

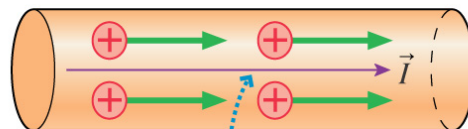
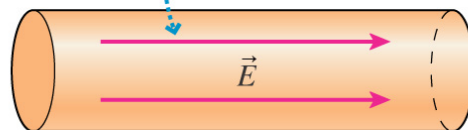
# PHY294H

- Professor: Joey Huston
- email: huston@msu.edu
- office: BPS3230
- Homework will be with Mastering Physics (and an average of 1 hand-written problem per week)
  - ◆ **Problem 29.77 (already assigned) will be the hand-in problem for 4<sup>th</sup> MP assignment (due Wed Feb. 10)**
  - ◆ **Help-room hours: 12:40-2:40 Tues; 3:00-4:00 PM Friday**
- Quizzes by iclicker (sometimes hand-written)
- Exam next Thursday: bring 1(-sided) 8.5X11" sheet of notes
  - ◆ practice exam available today
- Course website: [www.pa.msu.edu/~huston/phy294h/index.html](http://www.pa.msu.edu/~huston/phy294h/index.html)
  - ◆ lectures will be posted frequently, mostly every day if I can remember to do so

## Current

- Let me define the electric current  $I$ 
  - ◆  $\vec{I} = dQ/dt$  (in the direction of the  $\vec{E}$  field)
  - ◆ note that this is not the direction that the electrons move, but that's the convention we have
  - ◆ all of the effects that we are interested in are the same whether electrons move to the left or (fictitious) positive charges move to the right
  - ◆ unit of charge is the Coulomb (C)
  - ◆ unit of current is C/S or A (Ampere)
  - ◆  $1 \text{ A} = 1 \text{ C/s}$

Surface charges have created an electric field inside the wire.



The current  $\vec{I}$  is the rate at which the electric field seems to push *positive* charge through the wire.  $\vec{I}$  is in the direction of  $\vec{E}$ .

## Another dead guy

- Andre Ampere (1775-1836)
- We'll revisit him when we study the relationship between electric current and the magnetic field

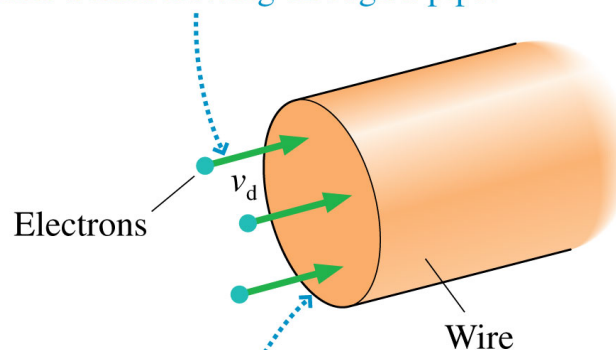


## Electron current

- We define the **electron current**  $i_e$  to be the number of electrons per second that pass through a cross section of the conductor.
- The number  $N_e$  of electrons that pass through the cross section during the time interval  $\Delta t$  is

$$N_e = i_e \Delta t$$

The sea of electrons flows through a wire at the drift speed  $v_d$ , much like a fluid flowing through a pipe.



The electron current  $i_e$  is the number of electrons passing through this cross section of the wire per second.

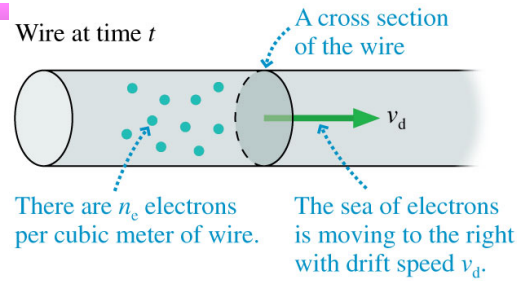
# Electron current

- If the number density of conduction electrons is  $n_e$ , then the total number of electrons in the shaded cylinder is

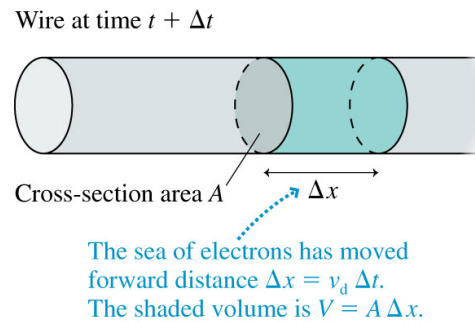
$$\begin{aligned} N_e &= n_e V \\ &= n_e A \Delta x \\ &= n_e A v_d \Delta t \end{aligned}$$

