PHY481 - Lecture 1 (Fall 2009)

Griffiths: Chapter 1 (up to roughly page 10)

A. A Brief History

Early observations of magnetic and electric properties include the lodestone which is a magnetic rock and amber that was considered special due to its ability to produce a strong static discharge.

More scientific ideas about charge and the flow of charge were developed by Gray, Franklin etc in the middle of the 18th century. Coulomb made these ideas concrete by measuring the force between charges and developing a formula to describe these forces (1780's). This formula is the basis of electrostatics (stationary charge distributions). However it is often easier to use a reformulation of this law to a form called Gauss's law to solve problems.

Ampere and Oersted noticed the fact that a DC current generates a magnetic field. Biot and Savart developed a formula to describe this. This occured in the 1820's. Analysis of the magnetic fields generated by DC currents is called magnetostatics. The Biot-Savart law, or Ampere's law is the basis of magnetostatics. Ampere also noticed that a time varying electric field also induces a magnetic field. Ampere's law describes this phenomenon (1820's).

Faraday observed that a time varying magnetic flux leads to an induced emf (1831). Joseph Henry also observed indications of this in 1830, but Faraday's experiments are considered definitive. Faraday's law describes these phenomenon and is the basis of electric motors and generators.

Heaviside and Gibbs wrote down the equations of electrostatics, magnetostatics, magnetic induction and a generalized form of Ampere's law in a unified form building upon two of Maxwell's papers from (1861-65). In the early literature the collection of these four equations, now known as Maxwell's equations, were therefore called the Hertz-Heaviside equations. Hertz was included because he did experiments verifying Maxwell's prediction of EM waves. However Einstein decided to call them the Maxwell-Hertz equations and the name stuck. Anyway they are considered a complete theory of classical electricity and magnetism or electromagnetism and can be considered the basis of this course.

Often an additional equation is added to this set - the Lorentz Force law. The Lorentz force law describes the way in which electric and magnetic fields effect a moving charge. This equation was developed by Lorentz (1892/1895).

The chronology of these discoveries and other physics discoveries is available at the history of physics from the aps

B. Basic concepts about charge

- There are two types of charge, positive and negative.
- Charge is conserved.
- Charge is quantized. $e = 1.6 \times 10^{-19} C$.

C. Basic concepts about materials

- Conductor - If there is a voltage across a conductor, current flows.
- Insulator - Even if there is a voltage across an insulator, current does not flow.
- Semiconductor = Insulator at low voltage and temperature.
- Superconductor = Conductor, for static charges and no applied magnetic field.

D. The first quantitative EM law: Coulomb's law (1780's)

The starting point in electrostatics is Coulomb's law, which gives the force between two stationary charges,

$$\vec{F} = k\frac{Qq}{r^2}\hat{r} = k\frac{Qq}{r^3}\vec{r} \tag{1}$$

- Q, q are stationary charges. Their units are coulombs (C)
- $\hat{r}$ is a unit vector along the line between the two charges.
- $\vec{r}$ is the vector distance between the two charges.
- $k = 9 \times 10^9 Nm^2/C^2 = 9 \times 10^9 kgm^3/C^2s^2$.
- $k = 1/4\pi\epsilon_0$. ϵ_0 is the permittivity of free space.

E. Gravitational force law : Measured by Cavandish 1790's

$$\vec{F_G} = -G\frac{Mm}{r^2}\hat{r} \tag{2}$$

- $G = 6.67 \times 10^{-11} Nm^2/kg^2$. The gravitational force is much weaker than the electostatic force.

Note that in these expressions $\vec{r} = \vec{r}_Q - \vec{r}_q$. Griffiths gives this difference a new symbol. I will write out the full expression whenever these is a possibility of confusion.

Example

Find the ratio of the magnitudes of the gravitational and electrostatic forces between two protons.

Solution The ratio of the magnitudes of the gravitational and electrostatic forces is,

$$\frac{F_G}{F} = \frac{GMm}{kQq} \tag{3}$$

For two protons, we have, $M = m = 1.67 \times 10^{-27} kg$ and $Q = q = 1.6 \times 10^{-19} C$. Plugging these numbers yields,

$$\frac{F_G}{F}(two\ protons) = \frac{6.67 \times 10^{-11}(1.67 \times 10^{-27})^2}{9 \times 10^9 (1.6 \times 10^{-19})^2} = 8.1 \times 10^{-35} \tag{4}$$

F. Most charge is "bound"

Due to the fact that the force between charges is so large, most charge is bound. That means that negative and positive charges are close to each other in regions where the total charge is almost zero. This raises the question. Why don't charges self destruct by crashing into each other and annihilating? The answer is in quantum mechanics, which provides an understanding of how atoms are stable. Atoms are composed of electrons orbiting the nuclei which contains the protons. This provides a stable configuration of bound charges. Within electrostatics bound charges still form dipoles. There are many more charge dipoles than there are free charges. Many electrostatic effects are due to dipoles rather than free charges, so we need to understand dipoles well.

No free magnetic charges have been observed, so that N and S magnetic poles always come in pairs. Nevertheless the search of free magnetic charge, ie magnetic monopoles, continues. The sources of magnetic fields are currents or moving charges.

G. Force between many charges - superposition

Force on a charge q due to many other charges, $Q_1, Q_2, ...Q_n$ is just the sum of the forces due to each of these charges, ie.

$$\vec{F}_{tot} = \sum_{i=1}^{n} k \frac{Q_i q}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i) \tag{5}$$

This is a vector sum, so the math can get messy. Here $\vec{r}_i$ is the position of the charge Q_i while $\vec{r}$ is the position of charge q.

