Nagy,

Tibor

Keep this exam **CLOSED** until advised by the instructor.

50 minute long closed book exam.

Fill out the bubble sheet: last name, first initial, **student number (PID)**. Leave the section, code, form and signature areas empty.

Two two-sided handwritten 8.5 by 11 help sheets are allowed.

When done, hand in your **test** and your **bubble sheet**.

Thank you and good luck!

Posssibly useful constant:

- g = 9.81 m/s^2

Posssibly useful Moments of Inertia:

- Solid homogeneous cylinder: $I_{CM} = (1/2)MR^2$
- Solid homogeneous sphere: $I_{CM} = (2/5)MR^2$
- Thin spherical shell: $I_{CM} = (2/3)MR^2$
- Thin uniform rod, axis perpendicular to length: $I_{CM} = (1/12)ML^2$
- Thin uniform rod around end, axis perpendicular to length: $I_{end} = (1/3)ML^2$

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Please, sit in row E.

$\boxed{\textit{1 pt}}$ Are you sitting in the seat assigned?

1.A◯ Yes, I am.

3 pt There are 147 steps between the ground floor and the sixth floor in a building. Each step is 16.7 cm tall. It takes 2 minutes and 50 seconds for a person with a mass of 66.9 kg to walk all the way up. How much work did the person do?
(in J)

2. **A**◯ 1.12×10^4 **B**◯ 1.26×10^4 **C**◯ 1.43×10^4 **D**◯ 1.61×10^4
E◯ 1.82×10^4 **F**◯ 2.06×10^4 **G**◯ 2.32×10^4 **H**◯ 2.63×10^4

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3 pt What was the average power performed by the person during the walk?
(in W)

3. **A**◯ 1.29×10^1 **B**◯ 1.71×10^1 **C**◯ 2.28×10^1 **D**◯ 3.03×10^1
E◯ 4.03×10^1 **F**◯ 5.36×10^1 **G**◯ 7.13×10^1 **H**◯ 9.48×10^1

$h = 16.7\ \text{cm} = 0.167\ \text{m}$: one step
$N = 147$: number of steps
$H = N \cdot h$: total height
$t = 2\text{min}50\text{s} = 170\text{s}$: duration of the climb

Energy balance:

$$\underbrace{KE_i}_{=0} + \underbrace{PE_i}_{=0} + W_{ext} = \underbrace{KE_f}_{=0} + PE_f$$

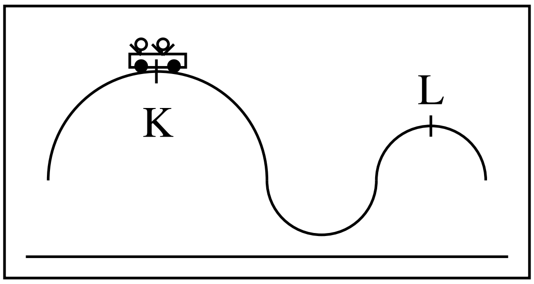
$$W_{ext} = PE_f$$

$$W = mgH = mgNh = 66.9 \cdot 9.81 \cdot 147 \cdot 0.167$$

$$W = 16.1\ \text{kJ} = 1.61 \cdot 10^4\ \text{J}$$

Power: $P = \dfrac{W}{t} = \dfrac{1.61 \cdot 10^4\ \text{J}}{170\ \text{s}} = 94.8\ \text{W}$

On a roller coaster ride the total mass of the cart - with passengers included - is 290 kg. Peak **K** is at 42.5 m above the ground, peak **L** is at 22.0 m. The speed of the cart at **K** is 15.5 m/s, at **L** it is 13.8 m/s. (The wheel mechanism on roller coaster carts always keeps the carts safely on the rail.)



