

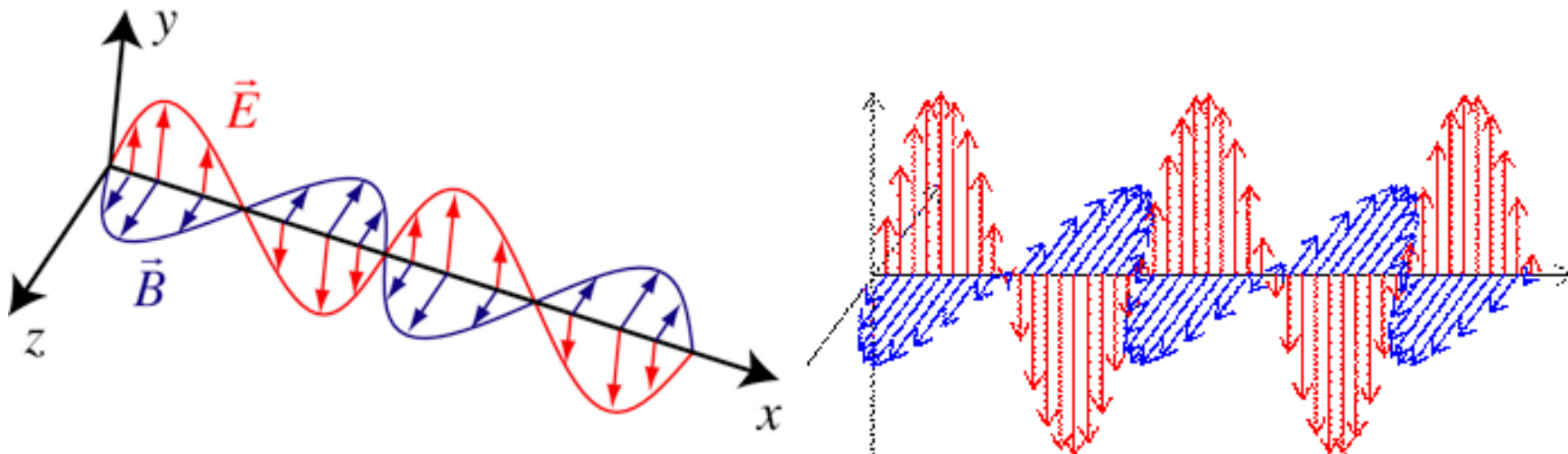


Electromagnetic Waves

Wave Solutions to Maxwell's Equations



- The electric field is always perpendicular to the direction the electromagnetic wave is traveling and is always perpendicular to the magnetic field
- The electric and magnetic fields are in phase
- The wave shown below is a snapshot in time
- The vectors shown represent the magnitude and direction for the electric and magnetic fields
- Now we must show that our proposed solution satisfies Maxwell's Equations



The Speed of Light

- Our solutions for Maxwell's Equations are correct if

$$\frac{E}{B} = c \quad \text{and} \quad \frac{E}{B} = \frac{1}{\mu_0 \epsilon_0 c}$$

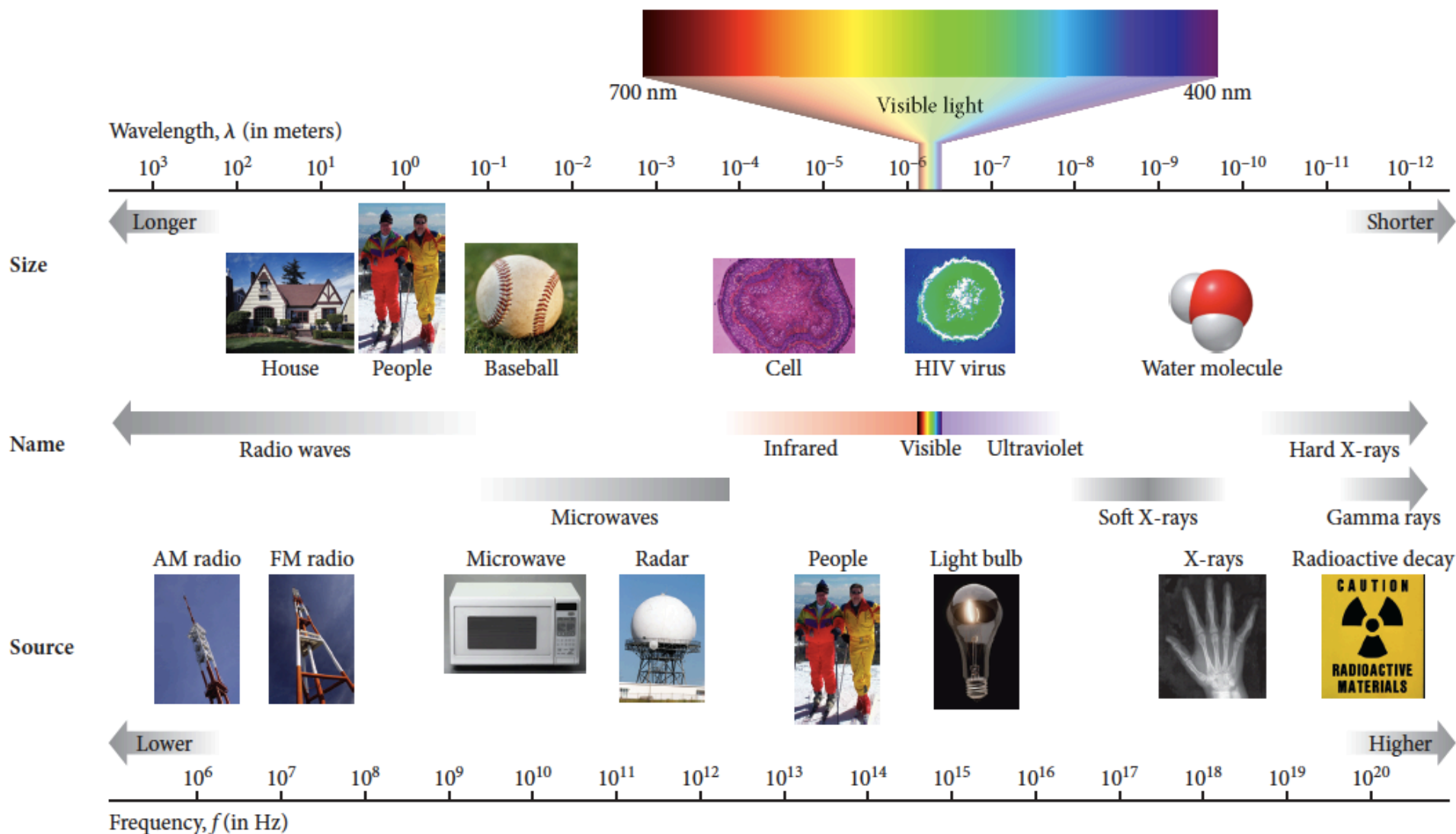
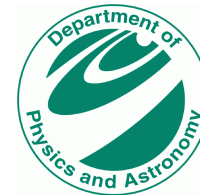
- We can see that

$$c = \frac{1}{\mu_0 \epsilon_0 c} \quad \Rightarrow \quad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

