Electric Field and Gauss's law

Announcements

- First exam is next Tuesday, January 28
  - 45 minute exam during lecture time
  - You can bring a 5" by 8" size cheat sheet note card (filled on both sides) (i.e. ½ letter size piece of paper)

- The following clickers are not yet registered. If your clicker number is in this list, email me
  - #07625336 #086EC2A4 #333DE4EA #389BEE4D #82916D7E #82BE6A56 #844E27ED #932F58E4 #95308326 #95349534

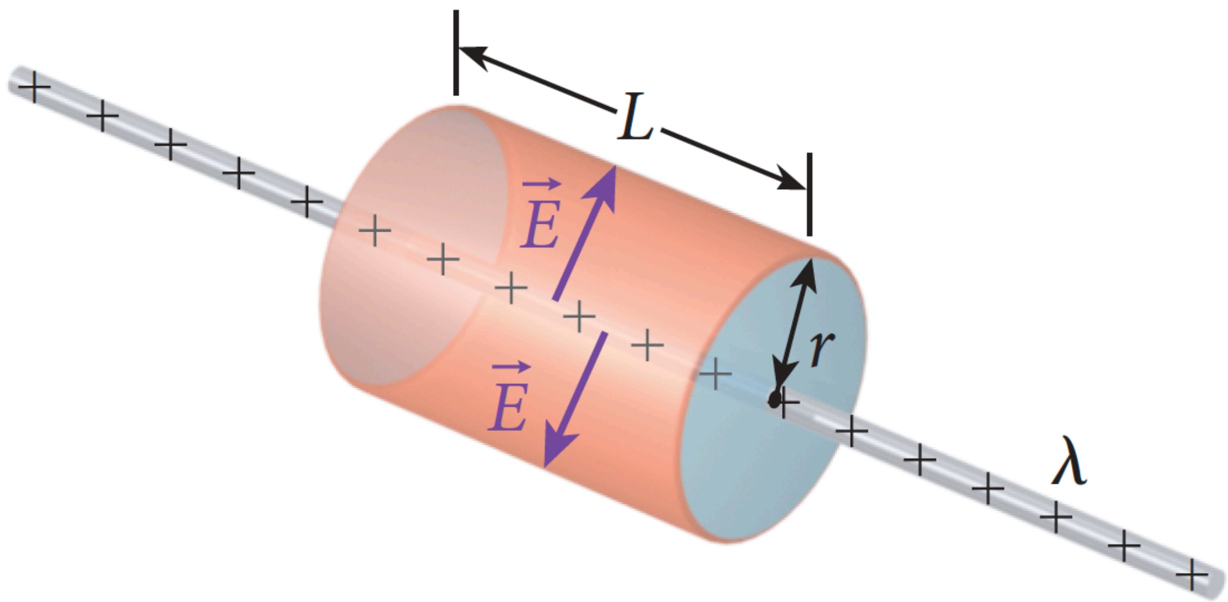
Shielding Demonstration

- We can demonstrate shielding in several ways
- We will place a volunteer in a wire cage and try to fry him/her with large sparks from a Van de Graaff generator
  - Charging the cage
  - Note that the shielding effect does not require a solid conductor
  - A wire mesh will also work, as long as you don't get too close to the open areas

Faraday Cage Extreme

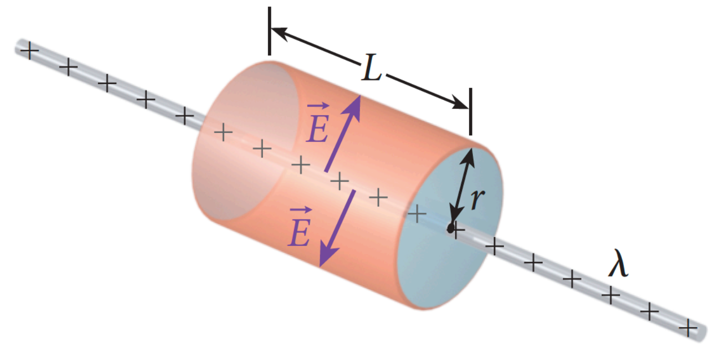
Cylindrical Symmetry

- Calculate the electric field from a conducting wire with charge per unit length λ using Gauss's Law
- Assuming a Gaussian surface in the form of a right cylinder with radius r and length L placed around the wire such that the wire is along the axis of the cylinder