- So the electron current is:

$$i_e = n_e A v_d$$



$v_d$  is very small, fractions of a mm/s



# Current

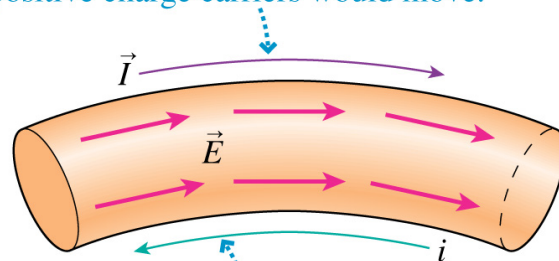
- $I = \Delta Q / \Delta t = e N_e / \Delta t = e i$

- ◆ where we defined  $i$  as the electron current
- ◆ each electron carries a charge  $e$ , so the current is the rate at which electrons move times the charge that each one carries
- ◆ as Andre Ampere would say, eh voila!

- Define current density  $J$

- ◆  $J = I / A = n e v_d$

The current  $\vec{I}$  is defined to point in the direction of  $\vec{E}$ . It is the direction in which positive charge carriers would move.



The electron current  $i$  is the motion of actual charge carriers. It is opposite to  $\vec{E}$  and  $\vec{I}$ .

## iclicker question

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A wire carries a current. If both the wire diameter and the electron drift speed are doubled, the electron current increases by a factor of

- A. 2.
- B. 4.
- C. 6.
- D. 8.
- E. Some other value.

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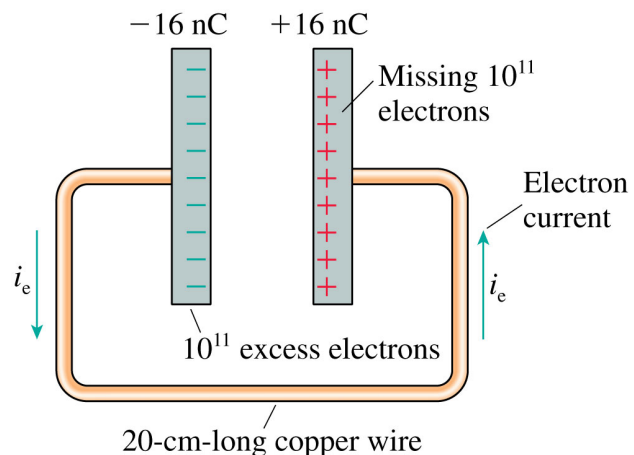
- A. 2.
- B. 4.
- C. 6.
- D. 8.  $i_e \propto Av_d$
- E. Some other value.

- In most metals, each atom contributes one valence electron to the sea of electrons.
- Thus the number of conduction electrons  $n_e$  is the same as the number of atoms per cubic meter.

**TABLE 30.1** Conduction-electron density in metals

Metal	Electron density ( $\text{m}^{-3}$ )
Aluminum	$6.0 \times 10^{28}$
Copper	$8.5 \times 10^{28}$
Iron	$8.5 \times 10^{28}$
Gold	$5.9 \times 10^{28}$
Silver	$5.8 \times 10^{28}$

- How long should it take to discharge this capacitor?
- A typical drift speed of electron current through a wire is  $v_d \approx 10^{-4}$  m/s.
- At this rate, it would take an electron about 2000 s (over half an hour) to travel 20 cm.

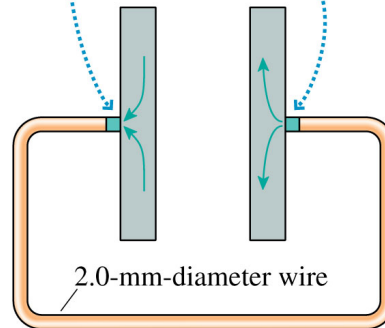


- But real capacitors discharge almost instantaneously!
- What's wrong with our calculation?

- The wire is *already full* of electrons!
- We don't have to wait for electrons to move all the way through the wire from one plate to another.
- We just need to slightly rearrange the charges on the plates *and* in the wire.

