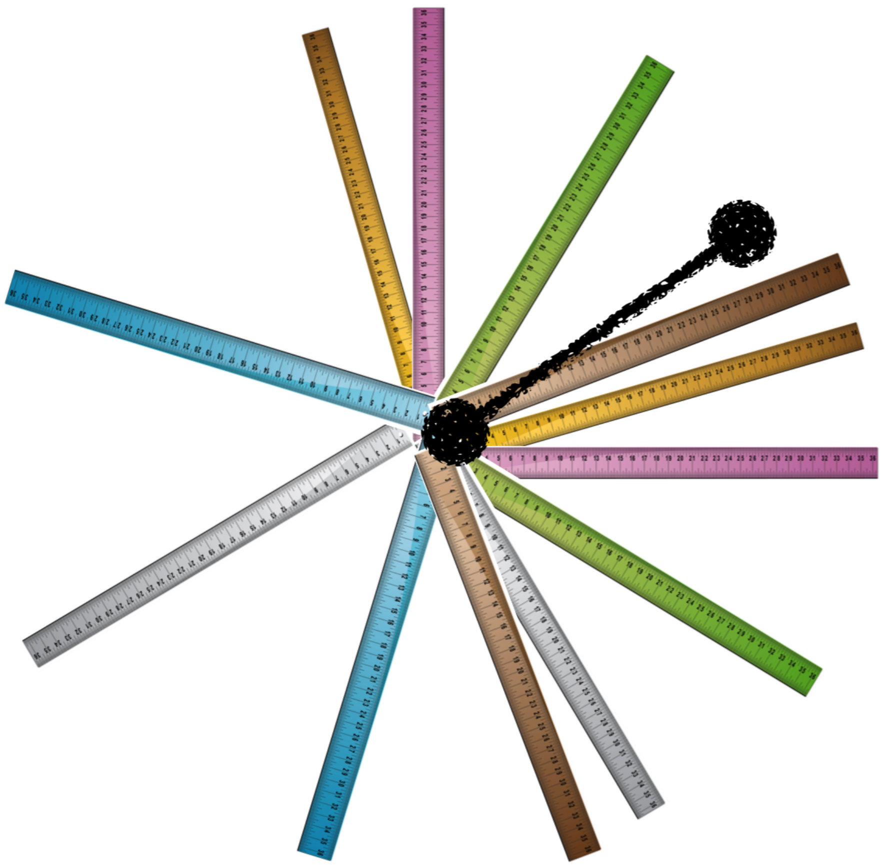

Figure 15: line3

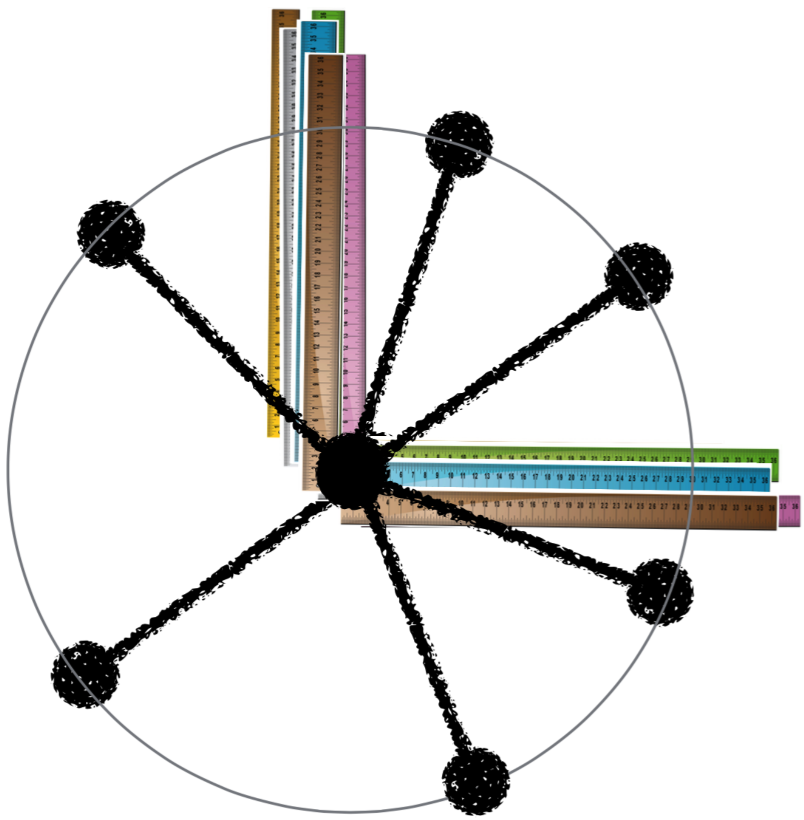

Figure 16: line4

$$L^2 = X^2 + Y^2 \tag{11}$$

Now take the yellow set of axes shown in Fig. 14 and measure the length of the line again. You'd say that it's

$$L'^2 = X'^2 + Y'^2 \tag{12}$$

right? But is the line any different? Of course not, so clearly

$$L = L'.$$

We could add more similarly rotated coordinate systems as in Fig. 15 and while the individual x and y coordinates of B would be different, they would all yield the same length, $L = L' = L'' = L''' = \ldots$ etc. We would say that the length of the line is **invariant** with respect to a rotation of the coordinate system. This is an important property of Space: lengths in space are constant, regardless of the reference frame from which they are viewed. Because, all of those different ruler combinations are just different reference frames.

One more step. Let's rotate the line and the rulers all about point A so that the ruler coordinate axes all overlap as in Fig. 16 and you can see that the B-end of each rotated line traces out a circle, which I'll call the **Invariant Curve**.[6] If we took this to 3 space dimensions, the Invariant Curve would actually be an Invariant Sphere. If we went to 4, or 5 or more space dimensions, the Invariant Curve would be an Invariant *Hypersphere*. The distinguishing feature of all of these curves, even beyond our familiar three dimensional space, is that the invariant quantity always looks like:

$$L^2 = x^2 + y^2 + z^2 + a^2 + b^2 + ...$$

The important thing about are those + signs for all of the coordinate combinations. Such a multidimensional space is called a **Euclidean Space** since it obeys all of the rules of geometry going back to Euclid in Hellenistic Greece.

5.2 Spacetime Invariants

But we're working in relativity now where we're beginning to mix space and time and so the natural question is whether there's an Invariant Curve...in spacetime? Is there some spacetime "length" that is always constant, regardless of the coordinate system, which is the same as asking whether there's something that *stays the same* even for co-moving, inertial observers?

Maybe since I've asked the question, you know that the answer is "yes" but it's a very subtle point. Let's make a baby. Um, methaphorically.

[6]Remember that Eq. **??** is actually the equation of a circle with radius L.

Causality and Babies

We need to set up a "geometry" of spacetime, which we'll represent as Cartesian coordinates but instead of axes which are both space, we need one of them to represent *time*. Now there are two problems with this:

- Time is not a space-ish coordinate, in the sense that time's unit—*seconds*—is not the same as length's unit, *meters*. So we need to get them both on the same footing and we'll choose space dimensions (meters) as our standard and turn our times into space-lengths by multiplying by c. In that way, ct (length/time times time) is has the units of length, while still *functioning* as a time coordinate. We'll plot ct as the x-axis in our spacetime diagrams just like we have before, except now it will be a length. So for a time interval of 1 second, the time-as-space amount would be $c \times 1$ second or $(3 \times 10^8 \text{ m/s}) \times 1 \text{ s} = 3 \times 10^8$ m. This is also the distance that light would travel in 1 second: 1 *light-second*.[7]
- Second, we can't really draw more than two dimensions on a flat surface, so we'll abstract all of the space coordinates into just one direction and plot that against ct.

Figure 17 is a representation of our spacetime axes, just like we've used before.

Now back to our question: remember what an invariant curve is. For different reference frames, that curve is what all observers would agree on. For our drawing of the line on the wall, each pair of rulers-as-axes represents a different observer. They differ in their rotational relationship around their common origin and each "observer" agrees that the line has the length L and that the ends of the different "L's" form the locus of a circle when the observers are put on the same footing. What's an invariant curve in *spacetime*?