Cylindrical Symmetry

- From symmetry we can see that the electric field will extend radially from the wire
- How?
  - If we rotate the wire along its axis, the electric field must look the same
  - Cylindrical symmetry



- If we imagine a very long wire, the electric field cannot be different anywhere along the length of the wire
  - Translational symmetry
- Thus our assumption of a right cylinder as a Gaussian surface is perfectly suited for the calculation of the electric field using Gauss's Law

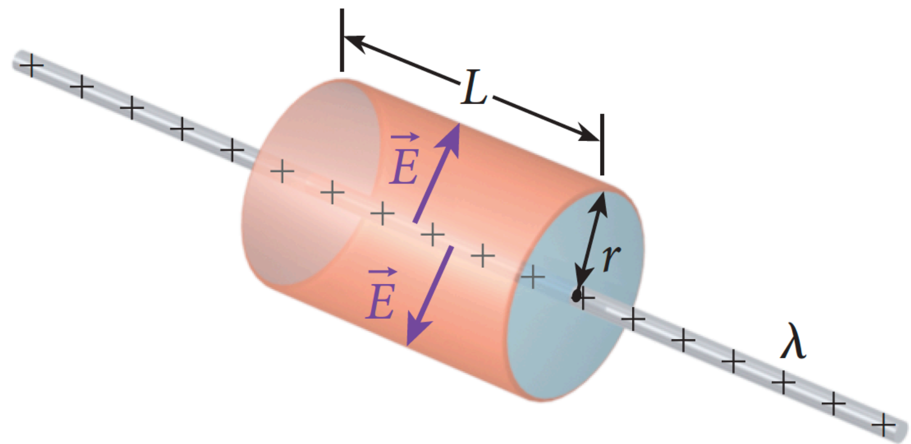
Cylindrical Symmetry

- The electric flux through the ends/caps of the cylinder is zero because the electric field is always perpendicular to the surface vector on the caps [cos(90°)=0]
- The electric field is always parallel to the surface vector on the wall of the cylinder so [cos(0°)=1]

$$\Phi = \oint\!\!\oint \vec{E} \cdot d\vec{A} = EA = E(2\pi rL) = \frac{q}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

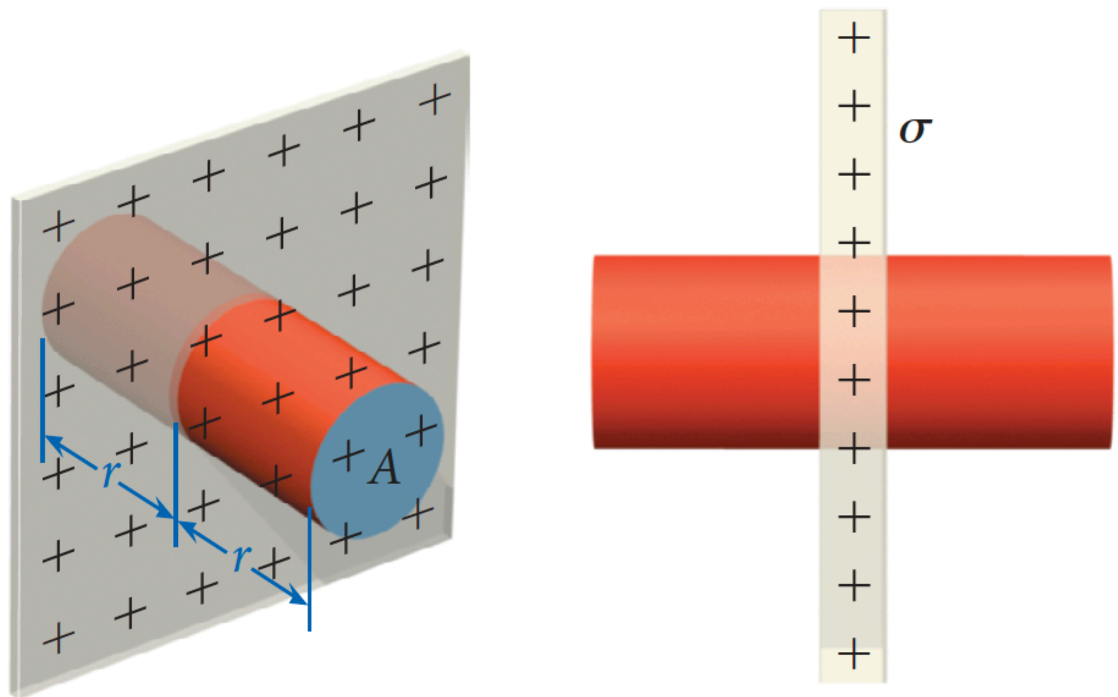
- Solve for the electric field

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}$$



Planar Symmetry, Non-Conductor

- Assume that we have a thin, infinite non-conducting sheet of positive charge



- The charge density in this case is the charge per unit area, σ
- From symmetry, we can see that the electric field will be perpendicular to the surface of the sheet