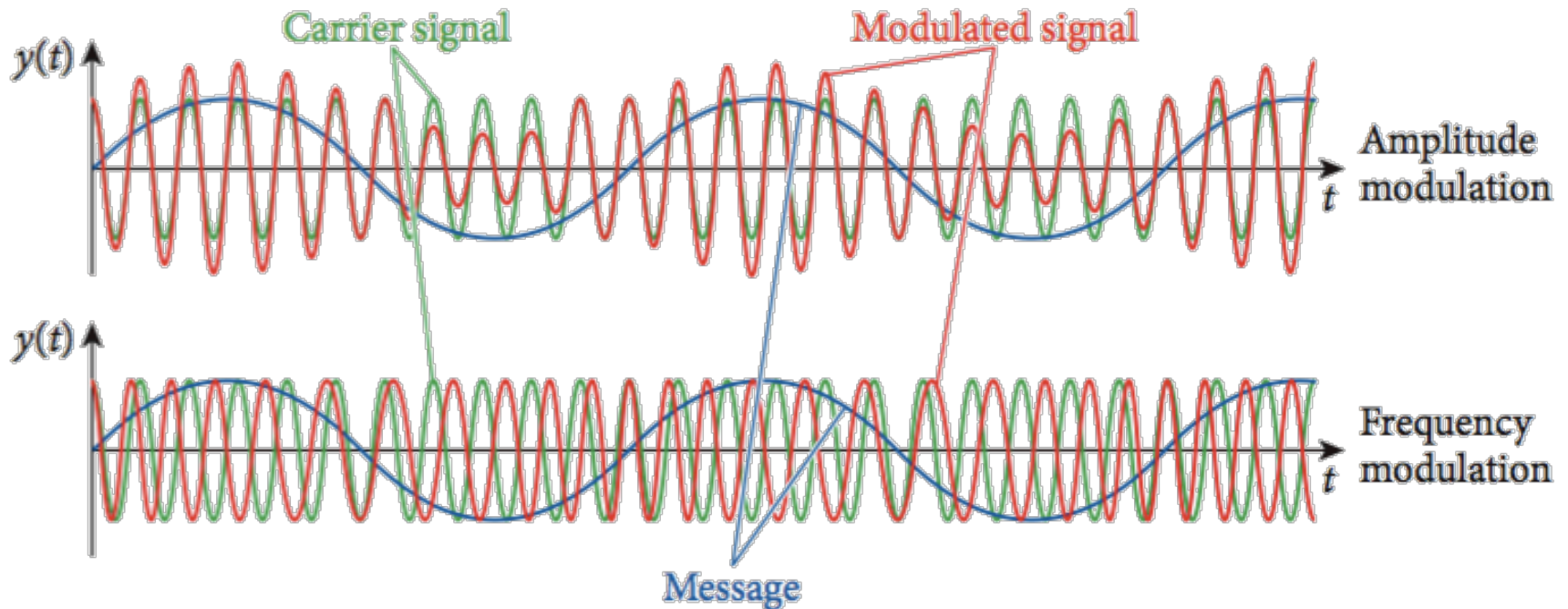
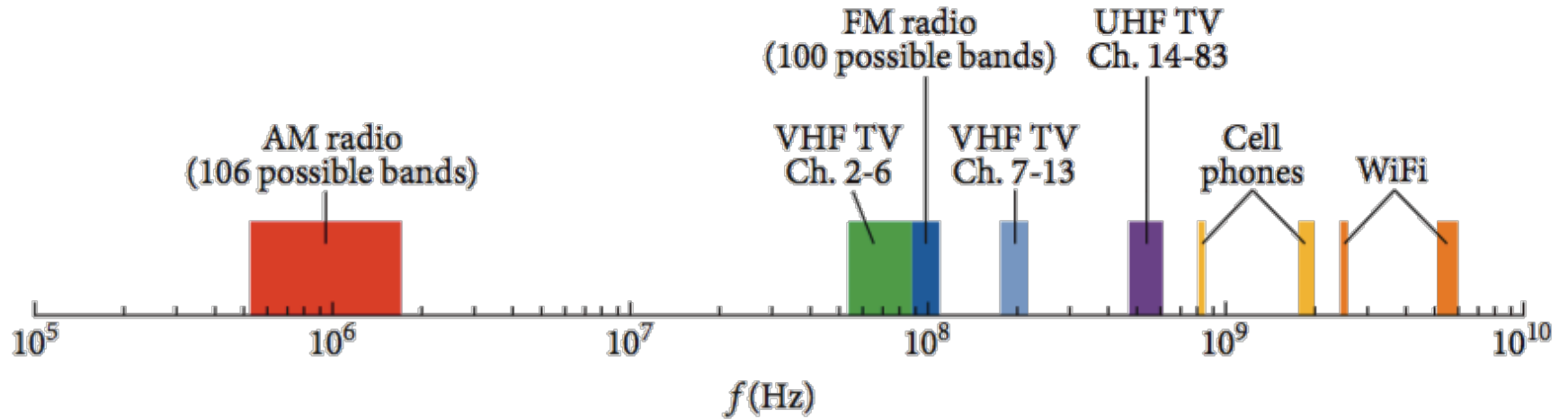
- Thus the speed of an electromagnetic wave can be expressed in terms of two fundamental constants related to electric fields and magnetic fields, the magnetic permeability and the electric permittivity of the vacuum
- If we put in the values of these constants we get

$$c = \frac{1}{\sqrt{(1.26 \cdot 10^{-6} \text{ H/m})(8.85 \cdot 10^{-12} \text{ F/m})}} = 2.99 \cdot 10^8 \text{ m/s}$$