The most natural thing would be to try a circle, just like we had for space, but now in spacetime coordinates, as shown. Since the time axis is horizontal and since we put $ct = 0$ at the center, everything to the right of the center is the *future* and everything to the left, is the *past*.

We'll start our clock (where $t = 0$) at the moment of the birth of a beautiful boy. We'll (Curiously) call him Benjamin (Button). We'll not do this on the airport sidewalk, but rather in a hospital room, Benjamin is born and then without moving in space, he cries. That's represented as point **A** on the plot: his space coordinates stay constant and his young time duration increases slightly.

Let's drive by the hospital in our car and think about what we might observe from within our frame of reference, the HF. The hospital is now the AF and is moving by us in the other direction.[8] We peer into the room and see that Benjamin was indeed born, but when he cries, in the HF coordinate system, he cries at point **B**, where he appears to have moved in space, so vertically on the diagram (since the whole hospital appears to us to have moved in space).

But now let's imagine another car-based observer moving the other direction relative to us. The same blessed event happens, but for that observer the hospital (the AF) is moving in the other direction. We can put that on our spacetime diagram. But it's strange: low look at point **C** which obviously doesn't make any sense. That other observer sees that Benjamin first cries... and *then he's born* since the crying happens to the left of $ct = 0$ where Benjamin cries before birth! Since this

[7] You've heard of "light-years" which is the same sort of thing, except it's the distance that light would travel in 1 year. It's a handy unit if you're an astrophysicist (or studying QS&BB).

[8] It might be useful to think of us on the airport sidewalk. To us, now the sidewalk is the HF and it's of course stationary with respect to us and the whole airport is moving relative to us and is the AF.

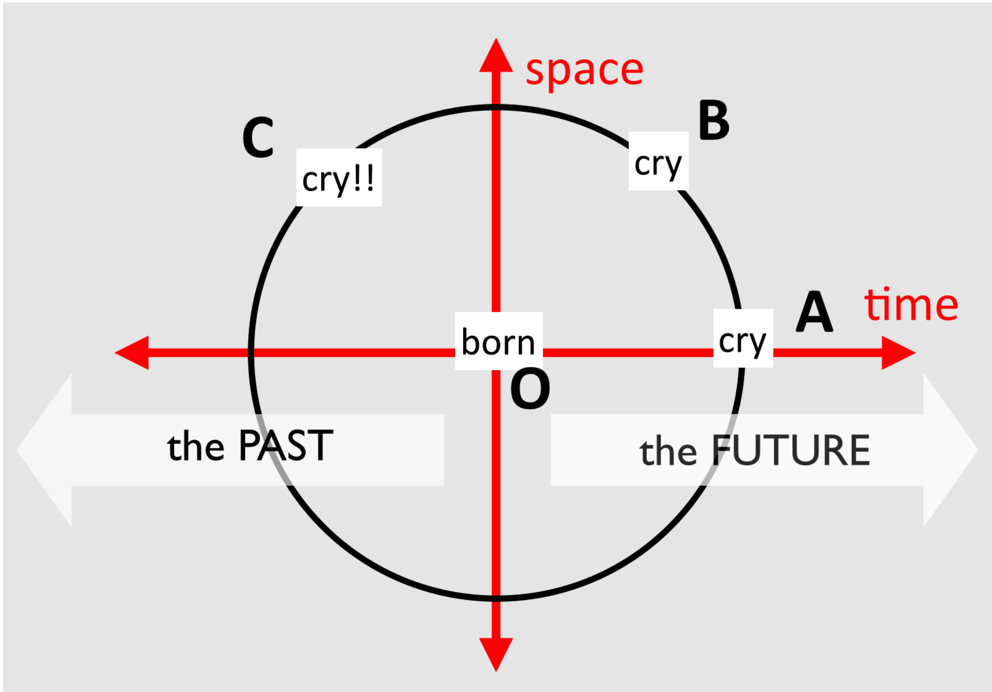

Figure 17: spacetimeborn1

creates a cause-effect reversal, our "Euclidian" assumption of a circle as the invariant spacetime curve must be wrong. We can't mess with Causality!

Space and Time: Doomed to Fade Away

This is where Einstein's critical mathematics teacher, Minkowski comes in because it was he who worked out the technical mathematical basis of Special Relativity and we've been using his terminology of "spacetime" all along.

Einstein later remarked that, "Since the mathematicians have grabbed hold of the theory of relativity, I myself no longer understand it." But he later came to understand the fundamental importance of Minkowski's work and publicly acknowledged that, but unfortunately only after Minkowski unexpectedly died at 45 years old.

6 Spacetime

Let's figure out what thinking in terms of spacetime implies. Figure 18 lays it out and introduces a new concept.[9] We take it that no signal or material object can move faster than the speed of light. So if we set up a Global coordinate system in spacetime, then this boundary corresponds to a line with a slope of 1.

$$\text{slope} = \frac{x}{ct} = 1$$

[9]Notice that I'm now plotting ct rather than t as per the bullet above.

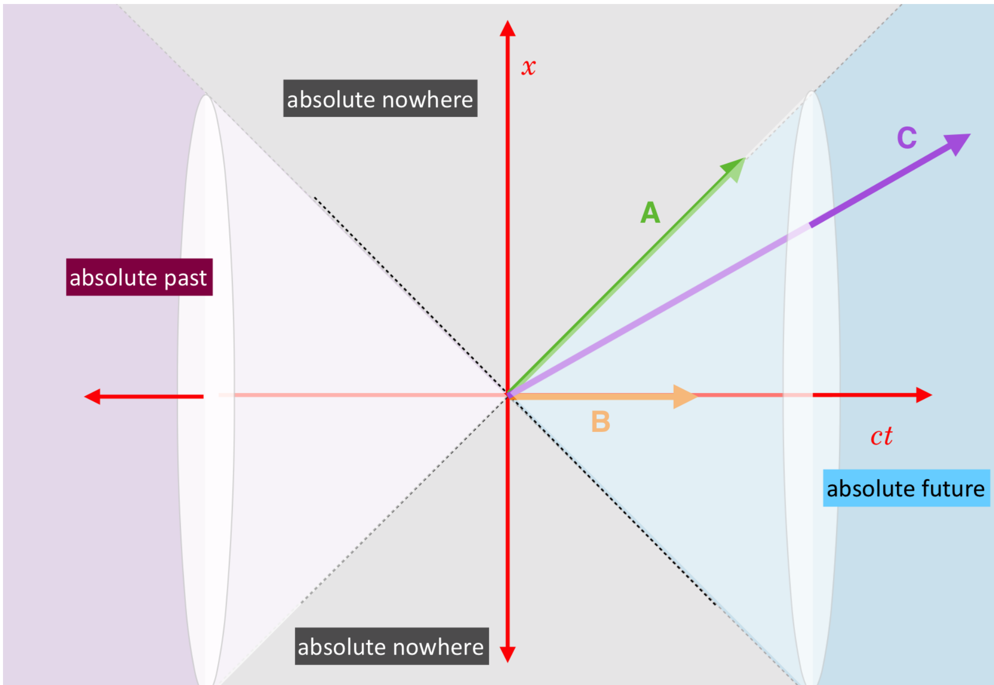

Figure 18: worldlines

which of course implies that

$$\frac{x}{t} = c.$$

This is in one space and one time dimension. If we expand to imagine two space dimensions, then these two boundary lines represent the surface of two cones oriented to the right (the future) and the left (the past) along the ct axis directions.[10]

So anything traveling at the speed of light[11] would travel in spacetime along the top or the bottom diagonal lines.[12]

A is a trajectory of a positively directed light beam.

Anything that's sitting still is still on a special spacetime trajectory, all of it's "motion" is along the ct direction, just in time. B is such a path. Finally, anything moving at speeds less than c would fall within the cone and C is such a trajectory. That's where we live in our everyday, sub-luminal lives.

These cones are special in Relativity and together they are called the **Light Cone**. Since nothing can travel faster than the speed of light, all real, physical trajectories must lie within the Light Cone and such trajectories are called the **Worldlines**. You can think that every object in the universe has a cone in spacetime attached to it that limits what the future might bring and what the past has been. I've drawn a worldline as a straight line, implying moving at a constant velocity,

[10] If we expand it all the way to our actual three space dimensions, we have to wrap our heads around the idea of a hyper-cone, which I'll not try to do myself. But feel free if you're so inclined!