4 pt How much energy is lost due to friction between the two peaks?
(in J)

4. **A** ◯ 4.52×10^4 **B** ◯ 6.55×10^4 **C** ◯ 9.50×10^4 **D** ◯ 1.38×10^5
E ◯ 2.00×10^5 **F** ◯ 2.90×10^5 **G** ◯ 4.20×10^5 **H** ◯ 6.09×10^5

Energy balance:

$$KE_i + PE_i = KE_f + PE_f + \Delta E_{th}$$

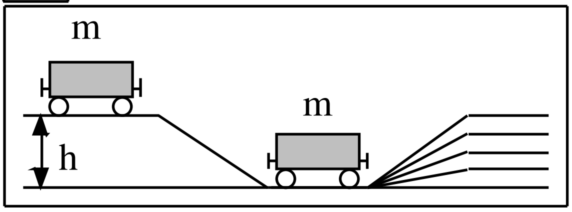
$$(KE_i - KE_f) + (PE_i - PE_f) = \Delta E_{th}$$

$$\Delta E_{th} = \frac{1}{2} M (v_i^2 - v_f^2) + Mg(h_K - h_L)$$

$$\Delta E_{th} = \frac{1}{2} \cdot 290 \cdot (15.5^2 - 13.8^2) + 290 \cdot 9.81 \cdot (42.5 - 22)$$

$$\Delta E_{th} = 6.55 \cdot 10^4 \text{ J} = 65.5 \text{ kJ}$$

3 pt A railroad cart with mass **m** is at rest on the top of a hill with height **h**. (See figure.)



Then it starts to roll down. At the bottom it collides with an identical cart. The two carts lock together. How high can they reach together? (Neglect any losses due to friction.)

5.A ◯ **h**, the original height.
B ◯ (3/4)**h**, three quarter of the original height.
C ◯ (1/2)**h**, half of the original height.
D ◯ (1/4)**h**, one quarter of the original height.
E ◯ Zero, they cannot climb any height.

When cart #1 rolls down, it converts potential energy to kinetic energy:

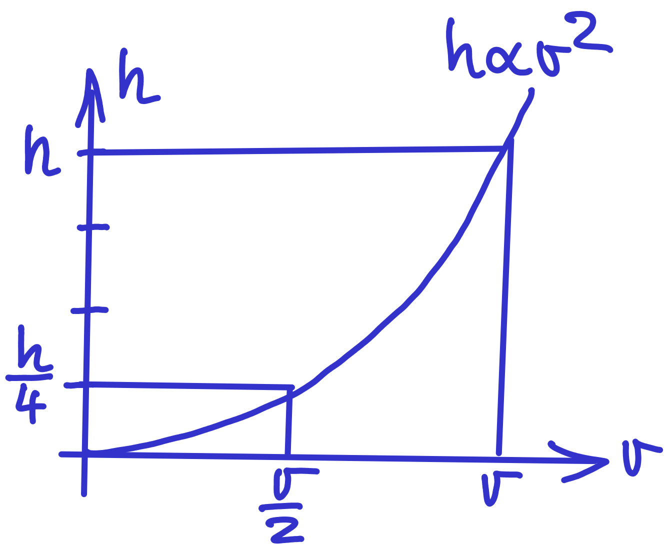
$$mgh = \frac{1}{2}mv^2 \Rightarrow \sqrt{2gh} = v.$$

When cart #1 collides with the resting cart #2, the velocity halves to $\frac{v}{2}$, beause the mass doubled to 2m.

If velocity v was reached coming from height h ($v = \sqrt{2gh}$), then how high will velocity $\frac{v}{2}$ take you?

Answer: $\frac{h}{4}$

$$h = \frac{v^2}{2g} \Rightarrow \boxed{h \propto v^2}$$



$\boxed{4\ pt}$ A 691 kg automobile slides across an icy street at a speed of 68.1 km/h and collides with a parked car. The two cars lock up and they slide together with a speed of 27.7 km/h. What is the mass of the parked car?
(in kg*)*

6. **A**◯ 8.61×10^2 **B**◯ 1.01×10^3 **C**◯ 1.18×10^3 **D**◯ 1.38×10^3
E◯ 1.61×10^3 **F**◯ 1.89×10^3 **G**◯ 2.21×10^3 **H**◯ 2.59×10^3