The Electromagnetic Spectrum



The Electromagnetic Spectrum



The Electromagnetic Spectrum



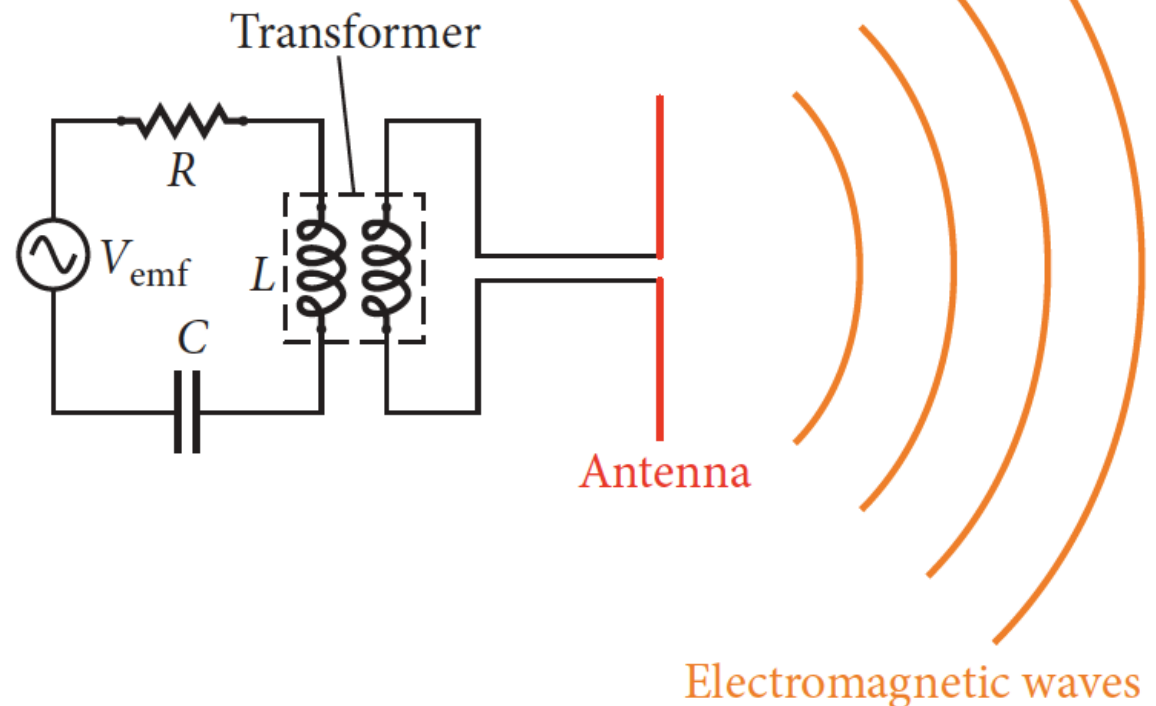
- Electromagnetic waves have wavelengths ranging from 1000 m to less than 10^{-12} m and frequencies ranging from 10^6 to 10^{20} Hz
- Ranges of wavelength and frequency have names that identify the most common application of those waves
- Visible light refers to electromagnetic waves ranging in wavelength from 400 nm to 700 nm
- The response of the human eye is peaked around 550 nm (green) and drops off quickly away from that wavelength
- Other wavelengths of electromagnetic waves are invisible
- However, we can still detect the electromagnetic waves
- For example, we can feel electromagnetic waves in the infrared (wavelengths just longer than visible up to around 0.1 mm) as warmth

Traveling Electromagnetic Waves

- Sub-atomic processes can produce electromagnetic waves such as gamma rays, X-rays, and light
- Electromagnetic waves can also be produced by an oscillator connected to an antenna
- Series RLC circuit, resonance frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- The connection between the circuit on the left and the circuit on the right is accomplished using a transformer



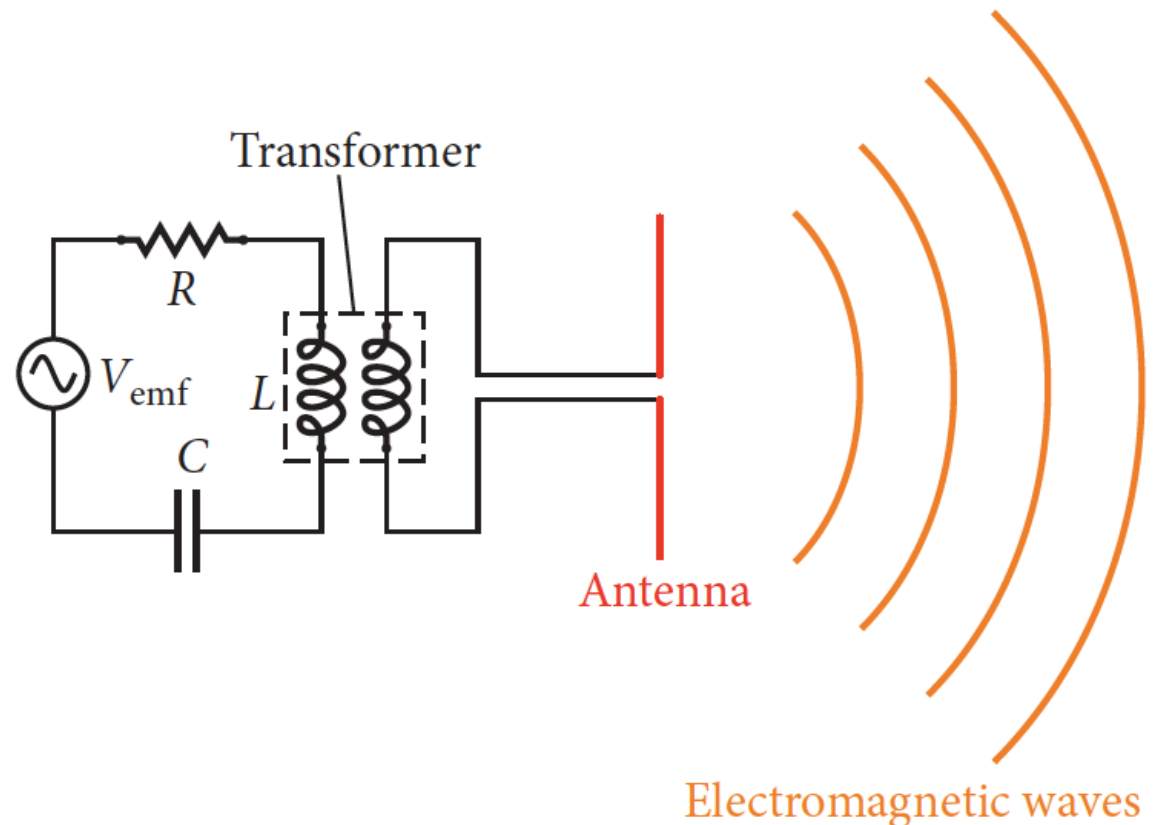
Traveling Electromagnetic Waves



- A dipole antenna is used to approximate an electric dipole
- The voltage and current in the antenna vary sinusoidally with time and cause charge in the antenna to oscillate with angular frequency ω of the circuit