[11] . . . um. That's only light.

[12] The top line would mean going in the $+x$ direction and along the bottom, in the $-x$ direction.

but real-life worldlines accelerate and decelerate and so they would trace out curves—but never with slopes steeper than 1.

What about outside of the Light Cone? Those are regions of space and time that are simply inaccessible to an observer and we call them the **Absolute Nowhere**, in order to make it sound spooky.

What Minkowski discovered was that there is indeed an Invariant Curve for relativity—a "spacetime length" that is the same for all observers in co-moving inertial frames of reference.

> **Wait.** *We've seen that times and lengths and even simultaneity are relative to a frame. How can there be something that's constant?*
>
> **Glad you asked.** *Indeed, separately time intervals and lengths do appear to be different from one frame to another, but separately. A particular combination turns out to be constant, just like x and x' or y and y' are individually different, but Eqs.* 12 *and* 11 *show that a particular combination is constant, namely as* L*. Watch what's next.*[a]
>
> [a] But we can't do anything about simultaneity! Sorry.

Remember, the invariant length for just space is

$$L^2 = x^2 + y^2.$$

The invariant length—called the Invariant Interval (or just "Interval" for short)—for spacetime in "(1,1) dimensions" (1 time and 1 space dimension) turns out to be:

$$s^2 = c^2t^2 - x^2. \tag{13}$$

That pesky minus sign makes all of the difference and Fig. 19 shows how. Equation 13 is the formula of a *hyperbola*, not a circle! Relativistic spacetime is hyperbolic and it's called **Minkowski Space**, quite different from Euclidean Space. Figure 19 shows this on the spacetime plot where the hyperbolae going left and right are the "real" Invariant Curves for our universe. Any trajectory or set of events will have spacetime points that must lie on a hyperbola, regardless of what reference frame they are in. Just like your wall drawings all have space points that lie on that circle of radius, L, a "length" in hyperbolic space is the same if it goes from 0 to any point on the hyperbolic curve.

This experience with Special Relativity is the first hint that unusual geometries figure into physics. Einstein reluctantly backed into it through his former teacher, but by the time he got to General Relativity, he rushed headlong into even stranger geometries.

We call that distance that always is the same—the one that lands on the hyperbola—the "Interval." It is the length of a line from the origin of the Worldline of an object to the surface of the hyperbola represented by Eq. 13. For space we called it L and for spacetime, we'll call the interval, s. Any arrow will do, and each one represents the space and time coordinates of inertial observers moving relative to one another, each being an AF, AF', AF", and so on. If an arrow lies on the horizontal axis, then that's special and that the object has not moved in position and that it has a value for the Interval of

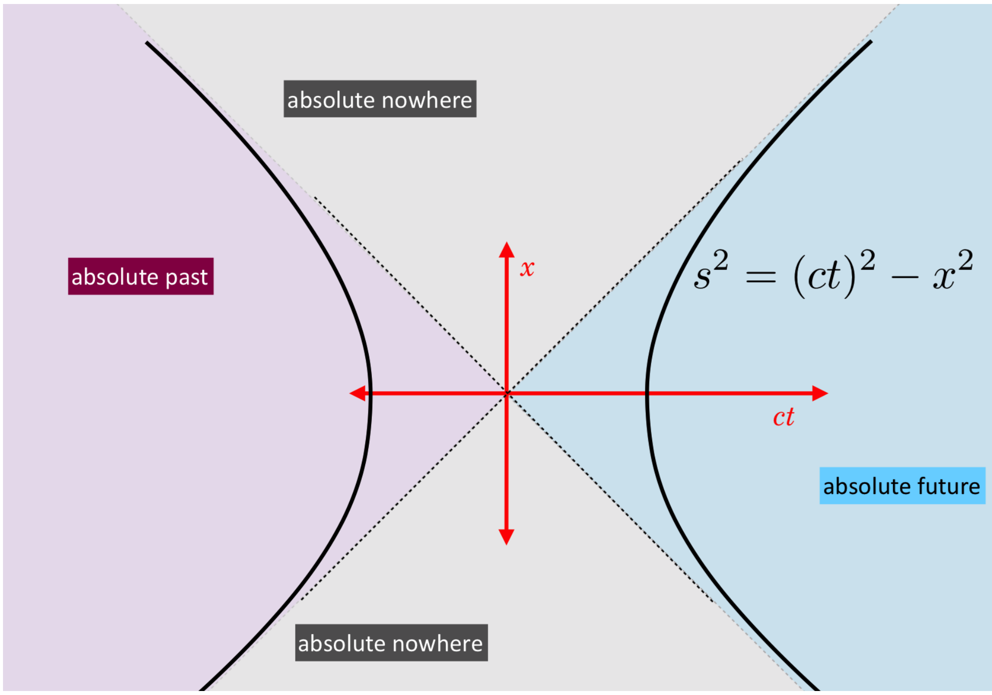

Figure 19: spacetimeinvariantnothing

$$s^2 = c^2t^2$$
$$s = ct \tag{14}$$

This is represents a unique frame: a worldline totally made up of time and hence from within the rest frame (the HF) of an object. Any motion relative to the rest frame involves a mixture of space and time coordinates that will satisfy Eq. 13 and become a family of different arrows. Let's go back to the airport.

It's a sad story. WearyTraveler on the moving sidewalk has spent his entire life there. From a small boy at the beginning of the trip until the current day, he's just moved along. His time is measured by the watch that he was given as a youngster and measures time in his frame. His space hasn't changed on the walkway.

Even more pathetic are the CouchPeople who have been sitting and watching WearyTraveler's life progress as he moves along in front of them. He's in their AF and they are still their own HF. They measure time with their clock. Figure 20 shows the sorry tale. Let's take this complicated figure apart:

- This is a picture of WearyTraveler's sorry life taken at two times: early and now.
- We see WearyTraveler as a child, at $T_A = 0$, which coincides with CouchPeople's original time, $T_H = 0$.
- We see that WearyTraveler never changed his position through his life: I've moved the coordinate axis so that WearyTraveler is standing at his origin. This way $x_A = 0$ that he was at as a child is still the same distance, $x_A = 0$ as an adult.

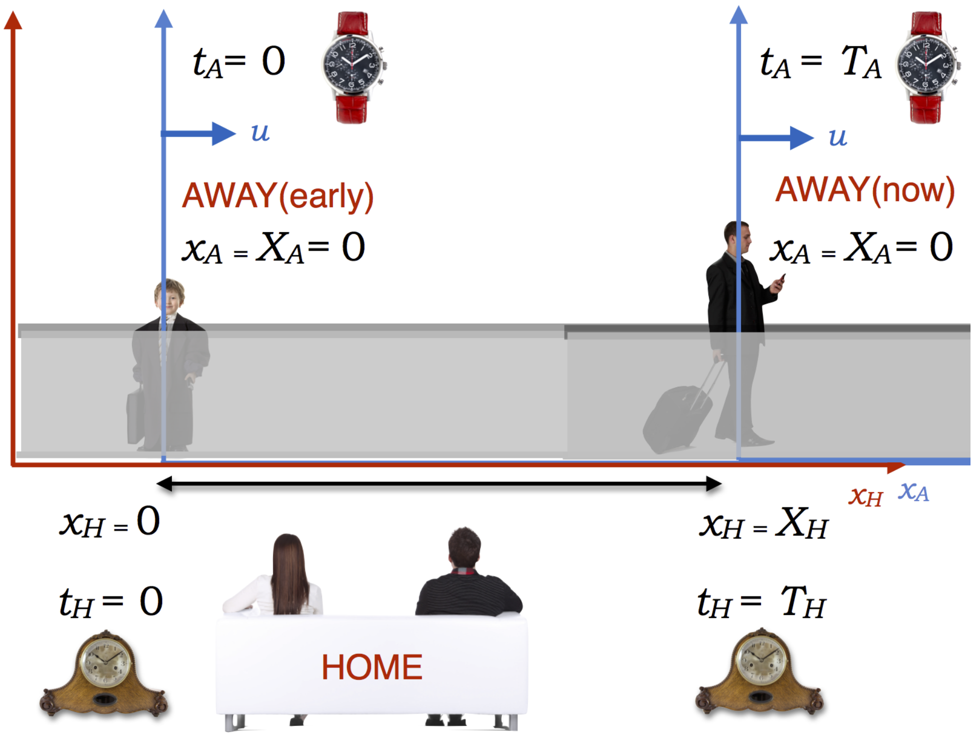

Figure 20: life

- But we see that in the airport, WearyTraveler appears to have moved from$x_H = 0$ toX_H.