Planar Symmetry, Non-Conductor

- To calculate the electric field using Gauss's Law, we assume a Gaussian surface in the form of a right cylinder with cross sectional area A and height $2r$, chosen to cut through the plane perpendicularly
- Because the electric field is perpendicular to the plane everywhere, the electric field will be parallel to the walls of the cylinder and perpendicular to the ends of the cylinder
- Using Gauss' Law we get

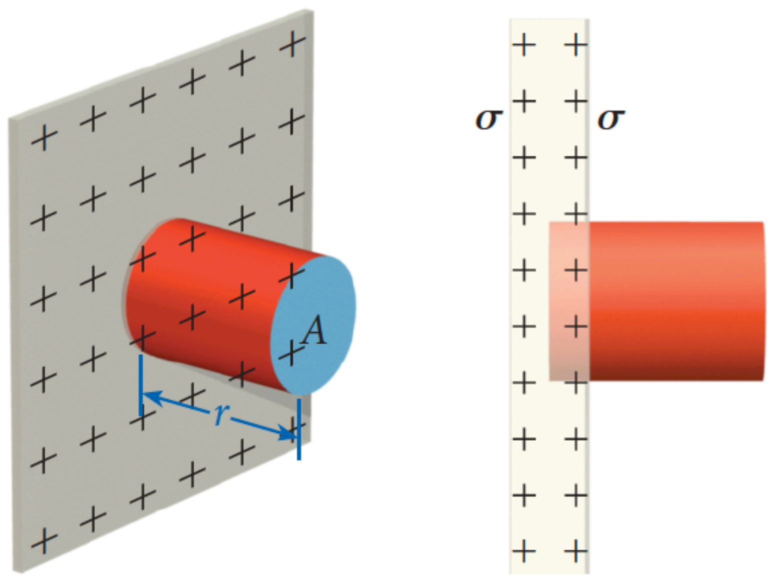
$$\Phi = \oiint \vec{E} \cdot d\vec{A} = EA + EA = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

- The electric field from an infinite non-conducting sheet is

$$E = \frac{\sigma}{2\varepsilon_0}$$

Planar Symmetry, Conductor

- Assume that we have a thin, infinite *conductor* (metal plate) with positive charge

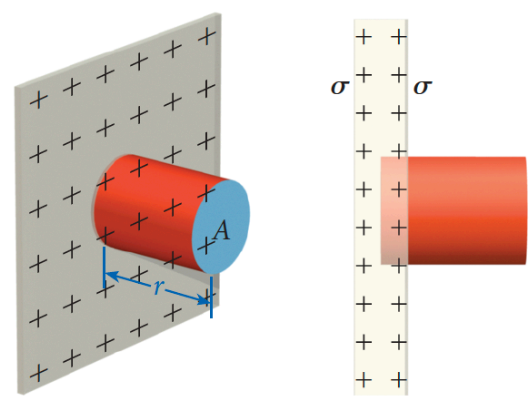


- The charge density in this case is also the charge per unit area, σ, but it's on both surfaces, there is equal surface charge on both sides
- From symmetry, we can see that the electric field will be perpendicular to the surface of the sheet

Planar Symmetry, Conductor

- To calculate the electric field using Gauss's Law, we assume a Gaussian surface in the form of a right cylinder with cross sectional area *A* and height *r*, chosen to cut through one side of the plane perpendicularly
- The field inside the conductor is zero so the end inside the conductor does not contribute to the integral
- Because the electric field is perpendicular to the plane everywhere, the electric field will be parallel to the walls of the cylinder and perpendicular to the end of the cylinder outside the conductor
- Using Gauss's Law we get the electric field from a charged flat conductor

$$EA = \frac{\sigma A}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{\sigma}{\varepsilon_0}$$