- The electromagnetic waves created by moving charges travel from the antenna with speed c and frequency $f = \omega/(2\pi)$

- Wavelength $\lambda = \frac{c}{f}$



Traveling Electromagnetic Waves

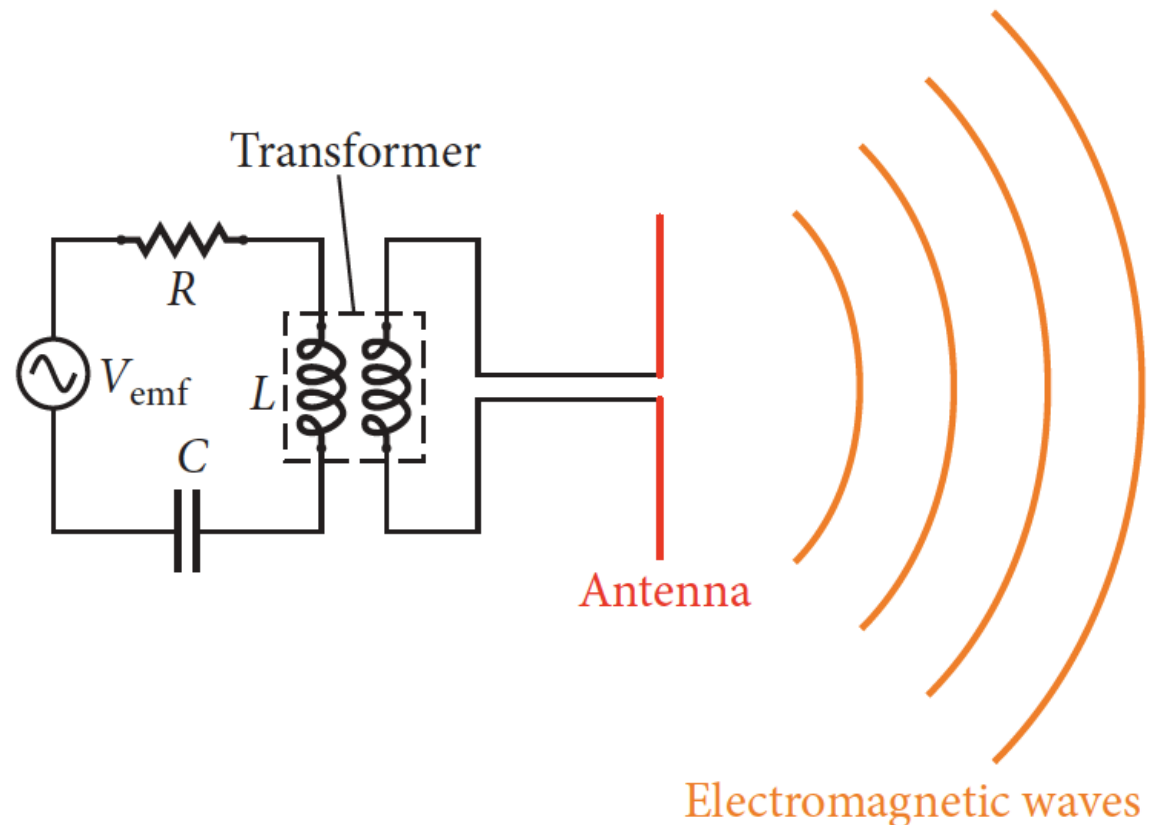


- We can think of these traveling electromagnetic waves as wave fronts spreading out spherically from the antenna
- However, at a large distance from the antenna, the wave fronts will appear to be almost planar

- So we can think of the electromagnetic wave in terms of our assumed form

$$E(\vec{r}, t) = E_{\max} \sin(kx - \omega t)$$

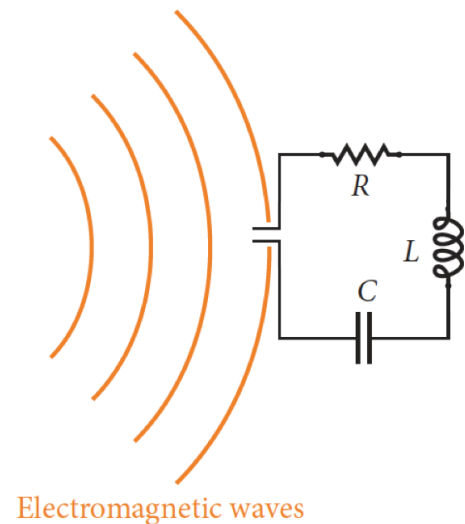
$$B(\vec{r}, t) = B_{\max} \sin(kx - \omega t)$$



Traveling Electromagnetic Waves



- If we now place a second RLC circuit in the path of these electromagnetic waves and tune it to the same frequency $\omega = \omega_0$ as the emitting circuit, then voltage and current will be induced in this second circuit
- These induced oscillations are the basic idea of radio transmission and reception
- If the second circuit has $\omega_{0,2} = 1/\sqrt{LC} \neq \omega_0$ smaller voltages and currents will be induced, providing selective tuning for different frequencies
- This principle of transmission of electromagnetic waves is used to transmit wireless signals



Energy Transport



- Electromagnetic waves carry energy
- In the sunlight, we feel warmth ... staying in the sunlight for too long will lead to sunburn
- These phenomena are related to electromagnetic waves emitted from the Sun
- Energy generated by nuclear reactions in the sun is carried by the electromagnetic waves
- The rate of energy transported by an electromagnetic wave is usually defined as

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

- This quantity is called the Poynting vector after British physicist John Poynting who first discussed its properties