What do they agree on? They agree on the value of the Interval, which as an increment (squared) in spacetime is:

$$\Delta s = (ct_H)^2 - (\Delta x_H)^2 = (ct_A)^2 - (\Delta x_A)^2$$
$$\Delta s = (ct_H)^2 - (\Delta x_H)^2 = (ct_A)^2$$

We've already thought about this situation and we saw different space and time interpretations by each frame. But now we can see that they agree completely on the value of the Interval. Figure 21 shows the situation. Here the arrow along the horizontal axis is the AF

determination. It's the time ($\times c$) of the WearyTraveler, in his frame. The other arrow measures the Interval for the AF as measured by the HF. These lengths are the same in a hyperbolic space!

7 Why Don't We Live Relativistically?

If relativity is right, why don't we see relativistic effects in everyday life? Let's look at the interval again, for two different frames, a HF and an AF. The interval would be the same for each:

$$s^2 = (ct_A)^2 - x_A^2 = (ct_H)^2 - x_H^2$$

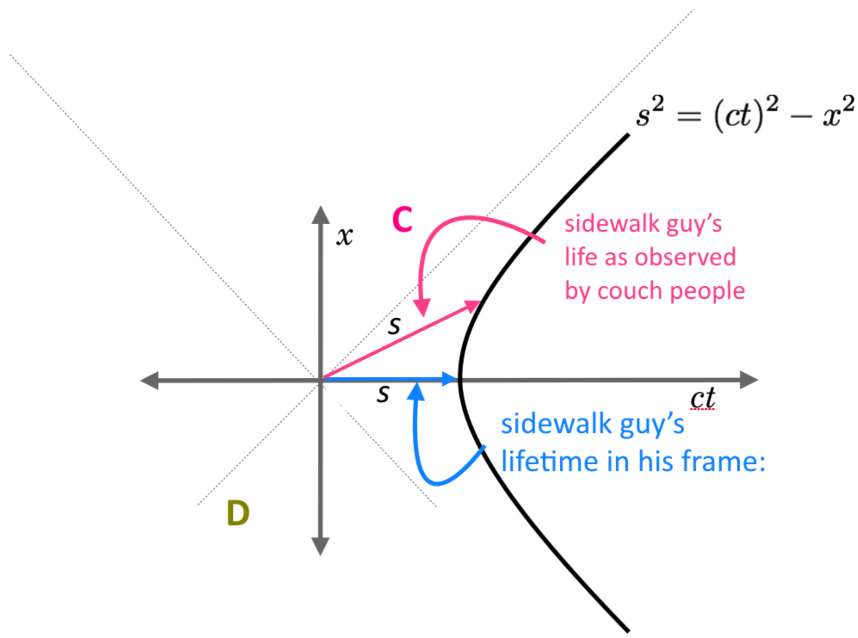

Figure 21: spacetimeinvariant

So each represents two different lines on the spacetime plot, pointing at the hyperbola. How about the tiniest time interval that a human might deal with... a single second in my HF.

$$ct_H = 3 \times 10^8 \text{m/s} \times 1 \text{ s} = 3 \times 10^8 \text{m} = 300,000 \text{ km}$$

This is almost the distance from the Earth to the Moon, so on human terms *the time piece of the interval dwarfs normal human-measure distances* that one might encounter. Since normal human speeds are tiny compared with the speed of light, then any time interval in the HF is going to be very close to the value as observed for the AF. So

$$s^2 \approx (ct_A)^2 \approx (ct_H)^2$$

That, in turn, means that the interval arrows for AF and HF in spacetime Fig. 22 are very close to one another and so everyday fames would be very similar and the time-space mixing would be negligible.

8 Simultaneity, Or Something

I've followed Einstein's thinking in reverse from his actual inspiration. Let's fix that now.

8.1 A Storm Broke Loose in My Mind

He was fixated on the electromagnetism contradictions and later recalled that after a night of thinking about them—when he was about to give up—that "a storm broke loose in my mind." He

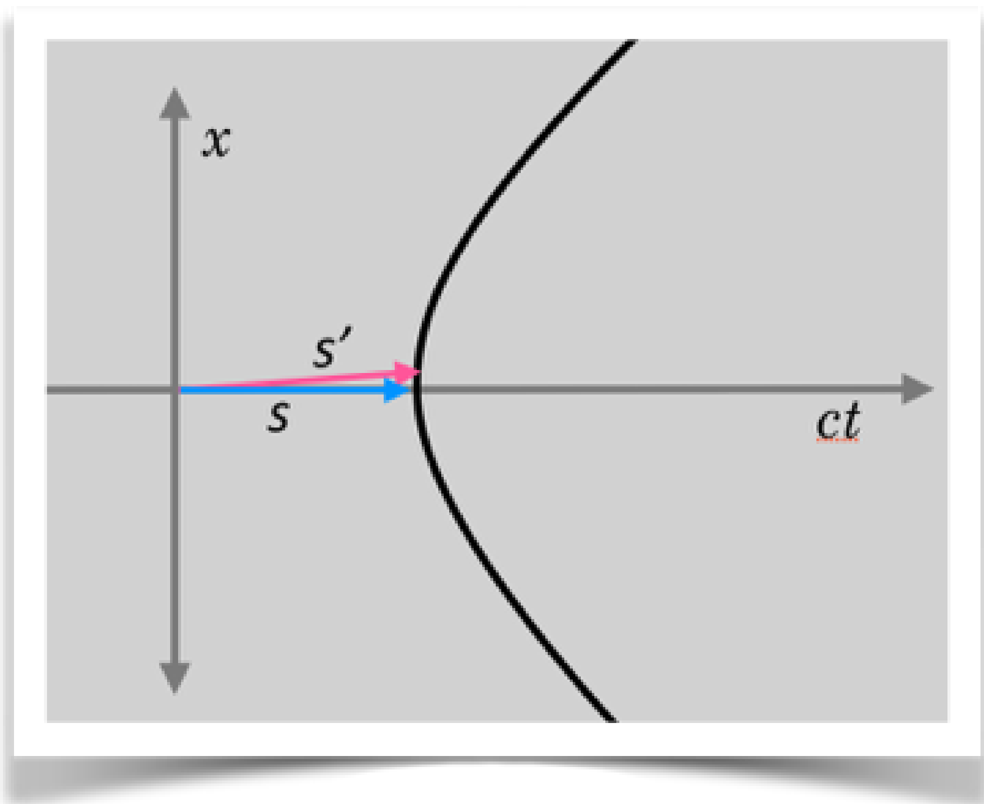

Figure 22: longtime

suddenly realized that the problem he faced was not about reference frame *speeds*. It was not about *Maxwell's Equations* and c. No. The problem that he was wrestling with was **Time** itself. Here's the thought that lit up his brain:

RIP, Simultaneity

Our blessed, Benjamin-birth tiptoed up to a philosophically dangerous situation, namely that a basic assumption about nature is that causes come before effects, and not the other way around. As silly as that sounds, Einstein had to make a serious philosophical leap in order to preserve it.

Figure **??** shows our airport folks yet again. Not yet tired of their light-measuring machine, they're at it again, this time on the sidewalk alone. They've rigged a light bulb that WearyTraveler has carefully and precisely placed midway between his two light sensors, turning them both to aim at the bulb. Here's the experiment: when the bulb is turned on, what is the difference in times for the light to reach the two sensors?

In the rest frame of the bulb, apparatus, and WearyTraveler... it's obvious. The light should reach both sensors at the same time. In the frame of the airport, it's different: The right hand sensor is running away from the light beam, so it would take the light more time to catch up with it. And the left hand sensor is coming toward the light beam, so it would reach quicker. So WearyTraveler would say, "Simultaneous!" But CouchPeople would say, "No! Left hits before Right.