Conservation of linear momentum:

$$m_1 v_i + m_2 \cdot 0 = (m_1 + m_2) v_f$$

$$m_1 v_i = m_1 v_f + m_2 v_f$$

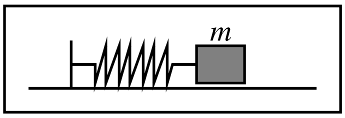
$$m_1 v_i - m_1 v_f = m_2 v_f$$

$$m_1 \frac{v_i - v_f}{v_f} = m_2$$

$$691 \frac{68.1 - 27.7}{27.7} = m_2$$

$$m_2 = 1.01 \cdot 10^3 \text{ kg}$$

2 pt A mass of $m = 1.03$ kg connected to a spring oscillates on a horizontal frictionless surface as shown in the figure.



$x = A \cdot \cos(\omega t)$

$A = 0.173\,m \; ; \; \omega = 2.42 \frac{rad}{s}$

The equation of motion of the mass is given by
$x = 0.173 \cos(2.42t)$
where the position x is measured in meters, the time t in seconds. Determine the period of the motion.
(in s*)*

7. **A**◯ 1.62 **B**◯ 1.90 **C**◯ 2.22 **D**◯ 2.60 **E**◯ 3.04 **F**◯ 3.55 **G**◯ 4.16 **H**◯ 4.87

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2 pt What is the maximum speed reached by the mass?
(in m/s*)*

8. **A**◯ 2.89×10^{-1} **B**◯ 4.19×10^{-1} **C**◯ 6.07×10^{-1} **D**◯ 8.80×10^{-1}
E◯ 1.28 **F**◯ 1.85 **G**◯ 2.68 **H**◯ 3.89

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2 pt Determine the spring constant.
(in N/m*)*

9. **A**◯ 4.72 **B**◯ 5.34 **C**◯ 6.03 **D**◯ 6.82
E◯ 7.70 **F**◯ 8.70 **G**◯ 9.84 **H**◯ 1.11×10^{1}

$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2 \cdot 3.14}{2.42} = 2.60\,s$$

$$v_{max} = A \cdot \omega = 0.173 \cdot 2.42 = 0.419\,m/s$$

$$k = m\omega^2 = 1.03 \cdot 2.42^2 = 6.03\,N/m$$

How to remember: $k = m\omega^2$

$$\underbrace{kA}_{F_{max}} = \underbrace{m\,A\omega^2}_{m \cdot a_{max}}$$

$\boxed{\textit{4 pt}}$ The period of a mass-spring oscillator is 2.90 s. Every time the oscillator completes a full period, the amplitude of the oscillation gets reduced to 92.2 percent of the previous amplitude. How much time does it take for the amplitude to decay to 44.1 percent of its original initial value?
(in s*)*

10. **A**◯ 7.03 **B**◯ 9.34 **C**◯ 1.24×10^1 **D**◯ 1.65×10^1
E◯ 2.20×10^1 **F**◯ 2.92×10^1 **G**◯ 3.89×10^1 **H**◯ 5.17×10^1

$T = 2.90\,s$: period

Amplitude reduction: $92.2\% = 0.922$

$$A_{n+1} = 0.922 \cdot A_n = f \cdot A_n$$

Initial amplitude: $A_i = 100\% = 1.0$

Final amplitude: $A_f = 44.1\% = 0.441$

$$0.922^N = 0.441$$

$$N \cdot \ln 0.922 = \ln 0.441$$

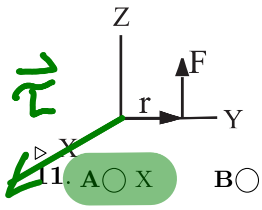
$$N = 10.08$$: "number" of oscillations

Total time : $t = N \cdot T = 10.08 \cdot 2.90$

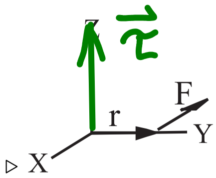
$$t = 29.2\,s$$

$\boxed{\textit{6 pt}}$ A body (not shown) has its center of mass (CM) at the origin. In each case below give the direction for the torque τ with respect to the CM on the body due to force **F** acting on the body at a location indicated by the vector **r**.