1. The  $10^{11}$  excess electrons on the negative plate move into the wire. The length of wire needed to accommodate these electrons is only  $4 \times 10^{-13}$  m.

3.  $10^{11}$  electrons are pushed out of the wire and onto the positive plate. This plate is now neutral.

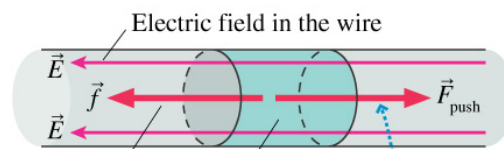
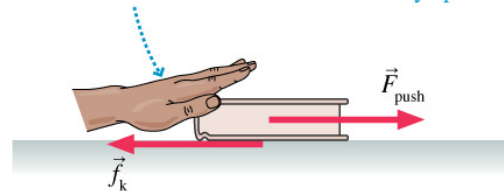


2. The sea of  $5 \times 10^{22}$  electrons in the wire is pushed to the side. It moves only  $4 \times 10^{-13}$  m, taking almost no time.

## Pushing electrons

- As we discussed before, there needs to be an electric field created inside the wire in order for the electrons to have a net velocity in a particular direction

Because of friction, a steady push is needed to move the book at steady speed.

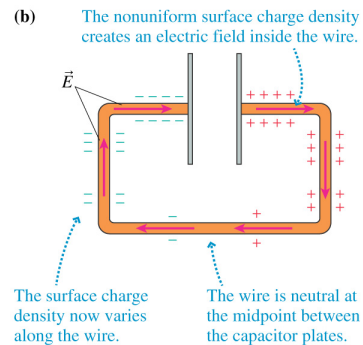
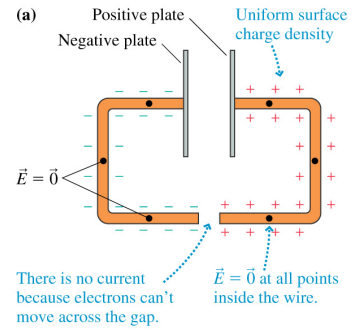


Retarding force due to collisions      Sea of electrons

Because of collisions with atoms, a steady push is needed to move the sea of electrons at steady speed. Electrons are negative, so  $\vec{F}_{\text{push}}$  is opposite to  $\vec{E}$ .

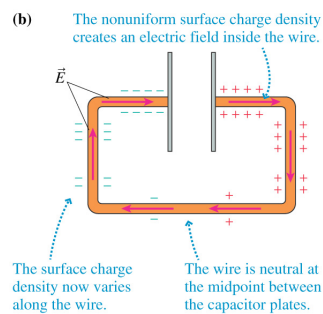
# Let's go back to our two charged plates

- With no connection in the middle, the charges distribute themselves over the surface of every conductor (the wires and the plates)
  - ◆ remember no excess electrons in the interior of a conductor in equilibrium situations
- If I suddenly connect the two ends of the wires, then the electrons near the negative (previously) end move onto the positive (previously) end
- Now there's a non-uniform distribution of charge and a non-equilibrium situation
  - ◆ not static so electrons throughout the conductor

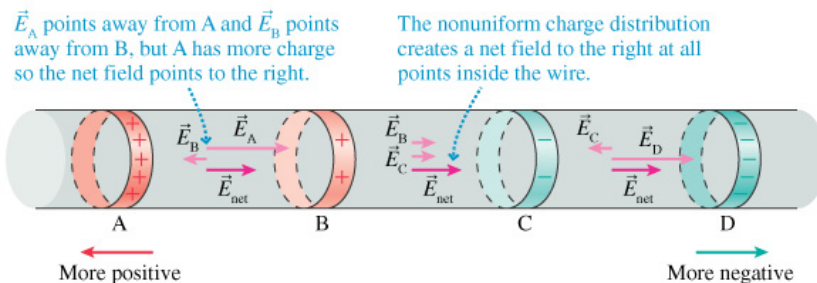


# Electric field

- Think about what's happening right after I connect the two wire ends
- Just consider 4 separate rings of the wire