Can't they calibrate their equipment to take into account the motion? Here's how a classical physicist before Einstein might have thought of this kind of circumstance. The appearance of the lack of simultaneity can be fixed if one just added or subtracted the speed of light and the speed of the sidewalk. Think of it this way. Suppose we've got a tug and a distance L behind it, a barge. The captain in the tug wants to synchronize his watch with the sailor in the barge. When the sailor sees midnight on her watch she yells to the captain, "Midnight!" The captain hears her but does he then set his watch to midnight? No, he knows that by the time he's heard her announcement, some time has passed since the speed of sound is finite and actually humanly slow. So he calculates how much time it would take and subtracts that time from his watch and sets it so that it would match the sailor's watch. That works just fine if they're sitting at port. But what if they're steaming ahead... now he has to adjust the time not just for the distance between tug and barge, but also for the fact that the tug is going away from the barge, so it would take still longer for the sound to reach him. He can do that because!... he knows the speed of sound in air, which is where the sound propagates. It's an absolute reference frame for sound: the speed of sound is constant only there and apparently different for other moving frames. But you can always calculate things back and synchronize watches, etc. In this situation.

> **Wait.** *So why can't WearyTraveler and CouchPeople make that same kind of re-calibration for the relative speeds of light and sidewalk?*
>
> **Glad you asked.** *Haven't you been listening? Sorry. Too snarky. The difference is that there's no analog of the air, in which the speed of light is c and only there is it* c. *Since the speed of light is the same value in all inertial frames, there is no way to make that correction.*

The concept of simultaneity is a relative concept: two events that are simultaneous for one inertial frame are not simultaneous in another. Nobody is right and nobody is wrong and this has consequences for what it means to make a measurement, a situation that Einstein called out in the opening paragraphs of his 1905 paper. There, he almost patronizingly notes,[13]

> "We have to take into account that all our judgments in which time plays a part are always judgments of simultaneous events. If, for instance, I say, 'That train arrives here at 7 o'clock,' I mean something like this: 'The pointing of the small hand of my watch to 7 and the arrival of the train are simultaneous events.'"

This simultaneous event—the clock pointing to 7 when the train arrives—is the case only for observers standing right with you in your frame of reference. Any other relatively moving observer would disagree. Their simulteneity-watch-setting would be different, no worse, no better.

Here's the shocking reasoning: If you cannot rely on things being "simultaneous" then you cannot agree on the Time of an event. If you can't agree on the time of an event, then Time suffers a humiliating demotion from something Absolute to something relative. This is what so excited Einstein. Nobody had before him thought of the possibility that time was not as Newton insisted...that it didn't just flow absolutely from past to future independent of anything external to it.

> Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year. Isaac Newton, Principia

> People like us, who believe in physics, know that the distinction between past, present, and future is only a stubbornly persistent illusion, Einstein in correspondence with the family of his deceased friend, Michele Besso.

> An analysis of the concept of time was my solution. Time cannot be absolutely defined, and there is an inseparable relation between time and signal velocity. Einstein

8.2 What is Time?

> If you do not ask me what is time, I know it. When you ask me, I cannot tell it. Saint Augustine

This was a profound discovery for Einstein. He realized that as a result there is **no special meaning to the notion of the time of an event**. When he realized this everything about relativity flowed in a couple of weeks from that awakening. He wrote his paper, gave it to his wife to check for mistakes, and then took to his bed out of sheer exhaustion.

Later he defined time: It's what a clock reads. Nothing more, nothing less. If what clocks read differ for different observers, then time is no longer an absolute.

[13] This was a very strange paper. One goes many pages into it before running into any mathematics. The first half-dozen pages are stories and careful definitions of what it means to make a measurement. Presumptive for a rookie, unknown scientist.

Now

What's "now" for you? It's our own personal notion—that we all share—that we're a part of a big universe in which things are happening... now. But given two aspects of our discussions, this is a highly complicated idea. First, signals about what's in our now-universe are not instantaneous, but they can only reach us at the finite speed of light, or less. So when I look in the mirror, I'm not seeing my face now... I'm seeing my face about 2 nanoseconds ago: 1 ns for the light to bounce off my face to the mirror and another nanosecond for it to bounce from the mirror to my eyes.[14]

In fact, nothing outside of my immediate place shares my now as far as I can determine it. If I look across the room *now* what I see is what the room looked like a few nanoseconds ago. If I gaze at the moon *now* I see what the moon looked like a few minutes ago. If I look at the Andromeda galaxy *now* I see what it looked like 2 million light years ago. So the finite speed of light adds a complication to what we can say about the nature of reality, *now*. That's sort of trivial when you think about it. Troubling, but trivial.

The other aspect of what we've just talked about is that even if we know how far away the other side of the room is—and I can unambiguously measure that—and I know the speed of light, I can arrange the objects that I look at now in what you might think is a proper time-sequence. But the inability to unambiguously identify my *now* as the right one and a *now* from the International Space Station, or an Andromedian observer is even more troubling. They're all proper nows and all of them are legitimate. Each is right and each is different.

In fact, it's even worse. We have this idea that time "flows" and we along with it. Our past is determined—I can't undo that yellow Mustang purchase in 1973—but my future is still to happen (to me) and I can avoid such a purchase next year. But my personal time seems to proceed (at an increasingly urgent clip) from past to future, passing through *now*.

We've seen that space and time are now two sides of the same thing we call spacetime. And we've seen that electric and magnetic phenomena are but two sides of the same thing we call electromagnetism. The consequences of the special theory of relativity seem to be unifying heretofore, different things. But as much as we think of spacetime as a single thing, we can't shake this idea that time is somehow special. After all, I can walk in the positive x direction and I can reverse myself and walk in the negative x direction. As much as I'd like to change my mind about that yellow Mustang, I can't myself walk in the negative time direction.

But! The underlying rules of physics are agnostic about the direction of time. Reverse the sign of time in the equations and the world would be indistinguishable from the other direction. If make a movie of aiming a pool ball so that it collides with another? If I play it back you can't tell whether I'm playing the film forwards or backwards... and the rules of physics describe both. So in physics, the direction of time is not so insistently "forward" as it seems to be for us personally. We'll see instances of this being a legitimate interpretation of real happenings when we get into quantum mechanics.

Suffice it to say, Einstein was the first in history to raise legitimate questions about the nature of time.

[14] And, I guess some number of microseconds for the light collection on my retina to be processed by my visual cortex and recognized by my brain. But that's not my concern here.

8.3 What about Causality?

You might be able to think of a circumstance in which maybe an airport officer on her Segway might be going by this little sidewalk parade in the other direction. Might she expect to see the light show in the opposite order? This might conjure up the idea of a problem of causality—that maybe we're back to crying before being born just by virtue of relative motion? Let's think about it slightly differently.

Suppose that we arrange for two light bulbs to fire simultaneously inside of WearyTraveler's frame on the sidewalk. A relatively moving observer (like CouchPeople, or SegwayCop) might say that the left hand bulb fired first, or the right hand bulb fired first. But relativity prevents one from actually observing a reversal of the order of events. . . the time spacing is different, sure. But not which came first and then second. Can't change that. Why? Because in order for that reversal to happen, the bulbs would have to be spaced so far apart that a signal could not reach to the observer or the signal would have to travel faster than the speed of light.[15] It would have a worldline that would be more than that light-speed-limiting 45 degree line on Fig. 18. Can't happen. Relativity is safe.

But just as it takes two to tango, we've seen that it still takes both space and time to make a velocity. c is a velocity, and so a distance divided by a time. In our Galilean transformations space quantities change between frames. But as we saw in detail if there's a special velocity that never changes, and if the space coordinates change, then time cannot be above the fray. Time's "marching on" had to be different for different observers and time coordinates are going to adjust in any comparison of events between co-moving frames of reference, just like space coordinates do.

9 So, What About That Ether?

When Einstein was doing his Bern-thing, he was in relative isolation. He didn't have access to a university library and so he had to rely on whatever passed through the patent office and whatever he remembered or could get through the mail. For many years it has been a matter of controversy as to just what he knew about research in the areas that touched his eventual Theory of Relativity. Let's go out west.