Energy Transport



- The magnitude of the Poynting vector is related to the instantaneous rate at which energy transported by an electromagnetic wave over a given area
 - more simply, the instantaneous power per unit area

$$S = |\vec{S}| = \left(\frac{\text{power}}{\text{area}} \right)_{\text{instantaneous}}$$

- The units of the Poynting vector are W/m^2
- For an electromagnetic wave, in which B is perpendicular to E , we can write

$$S = \frac{1}{\mu_0} EB$$

Energy Transport



- We know that the magnitude of the electric field and the magnetic field are directly related via $E/B = c$
- We can express the instantaneous power per unit area of an electromagnetic wave in terms of the magnitude the electric field or the magnetic field
- However, it is easier to measure an electric field so we express the instantaneous power per unit area as

$$S = \frac{1}{c\mu_0} E^2$$

- We can now substitute our sinusoidal form for the electric field and obtain an expression for the transmitted power per unit area

$$E = E_{\max} \sin(kx - \omega t)$$

Energy Transport



- The usual method of describing the power per unit area in an electromagnetic wave is the intensity I of the wave is

$$I = S_{\text{ave}} = \left(\frac{\text{power}}{\text{area}} \right)_{\text{ave}} = \frac{1}{c\mu_0} \left[E_{\text{max}}^2 \sin^2(kx - \omega t) \right]_{\text{ave}}$$

- The units of intensity are the same as the units of the Poynting vector, W/m^2
- The average of $\sin^2(kx - \omega t)$ over time is $1/2$
- So we can express the intensity as

$$I = \frac{1}{c\mu_0} E_{\text{rms}}^2 \quad \left(E_{\text{rms}} = E_{\text{max}} / \sqrt{2} \right)$$

Example problem: Plane wave



- The components of the electric field in an electromagnetic wave traveling in vacuum are described by $E_x = 0$, $E_y = 0$, and $E_z = 4.31 \sin(kx - 1.12E8 t)$ V/m, where x is measured in meters and t in seconds.

Calculate the intensity of the wave (in W/m^2).

Answer: The intensity is computed from the magnitude of the electric field as

$$I = \frac{1}{c\mu_0} E_{\text{rms}}^2 \quad \left(E_{\text{rms}} = E_{\text{max},z} / \sqrt{2} \right)$$

Thus $E_{\text{rms}} = 4.31/\sqrt{2}$ V/m = 2.34 V/m

Thus the intensity is

$$I = 0.0247 W / m^2$$

Radiation Pressure



- When you walk out into the sunlight, you feel warmth, but you do not feel any force from the sunlight
- Sunlight is exerting a pressure on you, but that pressure is small enough that we do not notice it
- This type of radiation is not the same as radioactive radiation resulting from the decay of unstable nuclei
- Let's calculate the magnitude of the pressure exerted by these radiated electromagnetic waves

Radiation Pressure

- Electromagnetic waves have energy
- Electromagnetic waves also have linear momentum
- Let's assume that an electromagnetic wave is incident on an object and that this object totally absorbs the wave over some time interval Δt
- The object will then gain energy ΔU over that time interval from the radiated electromagnetic waves and we can relate the change in momentum of the object Δp to the change in energy by

$$\Delta p = \frac{\Delta U}{c}$$

- The change in momentum of the object will be in the same direction as the incident electromagnetic wave

Radiation Pressure

- Poynting vector S $S = \frac{1}{c\mu_0} E^2 = \frac{\text{Power}}{\text{area}}$
- Electromagnetic wave incident on an object
- Force is

$$F = \frac{\Delta p}{\Delta t} = \frac{IA}{c} \quad \text{with} \quad I = \frac{1}{c\mu_0} E_{\text{rms}}^2 \quad \left(E_{\text{rms}} = E_{\text{max}} / \sqrt{2} \right)$$

- Intensity A , speed of light c
- If wave is absorbed, pressure is

$$p_r = \frac{I}{c}$$

- If wave is reflected, pressure is

$$p_r = \frac{2I}{c}$$

Radiation Pressure from a Laser Pointer



- A green laser pointer has a power of 1.00 mW
- We shine the laser pointer perpendicularly on a white sheet of paper, which reflects the light
- The spot of the laser pointer is 2.00 mm in diameter

- Question:
 - What force does the light from the laser point exert on the paper?
- Answer:
 - The intensity of the light from the laser pointer is given by

$$I = \frac{\text{power}}{\text{area}} = \frac{1.00 \cdot 10^{-3} \text{ W}}{\pi (1.00 \cdot 10^{-3} \text{ m})^2} = 318 \text{ W/m}^2$$

Radiation Pressure from a Laser Pointer (2)



- The radiation pressure is equal to the force exerted by the light divided by the area over which it acts

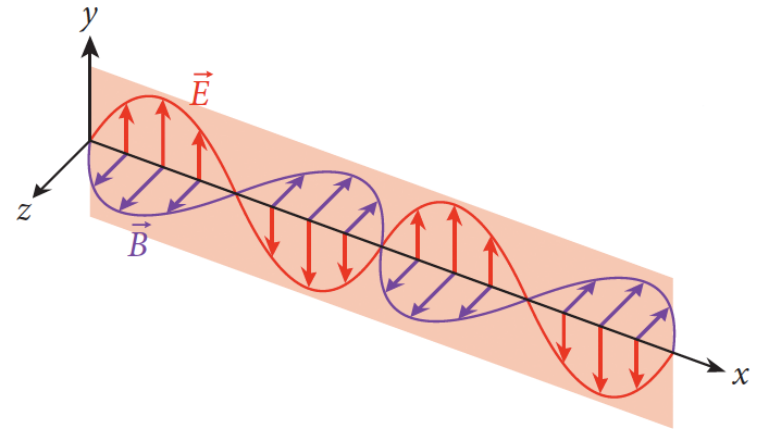
$$p_r = \frac{\text{force}}{\text{area}} = \frac{2I}{c}$$

- Thus the force exerted by the laser pointer light is

$$\text{force} = \text{area} \cdot \frac{2I}{c} = \pi \left(1.0 \cdot 10^{-3} \text{ m} \right)^2 \cdot \frac{2 \cdot 318 \text{ W/m}^2}{3.00 \cdot 10^8 \text{ m/s}} = 6.66 \cdot 10^{-12} \text{ N}$$