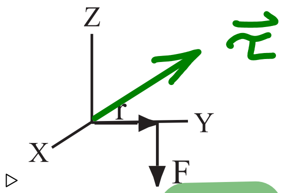
$\vec{\tau} = \vec{r} \times \vec{F}$



11. **A**◯ X **B**◯ -X **C**◯ Y **D**◯ -Y **E**◯ Z **F**◯ -Z

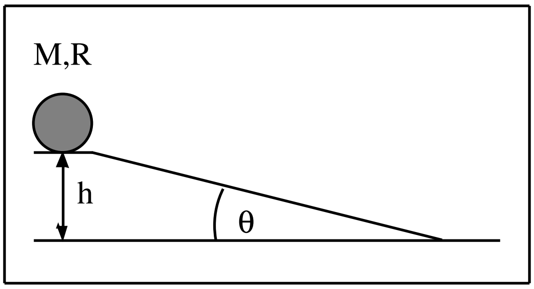


12. **A**◯ X **B**◯ -X **C**◯ Y **D**◯ -Y **E**◯ Z **F**◯ -Z



13. **A**◯ X **B**◯ -X **C**◯ Y **D**◯ -Y **E**◯ Z **F**◯ -Z

$\boxed{4\ pt}$ A solid, homogeneous cylinder with of mass of M = 2.45 kg and a radius of R = 19.1 cm is resting at the top of an incline as shown in the figure.



The height of the incline is h = 1.87 m, and the angle of the incline is θ = 10.9°. The cylinder is rolled over the edge very slowly. Then it rolls down to the bottom of the incline without slipping. What is the final speed of the cylinder? *(in* m/s*)*

14. **A**◯ 1.62 **B**◯ 2.03 **C**◯ 2.53 **D**◯ 3.17 **E**◯ 3.96 **F**◯ 4.95 **G**◯ 6.18 **H**◯ 7.73

Energy balance :

$$PE_i = KE_{t,f} + KE_{r,f}$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$I = \frac{1}{2}MR^2$: solid cylinder

rolling : $v = R\omega$ (no slip condition)

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\cdot\frac{1}{2}MR^2\cdot\frac{v^2}{R^2}$$

$$gh = \frac{1}{2}v^2 + \frac{1}{4}v^2$$

$$\sqrt{\frac{4gh}{3}} = v$$

$$v = \sqrt{\frac{4\cdot 9.81\cdot 1.87}{3}}$$

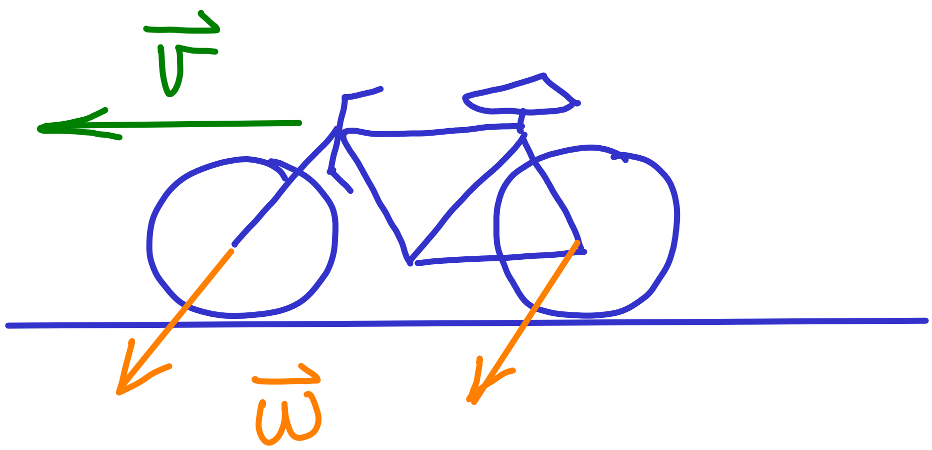
$$v = 4.95\ m/s$$

$\boxed{\textit{3 pt}}$ You ride your bicycle in the forward direction on a straight horizontal road. What is the direction of the velocity vector of your bicycle?