9.1 Albert Michelson

When someone measures something and their name is attached to it, that's a big deal in science and it happens every so often. Usually this indicates a significant discovery. When that experiment is a decade-long failure, well that's even more rare!

Albert Michelson (1852 - 1931) was born in Poland and his family moved to the United States when he was two years old. They were adventurous family—they went to the wild west and became merchants in various mining communities in California and Nevada. Michelson himself went to high school in San Fransisco, living with an aunt where he was a good student. His college education was unusual. He applied in a competition to the relatively new United States Naval Academy at Annapolis, Maryland and was rejected. So he did what anyone would do in such a situation, he got on a train in San Fransisco[16] and went to Washington to see the president. He was

[15] See Diagrammatica: Spacetime Diagrams for examples.
[16] He was one of the first transcontinental train passengers.

Figure 23: michelson

nothing, if not persistent, and President Ulysses S. Grant personally admitted him to Annapolis as a midshipman in 1869. He graduated and did two years at sea in the Navy and returned to the Academy as an instructor[17]. He had become a master experimentalist in the measurement of precision optical phenomena and perfected heroically precise techniques to measure the speed of light to very high precision. His expertise led him to study in Europe for a while and to return to making increasingly better measurements of c.[18] He was working on such a measurement using a one mile evacuated tube when he died in 1931.

Michelson was notoriously cranky and difficult. His first wife tried to have him commmitted and a maid sued for abusive treatment. He once had an argument about an experiment with a colleague in a hotel lobby that drew a crowd, maybe because they were loud and maybe because Michelson was still in his pajamas. He won the Nobel Prize in 1907, not for his measurements of c, but rather because of the most famous measurement of "zero" in all of physics and the device he invented in order to do it.

9.2 The "Michelson-Morely Experiment"

What Michelson decided to do when he resigned from the Navy and became a member of the faculty at the Case School of Applied Science, now called Case Western Reserve University in Cleveland, Ohio. There he teamed with Edward Morley (1838-1923) to do some of the most audacious experiments of their time: they tried to measure the speed of the Earth relative to the ether.

Remember, Michelson's life is not so far removed from James Maxwell and his theory of light. He and everyone around him believed that what he'd described were waves that "waved" in the invisible, but persistent ever-present ether. . . the Luminiferous Ether. Everyone presumed that the Earth and the planets were orbiting through this stuff and that we see the Sun, meant that the ether was jiggling as the light (and heat) from the Sun propagated to Earth. The ether was everywhere, but the Earth must be moving through it at some finite speed and Michelson and Morley set out to measure it.

Let's imagine a river that's flowing uniformly from left to right. If you were to take two identical motorized toy boats and set them going in the river, we could measure the river's speed by comparing their motions in two directions. Here's what we would need to know: the distance that each boat travels (W) and the speed that the boats go *relative to the water* (C). Let's say that the width of the river is W feet. One boat is sent racing downstream a distance W and then back upstream to the starting point. . . so it travels $2W$ feet total. Downstream, it would go *with* the current and so faster relative to the shore and in the return, it would fight *against* the current and go slower. If the river's speed is V, then the time downstream would be $t_{\text{down}} = \dfrac{W}{V+C}$ and the time to come back would be $t_{\text{back}} = \dfrac{W}{V-C}$. Since $V + C$ is bigger than $V - C$ and is in the denominator, then the time down is indeed smaller than the time back.

The other trip is across the river and in order to end up exactly opposite where the second boat starts, you'd aim a little upstream so that the river would carry the boat along as it goes across and the trajectory would be diagonal. The time to make this journey can also be calculated in

[17] He married the daughter of the head of the physics department!

[18] One of his most precise measurements was between two mountains separated by 22 miles in California, one being the Mount Wilson Observatory.

terms of V and $c$and precisely predicted. . . it will be faster than the down and up path. But the point is the following: if the water is not a flowing river, but a swimming pool, it wouldn't matter what direction you went, the times for the trips would be the same. Only if the water is flowing with a finite speed ("C") along with the first paths will the trips' durations be different in time.

Suppose instead of swimmers in a current, we have light in the ether. Since the Earth is moving through the Ether, a beam of light in that direction would be faster (or slower) relative to a beam that moved perpendicular to that Earth-ether "current." The speed of light is of course exceedingly fast and measurement of the absolute wavelength of light would be nearly impossible—after all, a red laser beam consists of light waves is about 650 nanometers, that's 650×10^{-9} m. But Michelson invented a clever device known ever since as the Michelson Interferometer. It's hard to measure the absolute wavelength, but if two beams are brought together so that they interfere and one is slightly out of phase relative to the other they would combine and make a visible interference pattern. The peaks in the interference are a) an indication that the waves are out of phase and b) an indirect measurement of that difference in phase. That quantity in turn could be used to determined the speed of the Earth relative to the Ether. Then Great Acclaim would await Michelson and Morley.

Figure 24 is a sketch of the idea. A source of light, A, sends it to a fancy mirror, H, which is designed to reflect half of the light and transmit the other half. So the reflected portion at A goes to B, reflects from another mirror M_1 and then passes back through H onto a telescope, T. Meanwhile the transmitted wave (dashed) goes through H at D, bounces from another mirror, H_2, comes back and follows the first beam from F to G at the telescope, T. Suppose the Earth is moving along the D-E length, then the speed of light in that D-E-F path will be faster than the speed in the A-B-C length and they would interfere at T.

The longer the path-length the more precise the measurement. In practice the experiment was very hard

But Michelson and Morley were even more clever. First, they mounted their device on a very heavy (large inertia, so resistant to acceleration from accident and so stable) circular platform that they floated in a big tub of Mercury (a heavy—now known to be dangerous—liquid) and they rotated it around the center so that they would eliminate any bias of any particular direction. Since any fringes would be a positive measurement, rotating it is fine. Also they set up a more complicated scheme than what I've sketched in Fig. 24 by creating many light paths with reflections before they were brought to interfere, in essence increasing the length of each path. Figure 25 is a drawing from their 1887 published paper and Fig. 26 is a photograph of their apparatus from the Case Western archives.

Their result: zero. zilch. nothing. nada. zip. diddly-squat. nil. Over and over, they measured no relative speed to the ether. It was a huge frustration and a crisis in physics and by 1887 everyone was panicking. The ether had to be there! Maxwell's Theory required it. Drastic measures were required: Michelson and Morley suffered over their equipment and the theory community began to make bizarre suggestions. One of the strangest was that the motion through the ether actually caused one of the arms of the apparatus to shrink. . . literally that the atoms[19] would be closer together and the length would be shorter.

This idea is called Lorentz-Fitzgerald Contraction after the two brave theoreticians who reluctantly proposed it. The amount of the contraction? Well, by now you probably can guess. It was precisely the same amount that comes out of Einstein's Length Contraction formula.

[19] Atoms? Only the brave (or the desperate) believed in atoms during the 1890s when this was proposed.