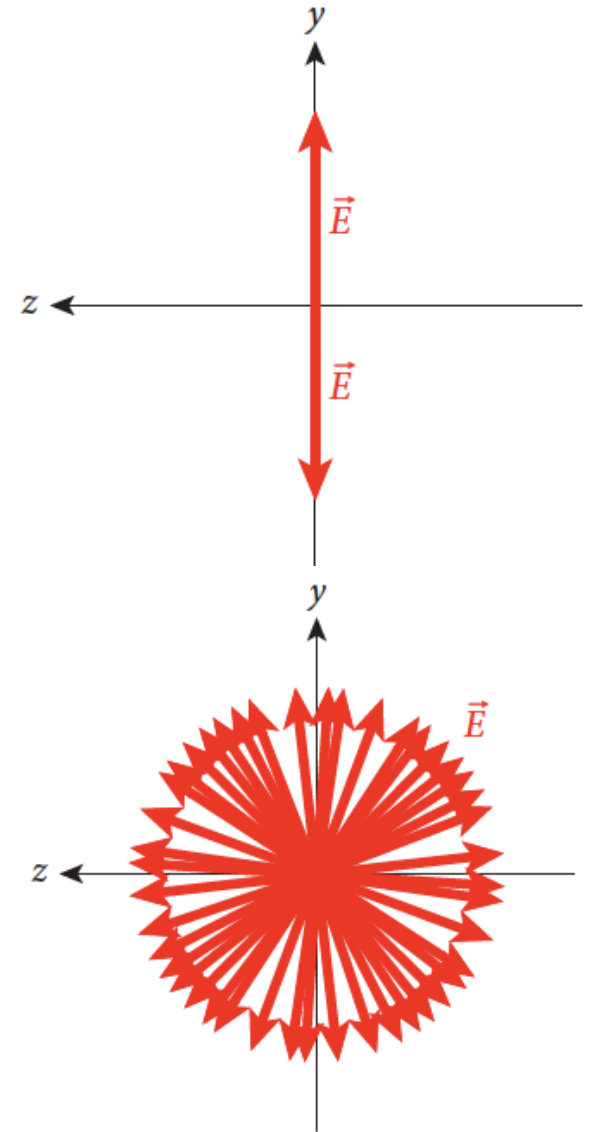
Polarization

- Consider the electromagnetic wave shown
- The electric field for this electromagnetic wave always points along the y -axis
- Taking the x -axis as the direction that the wave is traveling, we can define a plane of oscillation for the electric field of the electromagnetic wave as shown
- This type of wave is called a **plane-polarized wave** in the y -direction
- We represent the polarization of an electromagnetic wave by looking at the electric field vector in the y - z plane, which is perpendicular to the direction the wave is traveling



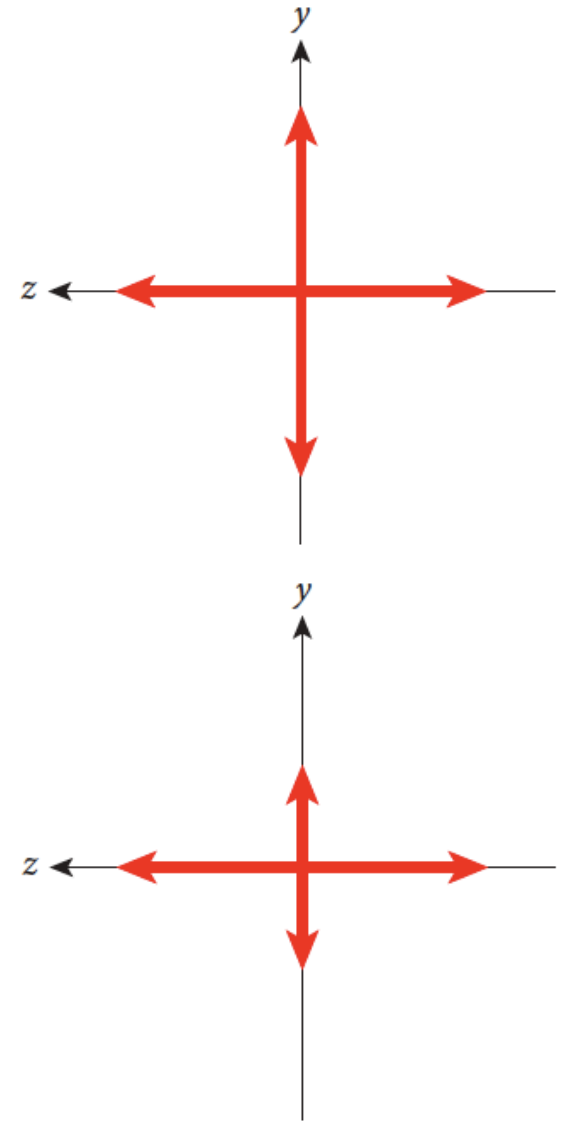
Polarization

- For plane polarized light, the electric field oscillates in the y -direction
- The electromagnetic waves making up the light emitted by most common light sources such as an incandescent light bulb have random polarizations
- Each wave has its electric field vector oscillating in a different plane
- This light is called **unpolarized light**
- We can represent the polarization of the light from an unpolarized source by drawing many waves with random orientations



Polarization

- We can represent light with many polarizations by summing the y -components and summing the z -components to produce the net y and z components
- For unpolarized light, we obtain equal components in the y - and z -directions
- If there is less net polarization in the y direction than in the z direction, then we say that the light is **partially polarized** in the z direction