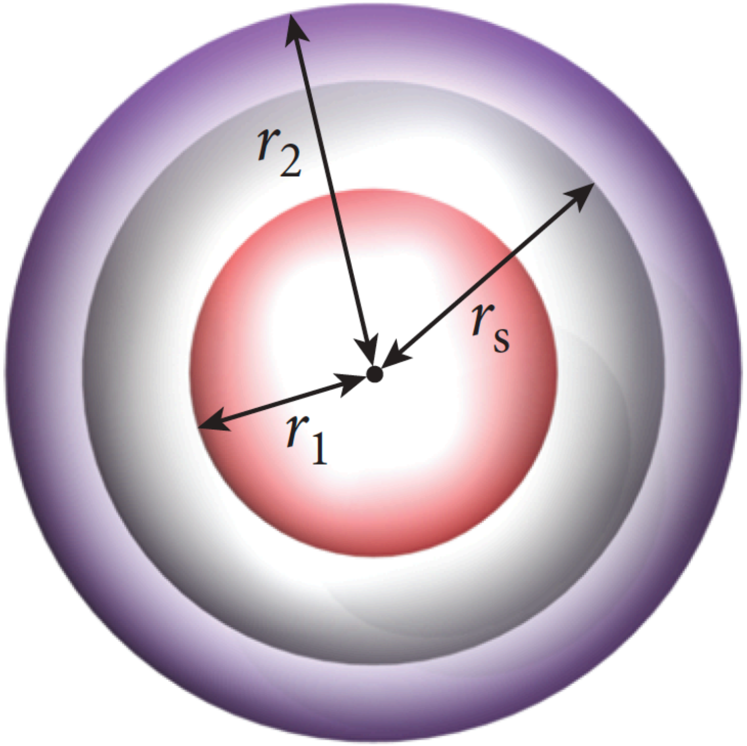
Special Symmetries

- We have applied Gauss's Law to a point charge and showed that we get Coulomb's Law
- Now let's look at more complicated distributions of charge and calculate the resulting electric field
- We will use a charge density to describe the distribution of charge
- This charge density will be different depending on the geometry

Symbol	Name	Unit
λ	Charge per length	C/m
σ	Charge per area	C/m^2
ρ	Charge per volume	C/m^3

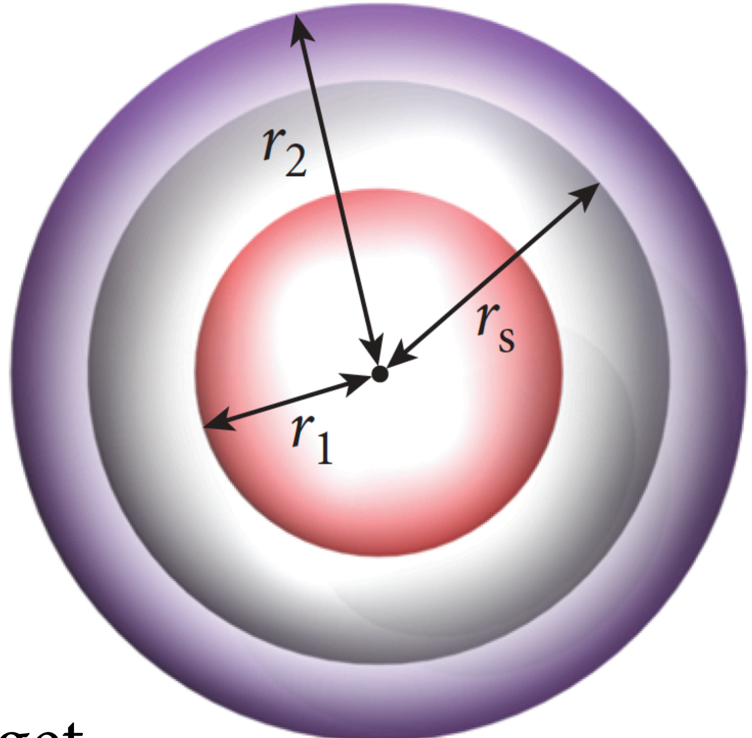
Spherical Symmetry

- Calculate the electric field from charge distributed as a **spherical shell**
- Assume that we have a **spherical shell** of charge q with radius r_S (gray)
- We will assume two different spherical Gaussian surfaces
  - $r_2 > r_S$ (purple) i.e. outside
  - $r_1 < r_S$ (red) i.e. inside