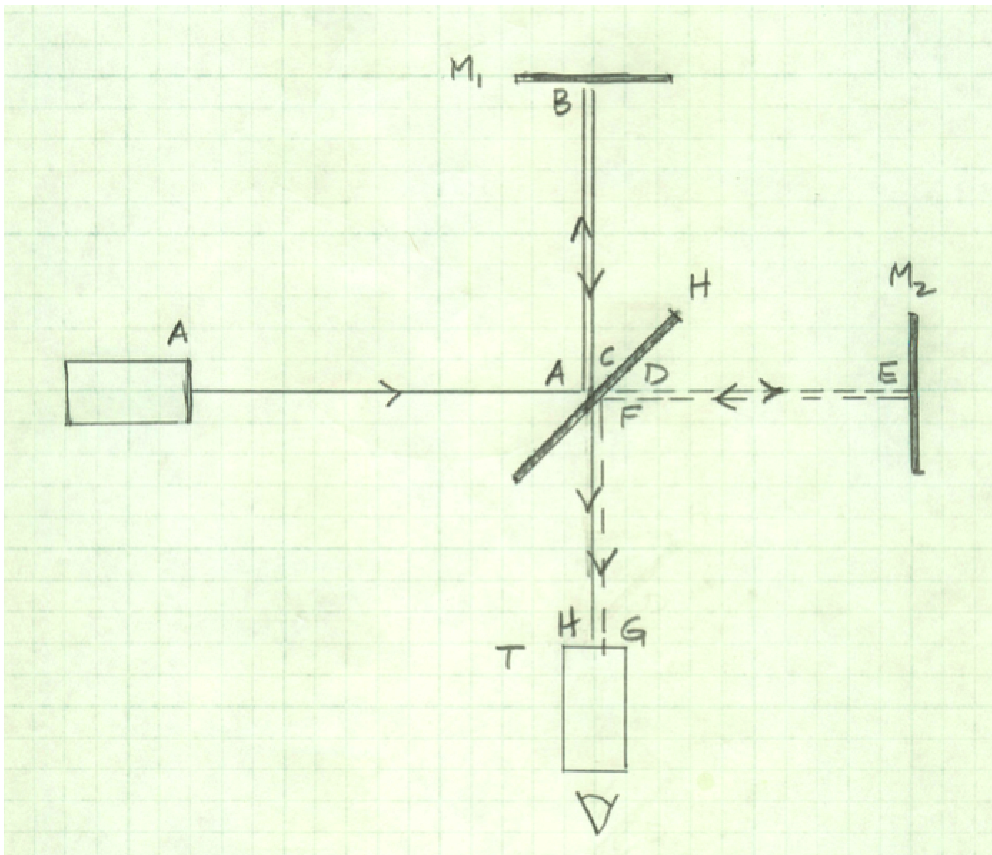

Figure 24: MMintme

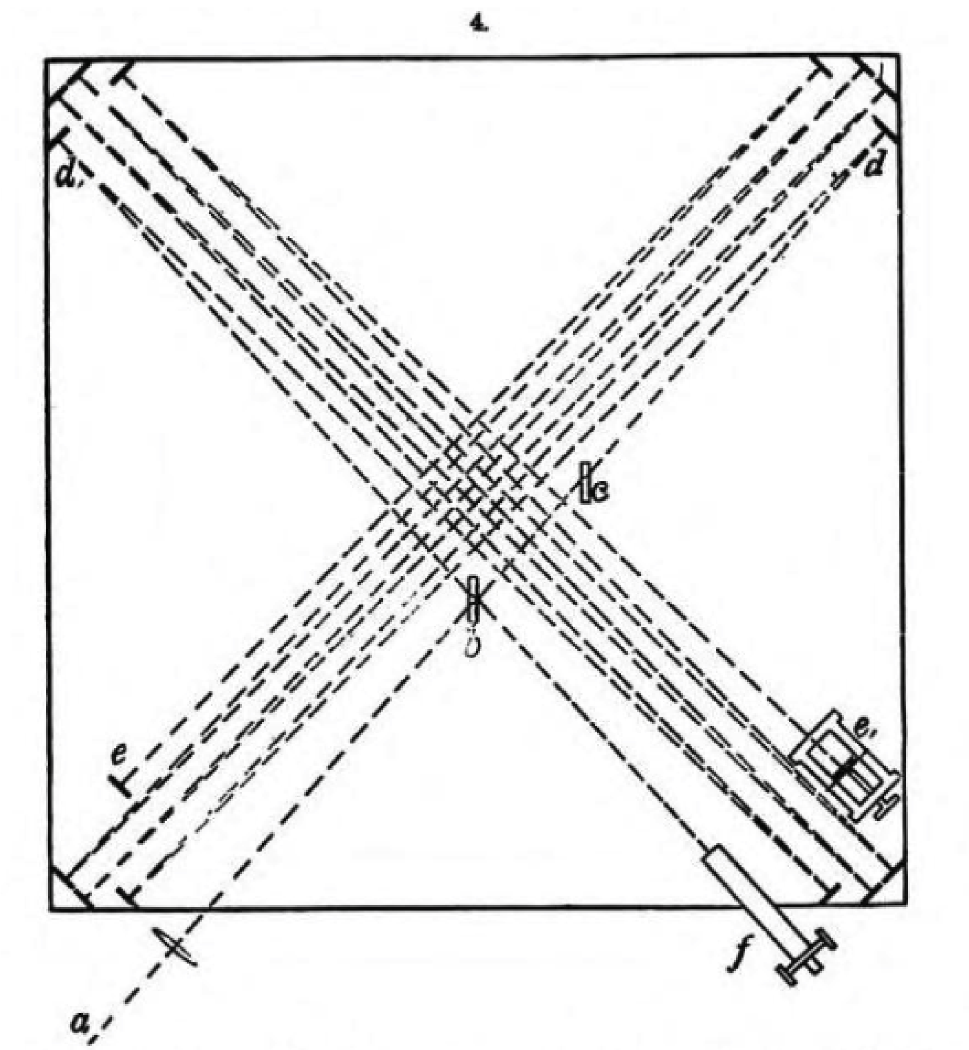

Figure 25: MMactual1

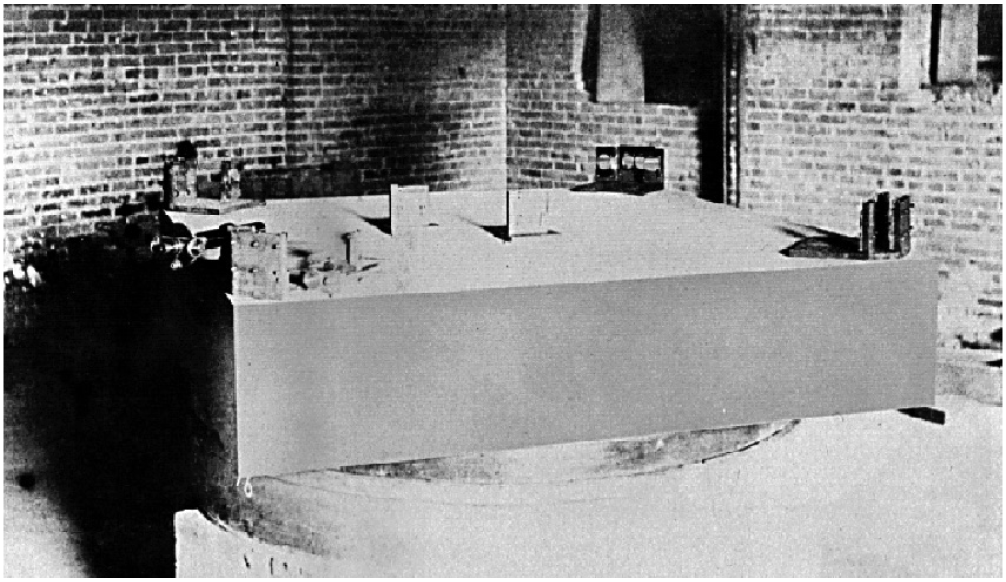

Figure 26: MMactual

9.3 The Superfluous Ether

In Einstein's paper there are no references to any other publication. This is highly unusual and it's even surprising that he got away with it. There's a heartfelt "thanks" to one of the Olympia Academy members, a buddy from the Patent Office,[20] but no reference to the work of Michelson's nor of Lorentz'. The question remains—in no small part because Einstein himself was not consistent in his recollections—did he know of the null Michelson Morley results? Or did he "predict them" after the fact (post-dict?)? The general conclusion of most historians is that he was aware of the null ether results but maybe not very familiar with the anxious theoretical work done in the previous 10 years to try to understand those results.

In any case, Einstein's conclusion was clear: he claimed that since there was no way to figure out if an observer is in a privileged reference frame—like Newton's Abosolute Frame, or the frame in which the ether presumably was stationary—then no such frame can exist. All frames, so to speak, are created equal. If you can't detect it, then you can't declare its existence, remember? They are equally likely and none can be picked out as the one that's *really* at rest. And so this 26 year old unknown patent clerk stated quite confidently in his 1905 paper:

> "The introduction of a 'luminiferous ether' will prove to be superfluous inasmuch as the view here to be developed will not require an 'absolutely stationary space' provided with special properties, nor assign a velocity-vector to a point of the empty space in which electromagnetic processes take place."

Quite remarkable.

[20]"In conclusion I wish to say that in working at the problem here dealt with I have had the loyal assistance of my friend and colleague M. Besso, and that I am indebted to him for several valuable suggestions."