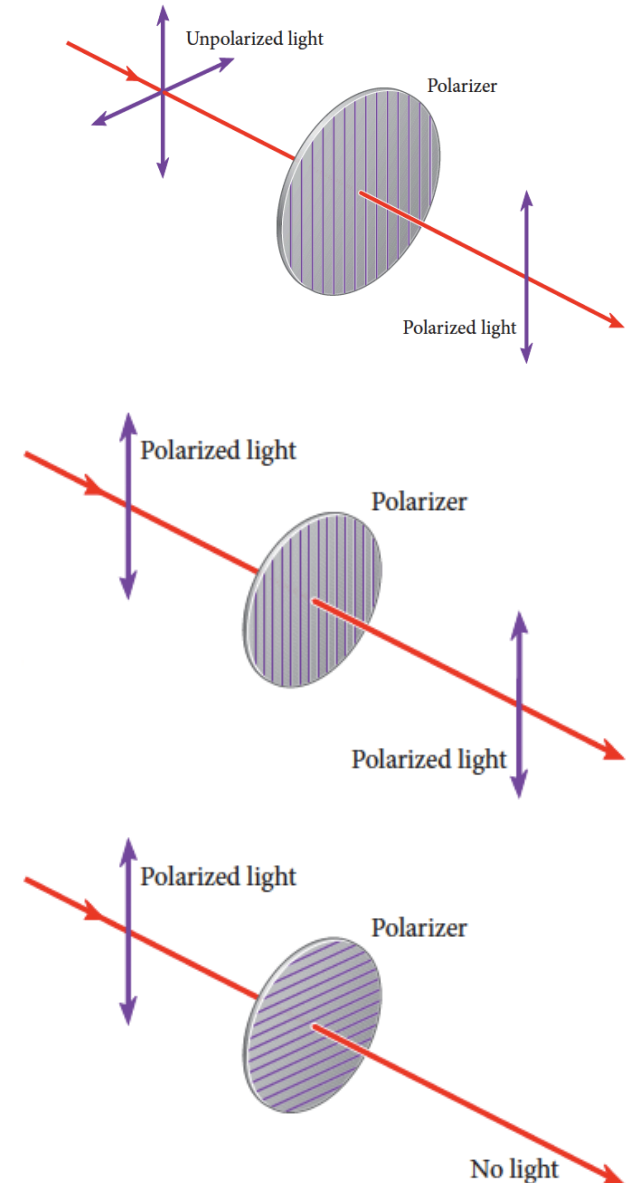


Polarizer

- We can change unpolarized light to polarized light by passing it through a polarizer
- A polarizer allows only one component of the polarization of the light to pass through
- One way to make a polarizer is to use a material that consists of long parallel chains of molecules that effectively only let light pass if it has a certain direction of polarization and block it if it has a polarization in the perpendicular direction
- We will discuss polarizers without taking into account the details of the molecular structure
- We will characterize a polarizer with a polarizing direction
- Unpolarized light passing through a polarizer will emerge polarized in the polarizing direction

Polarizer

- The components of the unpolarized light that have the same polarization as the polarizer are transmitted but those with polarization perpendicular to the polarizer are absorbed
- If light with polarization parallel to the polarizing angle is incident on the polarizer, all the light passes through
- If light with polarization perpendicular to the polarizing angle is incident on the polarizer, none of the light is transmitted



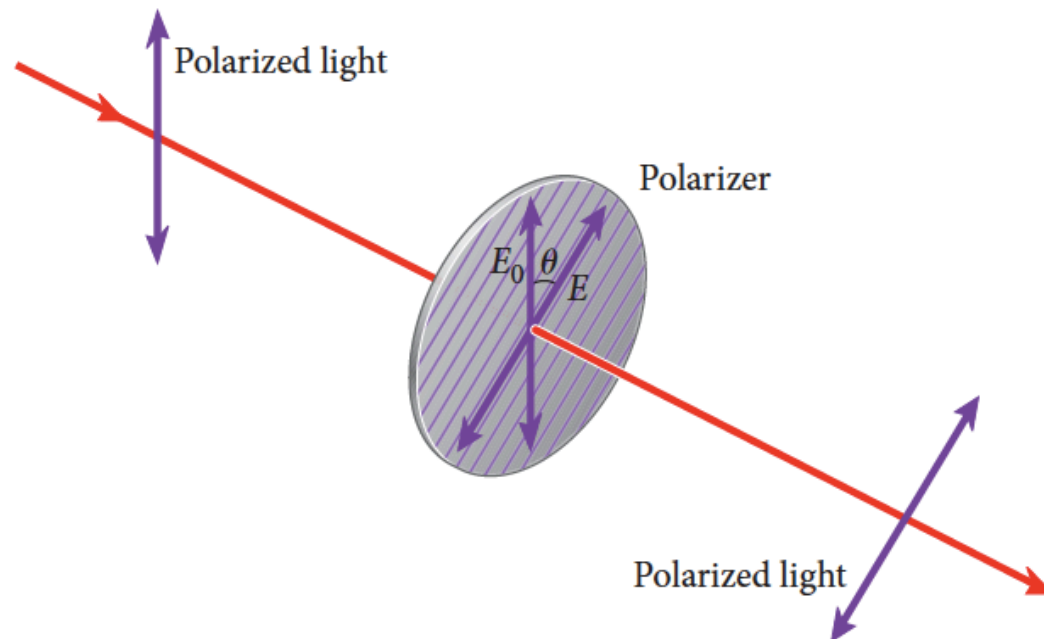
Intensity



- Now let's consider the **intensity** of the light that passes through a polarizer
- We begin with unpolarized light with intensity I_0
- Unpolarized light has equal components of polarization in the y and z directions
- After passing through a vertical polarizer only the y component of the polarization remains
- The intensity I of the light passing through the polarizer is given by
$$I = \frac{1}{2} I_0$$
- The unpolarized light had equal contribution from the y and z components and only the y components are transmitted by the vertical polarizer

Intensity

- Assume that polarized light passes through a polarizer and that this light has a polarization that is not parallel or perpendicular to the polarizing direction of the polarizer
- The angle between the incident polarization is θ

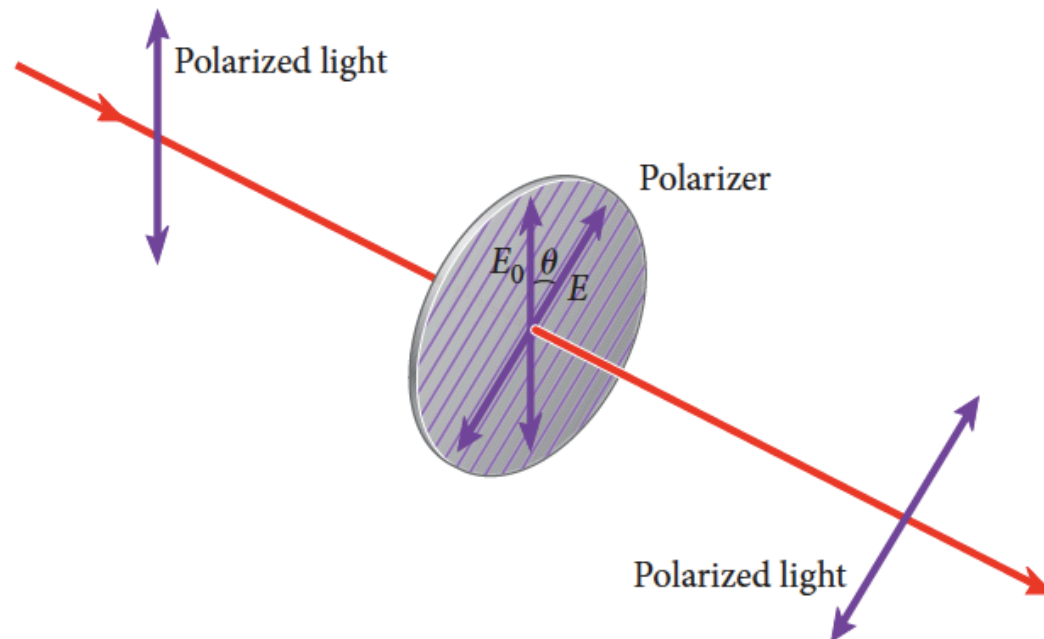


Intensity

- The electric field E of the transmitted light is

$$E = E_0 \cos \theta$$
- E_0 is the electric field of the incident polarized light
- The intensity of the light I_0 before the polarizer is given by