Spherical Symmetry

- Let's start with the Gaussian surface outside the sphere of charge, $r_2 > r_S$ (purple)
- We know from symmetry arguments that the electric field will be radial outside the charged sphere
- If we rotate the sphere, the electric field cannot change
  - Spherical symmetry
- Thus we can apply Gauss' Law and get

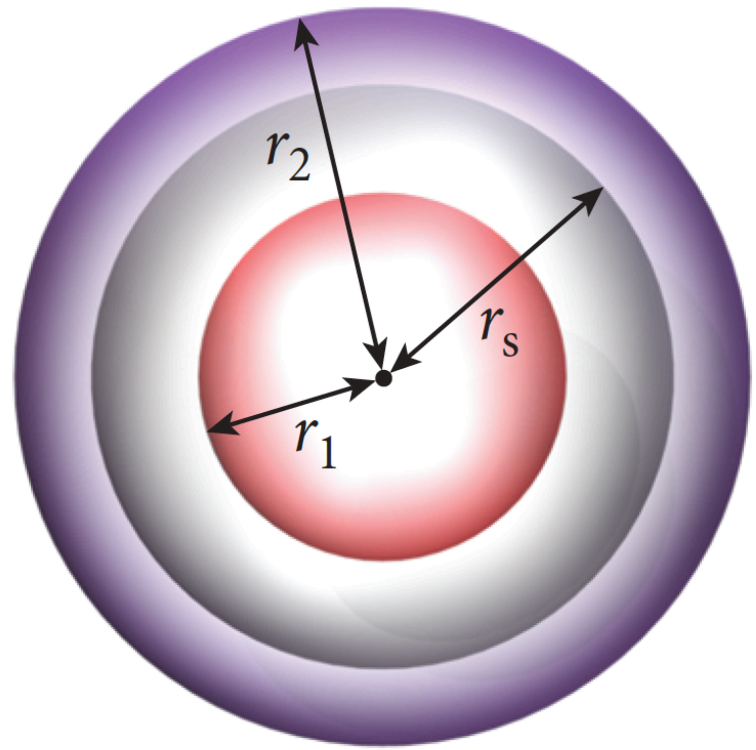


$$\oiint \vec{E} \cdot d\vec{A} = E\left(4\pi r_2^2\right) = \frac{q}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_2^2}$$

Spherical Symmetry

- Let's take the Gaussian surface inside the sphere of charge, $r_1 < r_S$ (red)
- The enclosed charge is zero so

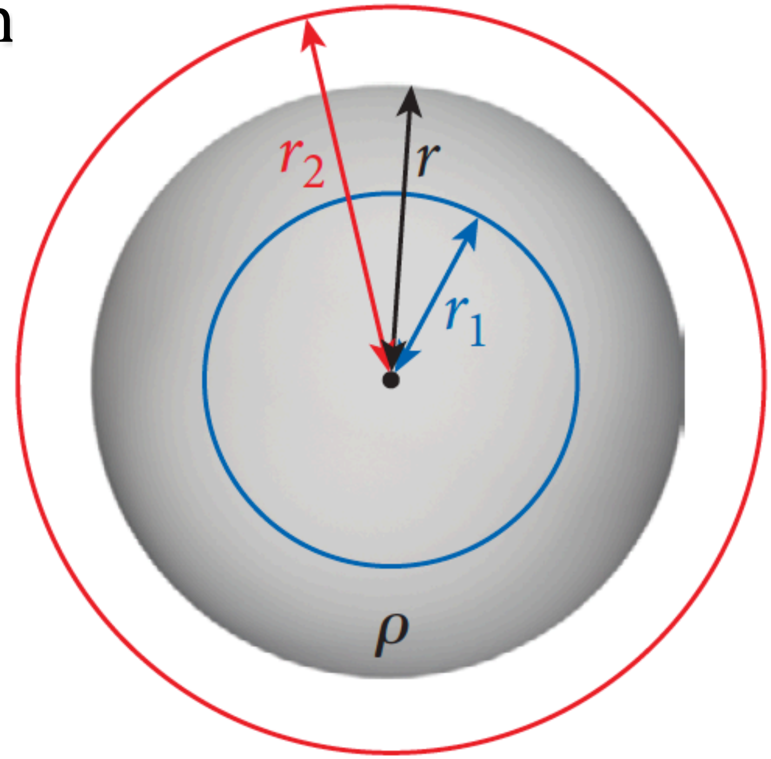
$$\oiint \vec{E} \cdot d\vec{A} = E\left(4\pi r_1^2\right) = 0 \quad \Rightarrow \quad E = 0$$