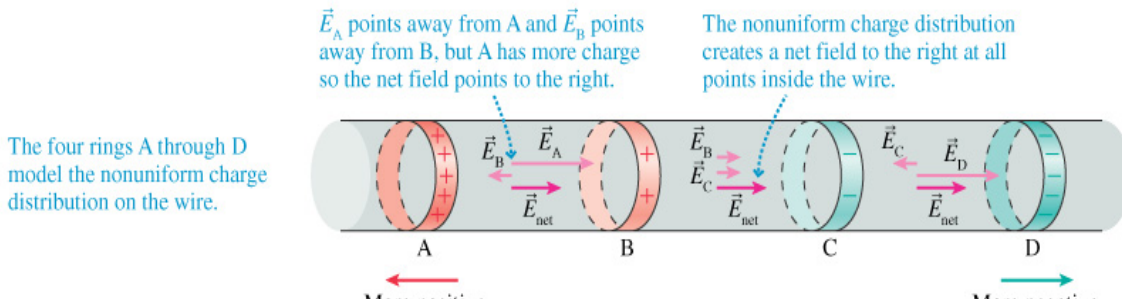
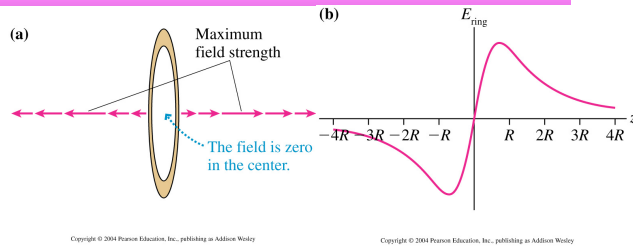


The four rings A through D model the nonuniform charge distribution on the wire.

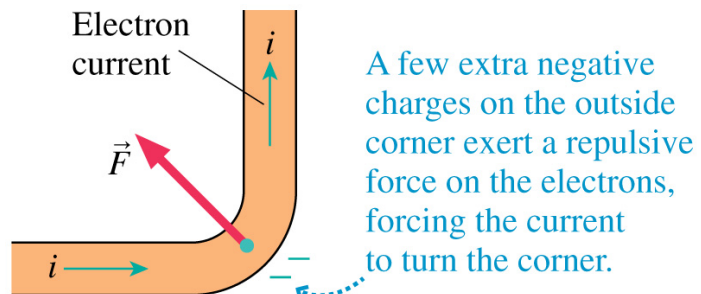


# Electric field

- We know the electric field from a ring of charge
- Each ring of charge contributes
- Because of the gradient, there's a net electric field going from the more positive end towards the more negative end



- This model even explains electrons turning corners





# Microscopic model of conduction

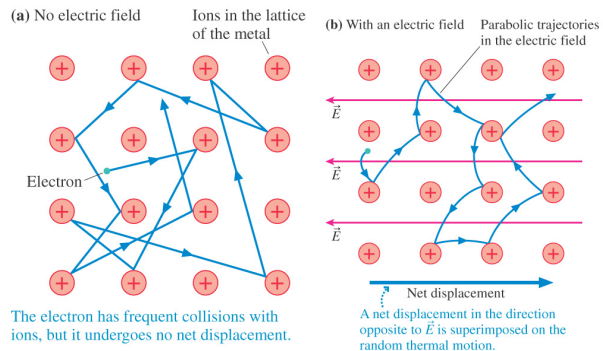
- Electrons are travelling at about  $10^5$  m/s
- Can think of them as behaving like gas molecules, travelling in straight lines between collisions
  - ◆ free electron or Drude model
  - ◆ doesn't take into account some quantum mechanical effects but good enough for the moment
- After an electric field is applied, the electrons are now following parabolic paths

For a gas of electrons, we can write

$$v_{av} = [3kT/m]^{0.5}$$

$$v_{av} = [3(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})/9.1 \times 10^{-31} \text{ kg}]^{1/2}$$

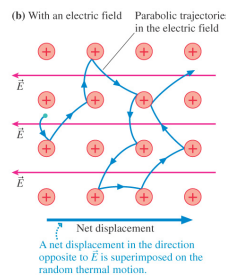
$$v_{av} = 1.2 \times 10^5 \text{ m/s}$$



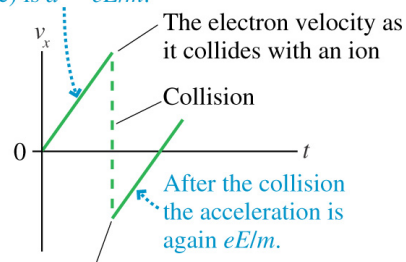
# Microscopic model of conduction

- Because of the electric field, the electron is going to experience an acceleration (between collisions) and the velocity in the direction of the electric field will look like