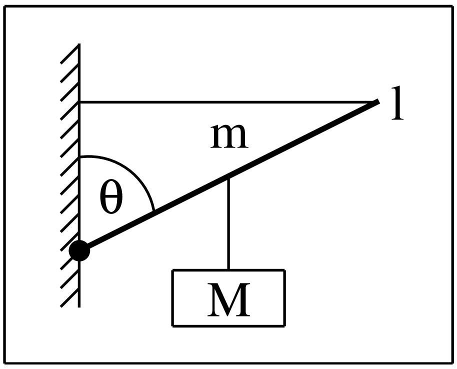
15.A◯ The velocity is zero.
B◯ down to the ground
C◯ to your right
D◯ forward
E◯ to your left
F◯ backward
G◯ up to the sky

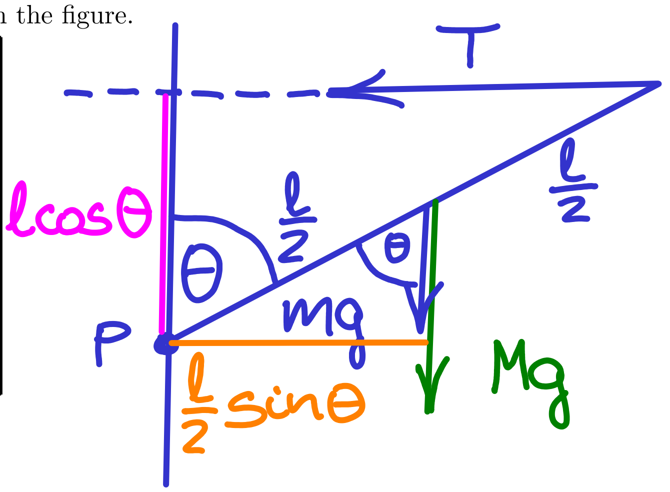
$\boxed{\textit{3 pt}}$ What is the direction of the angular velocity vector of your wheels?

16.A◯ to your right
B◯ backward
C◯ The angular velocity is zero.
D◯ up to the sky
E◯ forward
F◯ to your left
G◯ down to the ground



A crate with a mass of M = 75.5 kg is suspended by a rope from the midpoint of a uniform boom. The boom has a mass of m = 147 kg and a length of l = 8.79 m. The end of the boom is supported by another rope which is horizontal and is attached to the wall as shown in the figure.





3 pt The boom makes an angle of $\theta = 62.1°$ with the vertical wall. Calculate the tension in the vertical rope.
(in N*)*

17. **A**◯ 4.74×10^2 **B**◯ 5.93×10^2 **C**◯ 7.41×10^2 **D**◯ 9.26×10^2
E◯ 1.16×10^3 **F**◯ 1.45×10^3 **G**◯ 1.81×10^3 **H**◯ 2.26×10^3

$Mg = 75.5 \cdot 9.81 =$
$= 741\,N$

3 pt What is the tension in the horizontal rope?
(in N*)*

18. **A**◯ 1.76×10^3 **B**◯ 2.06×10^3 **C**◯ 2.41×10^3 **D**◯ 2.82×10^3
E◯ 3.30×10^3 **F**◯ 3.86×10^3 **G**◯ 4.52×10^3 **H**◯ 5.29×10^3

Balance of the torques:

$$T \cdot \ell\cos\theta = (mg + Mg)\,\frac{\ell}{2}\sin\theta$$

$$T = \frac{1}{2}(m+M)\,g\tan\theta$$

$$T = \frac{1}{2}(147 + 75.5)\cdot 9.81 \cdot \tan 62.1°$$

$$T = 2.06 \cdot 10^3\ N$$