The principle of superposition also applies when there is a continuous distribution of charge. For example charge distributions on rods, discs, spheres etc. However when treating these distibutions, the sum in Eq. (5) becomes an integral. In treating these problems, we define a small element of charge dQ. This is the amount of charge in a small part of the continuous charge distribution. We shall consider three cases:

- Lines: Then $dQ = \lambda dx$, where λ is the linear charge density.
- Surfaces: Then $dQ = \sigma dA$, where σ is the surface charge density.
- Volumes: Then $dQ = \rho dV$, where ρ is the volume charge density.

H. Maxwell's equations in vacuum (Chapter 7 of Griffiths)

1. Gauss' law for electric field - the basis of electrostatics (Chapters 2-3 of Griffiths)

The Electric flux, ϕ_E, through a closed surface, A is proportional to the net charge, q, enclosed within that surface. $d\vec{a} = da\hat{n}$.

$$\phi_E = \oint \vec{E} \cdot d\vec{a} = \frac{q}{\epsilon_0} \tag{6}$$

Differential form,

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \tag{7}$$

2. Gauss's law for magnetic field

The Magnetic flux, ϕ_B, through a closed surface, a is equal to zero.

$$\phi_B = \oint \vec{B} \cdot d\vec{a} = 0 \tag{8}$$

Differential form

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{9}$$

3. Generalized Ampere's law - Magnetostatics (Chapter 5 of Griffiths)

The path integral of the magnetic field around any closed loop, is proportional to the current enclosed by the loop plus the displacement current enclosed by the loop.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\phi_E}{dt} \tag{10}$$

Differential form

$$\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{11}$$

4. Faraday's law - magnetic induction (Chapter 7 of PS)

The emf induced in a closed loop, is proportional to the negative of the rate of change of the magnetic flux, ϕ_B, through the closed loop,

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt} \tag{12}$$

Differential form

$$\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{13}$$

5. Lorentz force law

The force on a charge moving with velocity $\vec{v}$ in an electromagnetic field is given by,

$$\vec{F} = q(\vec{E} + \vec{v} \wedge \vec{B}) \tag{14}$$

Chapters 4 and 6 of Griffiths cover electric and magnetic fields inside matter where some extensions of Maxwell's equations above are needed.

I. Vectors properties and vector operations

1. Consider vectors defined in a orthogonal basis set, with unit vectors $\hat{x}, \hat{j}, \hat{k}$ in the x,y and z directions respectively. Then a vector $\vec{A}$ may be written,

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z} = (A_x, A_y, A_z) \tag{15}$$

2. Vector addition and subtraction is carried out component by component, for example for vectors $\vec{A}$ and $\vec{B}$ we have,

$$\vec{A} + \vec{B} = (A_x + B_x, A_y + B_y, A_z + B_z) \tag{16}$$

Pictorially this corresponds adding vectors "head to tail".

3. Scalar multiplication is also carried out component-wise, so that,

$$c\vec{A} = (cA_x, cA_y, cA_z) \tag{17}$$

4. The length of a vector is,

$$A = |\vec{A}| = (A_x^2 + A_y^2 + A_z^2)^{1/2} \tag{18}$$

5 The dot product of two vectors is a scalar given by,

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = ABcos(\theta) = \vec{B} \cdot \vec{A} \tag{19}$$

Note that the length of a vector $\vec{A}$ is given by $A = (\vec{A} \cdot \vec{A})^{1/2}$

6. The cross product of two vectors gives a third vector which is perpendicular to the plane of the two starting vectors. This is a very important property as often we want to use the normal to a plane, for example in defining the normal to a surface. The cross product is given by,

$$\vec{A} \wedge \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = ABsin(\theta)\hat{n} = (A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x) \tag{20}$$

Note also that $\vec{A} \wedge \vec{B} = -\vec{B} \wedge \vec{A}$

7. Vector triple products

$$\vec{A} \cdot (\vec{B} \wedge \vec{C}) = \vec{C} \cdot (\vec{A} \wedge \vec{B}) = \vec{B} \cdot (\vec{C} \wedge \vec{A}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \tag{21}$$

This triple product gives the volume of a parallelepided, with the vectors $\vec{B}$ and $\vec{C}$ forming the base. The second triple product is,

$$\vec{A} \wedge (\vec{B} \wedge \vec{C}) = B(\vec{A} \cdot \vec{C}) - C(\vec{A} \cdot \vec{B}) \tag{22}$$

Note that using this identity all vector expressions containing cross products can be reduced to expressions containing at most one cross product.

8. A pseudoscalar and pseudovector (or axial vector). A pseudovector is like a vector except that it does not change sign under inversion, for example if $\vec{A}$ and $\vec{B}$ are vectors, then $\vec{p} = \vec{A} \wedge \vec{B}$ is a pseudovector as inversion of $\vec{A}$ and $\vec{B}$ has no effect on $\vec{p}$, whereas a true vector would change sign. Pseudoscalars may be formed by combining vectors and pseudovectors, for example the triple product.

9. Commutative (Exchange order). Scalar addition and multiplication are commutative, but subtraction and division are not. Vector addition and dot product are commutative, but subtraction and cross product are not. Associative (Move parentheses). Which operations are associative, e.g. $(\vec{A} \cdot \vec{B}) \wedge \vec{C})$ does not make mathematical sense. Note that division and subtraction are not associative. Distributive (Operate on a sum of terms inside a bracket). e.g. a scalar times a sum of vectors is distributive. Vector operations are distributive under multiplication by a scalar.

10. The position or displacement vector and the definition of a vector. The position vector in Cartesian co-ordinates is $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$. This vector may be transformed by a co-ordinate transformation, e.g. rotations or translations. A quantity $\vec{A} = A_x\hat{x} + A_y\vec{y} + A_z\vec{z}$ is a **vector** if it transforms in the same way as $\vec{r}$ under co-ordinate transformations.