- We find that the electric field is zero everywhere inside spherical shell of charge
- Thus we obtain two results
  - The electric field outside a spherical shell of charge is the same as that of a point charge
  - The electric field inside a spherical shell of charge is zero

Spherical Symmetry: Uniform Distribution

- Next, let's calculate the electric field from charge distributed throughout a spherical volume with a uniform charge density $\rho > 0$ and radius r



- We will assume two different spherical Gaussian surfaces
  - $r_2 > r$ (outside, red)
  - $r_1 < r$ (inside, blue)
- Let's start with the surface with $r_1 < r$
- From the symmetry of the charge distribution, the electric field is perpendicular to the Gaussian surface everywhere

Spherical Symmetry: Uniform Distribution

- Gauss's Law gives us

$$\oiint \vec{E} \cdot d\vec{A} = E\left(4\pi r_1^2\right) = \frac{q}{\varepsilon_0} = \frac{\rho\left(\frac{4}{3}\pi r_1^3\right)}{\varepsilon_0}$$

- Solving for E we find

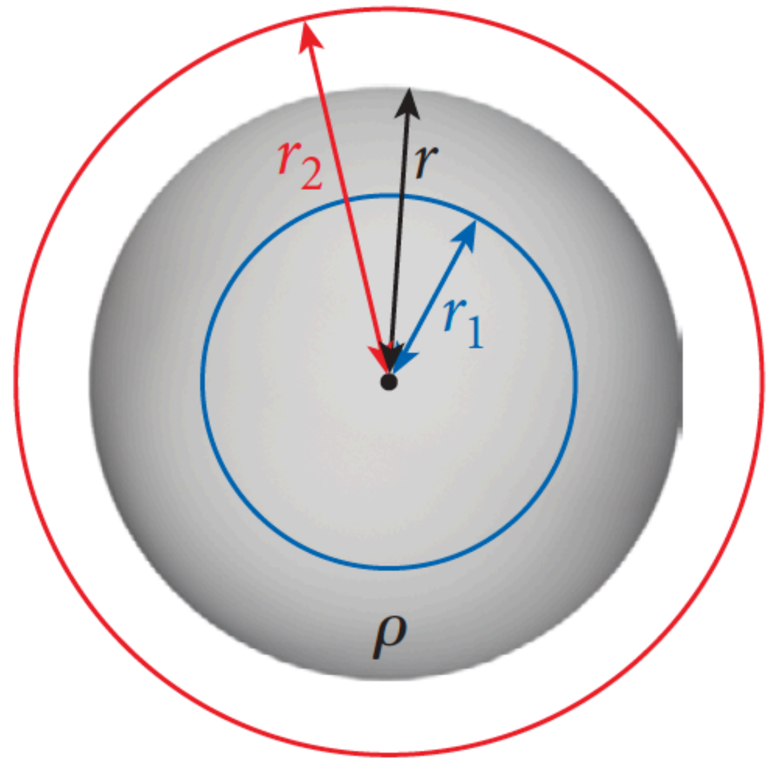
$$E_{\text{inside}} = \frac{\rho r_1}{3\varepsilon_0}$$

- The total charge on the sphere is

$$q_{\text{t}} = \rho V = \rho\left(\frac{4}{3}\pi r^3\right)$$

- The enclosed charge is

$$q = \frac{\text{volume inside } r_1}{\text{volume}} q_{\text{t}} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r^3} q_{\text{t}} = \frac{r_1^3}{r^3} q_{\text{t}}$$



Spherical Symmetry: Uniform Distribution

- Gauss's Law gives us

$$\oiint \vec{E}\cdot d\vec{A} = E\left(4\pi r_1^2\right) = \frac{q_t}{\varepsilon_0}\frac{r_1^3}{r^3}$$

- Solving for E we find

$$E_{\text{inside}} = \frac{q_t r_1}{4\pi\varepsilon_0 r^3} = \frac{kq_t r_1}{r^3}$$

- Consider $r_2 > r$

$$\oiint \vec{E}\cdot d\vec{A} = E\left(4\pi r_2^2\right) = \frac{q_t}{\varepsilon_0}$$

- Solving for E we find

$$E_{\text{outside}} = \frac{q_t}{4\pi\varepsilon_0 r_2^2} = \frac{kq_t}{r_2^2} \quad \text{same as point charge}$$