$$I_0 = \frac{1}{c\mu_0} E_{rms}^2 = \frac{1}{2c\mu_0} E_0^2$$



Intensity



- After the light passes through the polarizer, the intensity I is

$$I = \frac{1}{2c\mu_0} E^2$$

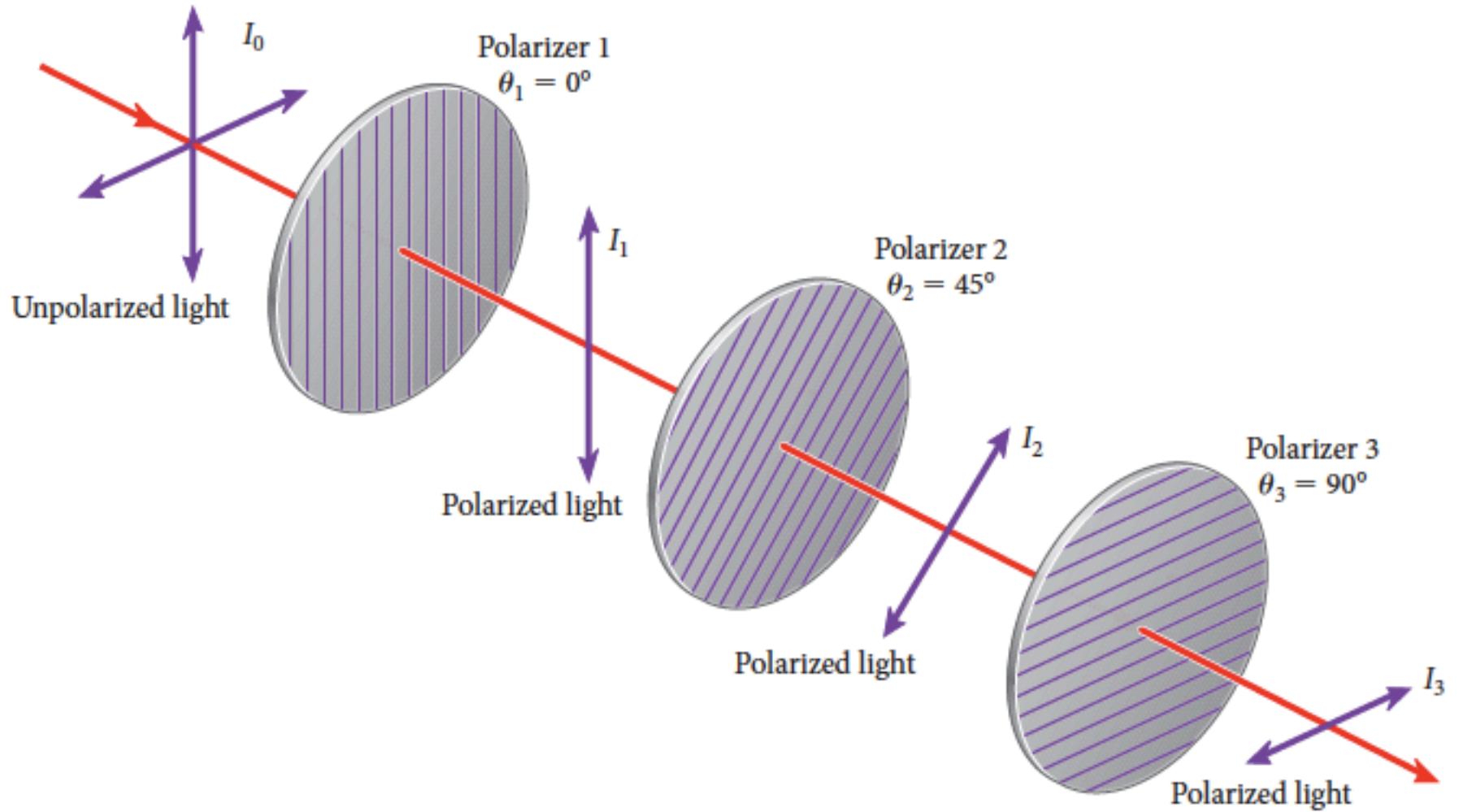
- The transmitted intensity in terms of the initial intensity is

$$I = \frac{1}{2c\mu_0} E^2 = \frac{1}{2c\mu_0} (E_0 \cos\theta)^2 = I_0 \cos^2 \theta$$

- This result is called the **Law of Malus**
- This equation only applies to the case of polarized light incident on a polarizer

Three Polarizers

SOLUTION



Circular Polarization

- Linear: “horizontal” & “vertical”
- Circular: “right circular” & “left circular”
- Can make circular polarization filters
 - Same rules as for linear polarization filters
 - Used for most recent 3D movies (Avatar, ...) and 3D TV sets
- Plane mirrors reverse circular polarization direction upon reflection (but leave linear polarization direction unchanged)
 - Used in outdoor LCD displays (ATM machines, ...) to reduce/eliminate glare from reflected sunlight

Applications of Polarization

- Sunglasses can have a coating that is polarized to block reflected light that is usually polarized
- LCD screens have an array of liquid crystals sandwiched between two polarizers whose polarizing angles are rotated 90° with respect to each other
- Normally, the liquid crystal rotates the polarization of the light between the two polarizers so that light passes through
- An array of addressable electrodes applies a varying voltage across each of the liquid crystals, causing the liquid crystals to rotate the polarization less, darkening the area covered by the electrode
- The screen can then display a large number of picture elements, or pixels, that produce a high resolution image