- ◆  $v_x = v_{ix} - eE/m \cdot \Delta t$
- ◆ acceleration is opposite the direction of the E field



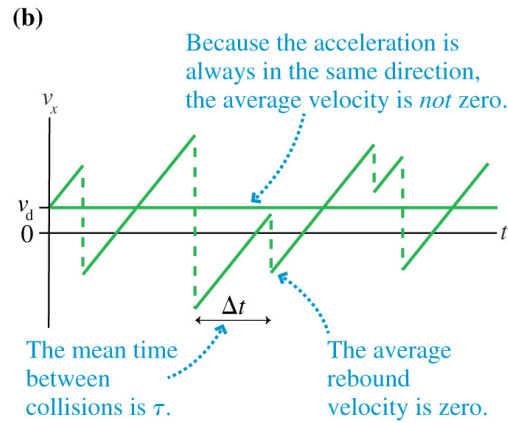
- (a) The acceleration between collisions (the slope of the line) is  $a = eE/m$ .



The electron rebounds with velocity  $v_{ix}$ .

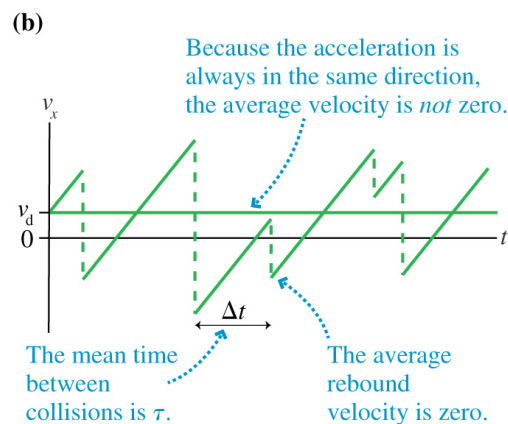
# Mean time between collisions

- Let the average time between collisions be  $\tau$
- Then we can write
  - ♦  $v_D = -eE\tau/m$
  - ♦ after collisions  $v_{ix} \sim 0$  so ignore first term
- But remember  $i$  (electron current) =  $nAv_d$ 
  - ♦ so  $|i| = (n\tau A/m) \cdot E$
  - ♦ the electron current is proportional to the strength of the electric field
  - ♦ important result that we will re-visit



## ...or equivalently

- Or equivalently can think of the collisions as being a drag force acting on the electrons
  - ♦  $ma = -eE - (a \text{ constant})v$
  - ♦ looking at units the constant must have units of mass/time
  - ♦  $ma = -eE - m/\tau v$
  - ♦ when  $v$  reaches terminal speed ( $v_d$ ),  $a = 0$
  - ♦  $v_d = -eE\tau/m$



# Conductivity and resistivity

- Define the conductivity of a material

$$\sigma = \text{conductivity} = \frac{n_e e^2 \tau}{m}$$

- The conductivity of a material characterizes the material as whole
- The current density  $J$  is related to the conductivity and the electric field by

$$J = \sigma E$$

- Can define the resistivity as the reciprocal of the conductivity->how difficult is it for the electrons to move

$$\rho = \text{resistivity} = \frac{1}{\sigma} = \frac{m}{n_e e^2 \tau}$$

# Conductivity and resistivity

**TABLE 30.2** Resistivity and conductivity of conducting materials

Material	Resistivity ( $\Omega \text{ m}$ )	Conductivity ( $\Omega^{-1} \text{ m}^{-1}$ )
Aluminum	$2.8 \times 10^{-8}$	$3.5 \times 10^7$
Copper	$1.7 \times 10^{-8}$	$6.0 \times 10^7$
Gold	$2.4 \times 10^{-8}$	$4.1 \times 10^7$
Iron	$9.7 \times 10^{-8}$	$1.0 \times 10^7$
Silver	$1.6 \times 10^{-8}$	$6.2 \times 10^7$
Tungsten	$5.6 \times 10^{-8}$	$1.8 \times 10^7$
Nichrome*	$1.5 \times 10^{-6}$	$6.7 \times 10^5$
Carbon	$3.5 \times 10^{-5}$	$2.9 \times 10^4$

\*Nickel-chromium alloy used for heating wires.