10 The Most Famous “Paradoxes” of Relativity

Michelson never doubted the ether and he wasn’t alone. This idea was so deeply ingrained that it was just too much for many to give up on. This put those older scientists in a tough spot and some lived a dual life, accepting the formalism of Relativity, but not holding to the basic consequences. It was not hard to begin to think of all kinds of “impossible” paradoxes that should render the theory crazy, but they have all found explanation, usually with a deep use of the rules of relativity itself.

I’ll discuss the tests that confirm Relativity in the next chapter, but we should skim through one of the more colorful challenges that was thrown down to the theory. And then dismiss it. The famous siblings:

Twins

If you want to call Relativity crazy, you don’t have to look any further than the famous Twin Paradox. Just ask Mr Google about it and you’ll get more than a million hits, which isn’t bad for an obscure mathematical conundrum of physics! I believe that one of the first challenges on this score came from the French physicist, Paul Langevin in 1911 and it goes like this:

Two twins are born on Earth and one of them gets into a spacecraft and goes to a distant star at a relativistic gamma of 100, $\gamma = 100$. His sister stays on Earth and lives her life. The spaceman twin then turns around and comes back to Earth at the same speed as before and when he arrives he has aged just two years… but he finds that his sister is long gone since 200 years have elapsed on Earth. Now there are two things that are troubling about this, three if you want to be sentimental. First the substitution for a ticking clock by a biological organism often hits people the wrong way. But biology is still chemistry and chemistry’s engines are electrical in the end and so the rules of physics, quite independent of thinking, breathing organisms still rule the day. A biological clock is a clock and the rule of physics still apply. No, the paradoxical part comes from asking the question the other way. We assumed in the statement of the problem that the Earth twin was at rest and we reached our conclusion by thinking about it as if the Earth frame was special.

But we know now that that’s not fair and so shouldn’t we be able to ask the question from the other perspective, as if the space twin was “at rest” and the Earth receded from it and then returned? And then wouldn’t it be the space twin who would be 200 years older and the Earth twin only 2 years older? That’s the “paradox” part of the Twin Paradox, and the answer is no.

If the twins were separated and then one left in the spaceship and they never re-connected, then indeed, neither frame would be special and the conclusions about the time-dilation would be reciprocal. But that’s not what happened in the story. The Earth twin remained in one rest frame for the whole time, while the space twin lived in *two* rest frames, the one going out and the one that returned. That makes all the difference. Now you could ask about the alternative assignment of the space twin being the HF and the Earth twin the AF. The Earth twin still stays in one frame, but in order for them to meet up, even if the space twin is at relative rest on the way out, in order to then catch up, he has to go even faster in a new rest frame than in the first verse of the story. The end result is still the same, he’s aged 2 years and the Earth has aged 200 years. What’s not relative is this: the Earth twin always stayed in one rest frame while, no matter how you tell the story, the space twin has to participate in two different reference frames. There’s no relativity about that. It’s not a paradox. Let’s put the ladder away.

10.1 Fitting in the Garage

One other tricky paradox. We have a garage that's G meters long and our ladder is L meters long and $L > G$. If we run at the garage holding the ladder and run really fast, then from the garage's perspective, the ladder is Lorentz Contracted and it fits. But from the ladder's perspective, the garage is Lorentz contracted, and so the ladder is even less of a fit than before. That's a paradox. Or is it.

Still have to write this...

10.2 Relativity From the Sky

Still have to write this...

Interstellar space is full of all kinds of particle debris and much of it bombards us constantly—we call them Cosmic Rays and they are a threat, an annoyance, and a useful tool. Many of the interstellar particles are protons which have been accelerated to very high speeds through a mechanism that we don't understand. The process of particles getting to Earth is a complicated one and not unlike the process that we use on Earth when we intentionally crash protons or electrons into matter in experiments. The protons hit the nitrogen in the atmosphere[21] and create enormous particle showers which can be miles across.[22]

Among the by-products of these collisions is the creation of a particle called the "muon" which is essentially a heavy electron. We'll meet the muon later as it plays an important role in the history of particle physics and its relationship to the electron is one of great interest. Most of the particles that we know about are essentially unstable. That is, they decay into other particles. One moment they're there and the next moment something else is there! We know of only two particles that are stable against decay—that is, our measurements searching for their decay lead to lifetimes that are longer than the lifetime of the universe. Lucky for us, as these two stable particles are the proton and the electron.[23]

The muon is relatively long-lived (on the scale of subnuclear things) and decays on average in about one and a half microsecond, 1.5×10^{-6}˜seconds.[24] So how far would a muon go on average if it were traveling at the speed of light?

QR question muon distance

Figure 27 shows the situation. The cascade of particles (I've only drawn in one of the hundreds of thousands of particles in the "shower" induced by the original proton.) leads to a steady rate of about 1 muon through your thumbnail every minute. The issue is the distance that the muons

[21] Did you know that most of the atmosphere is nitrogen and that oxygen is only the second most abundant element?

[22] We'll study these particles later.

[23] The neutron is unstable and decays by itself in about 10 minutes. But when it's bound into a nucleus, that decay is suppressed because? Relativity. Next chapter!

[24] This is the so-called "half life" of the muon. It means that if we start with 100 muons, after 1,5 microseconds there will be 50 left. After another 1.5 microseconds, 25... and so on.

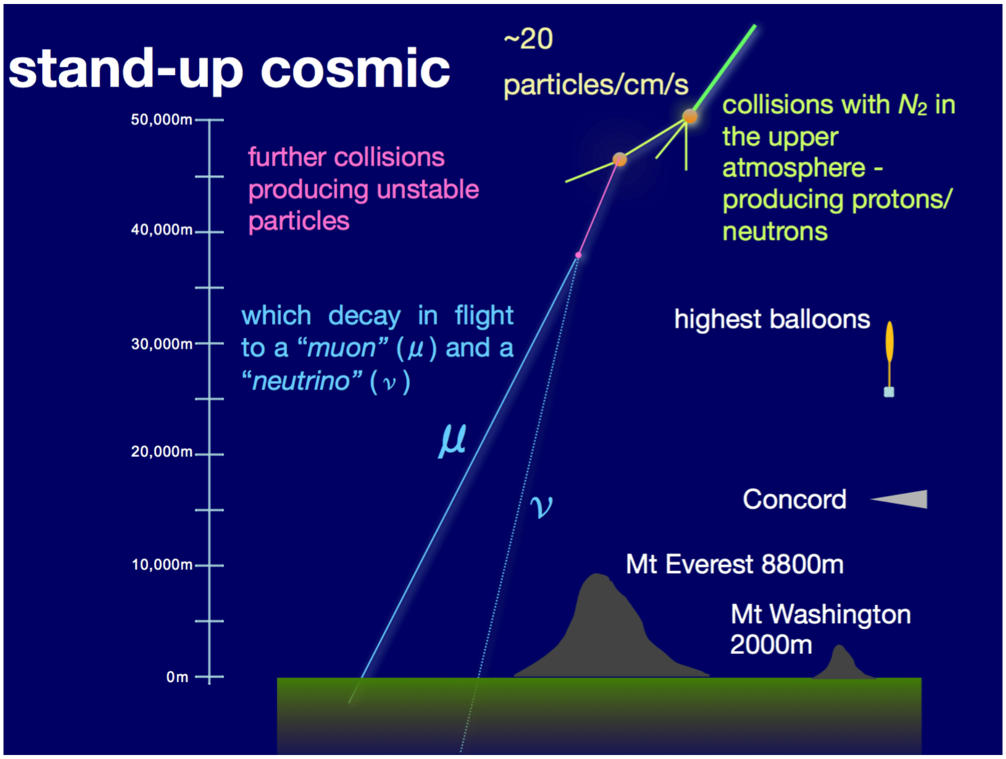

Figure 27: cosmic

must travel in order to reach the earth. As you saw in your calculation above, on average a muon will decay in less than 500 meters, yet they seem to make it to Earth——more than 50,000 meters! How does that happen? This is many, many “lifetimes” for a muon. If 100 muons are produced in the upper atmosphere, after only about 2000 meters, there would be less than 10 left. There’s still more than 45,000 meters to go!

We have the tools to understand this.

need to describe and explain the muon thing

Now discuss the tee shirt equation that I know you’ve been wanting to understand.